Analyzing WMAP Observation by Quantum Gravity

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and
“Focus on Quantum Gravity Research”
(Nova Science Publisher, NY, 2006), Chap.1
Motivation

WMAP has established the inflationary scenario of the universe. Various cosmological parameters have been determined precisely.

But, basic problems are remained:

• What is the inflaton field?
• What is the inflaton potential?

WMAP determined the initial conditions for cosmological perturbation theory. But, its origin is not understood yet.

The aim of this talk is to show an inflationary scenario of quantum gravity origin consistent with WMAP observation without introducing any artificial field.
The Model of Quantum Gravity

Our model is based on 3 fundamental conditions:

- quantum diffeomorphism invariance
  (=conformal invariance)
- finiteness
  (=renormalizability, no BH singularity)
- 4 space-time dimensions

These restrict gravitational action.
Renormalizable quantum gravity

\[ I = \int d^4 x \sqrt{-g} \left\{ -\frac{1}{t^2} C_{\mu\nu\lambda\sigma}^2 - bG_4 + \frac{m_{pl}^2}{16\pi} R - \Lambda \right\} + I_{\text{Matter}} \]

Weyl action, Euler density, Conformal inv. actions

Dimensionless

Conformal mode (non-perturbative)

Traceless tensor mode (perturbative)

Perturbation about conformal flat \((C_{\mu\nu\lambda\sigma} = 0)\):

\[ g_{\mu\nu} = e^{2\phi} (\hat{g}_{\mu\nu} + th_{\mu\nu} + \cdots), \quad tr(h) = 0 \]

Non-perturbative formulation of QG

Recent development and essential point! K.H., hep-th/0203260

\( R^2 \) is forbidden by Wess-Zumino condition.
Dynamics of Weyl action (traceless mode)

Asymptotic Freedom (AF)

\[ t_r^2(p) = \frac{1}{\beta_0 \log(p^2/\Lambda_{QG}^2)}, \quad \beta = -\beta_0 t_r^3 \]

Consequence of AF 1

New dynamical scale \( \Lambda_{QG} \) \( \rightarrow \) space-time phase transition from quantum to classical

Consequence of AF 2

At very high energies \( (t_r \rightarrow 0) \), \( C_{\mu\nu\lambda\sigma} \rightarrow 0 \)

\[ \text{cf. } F_{\mu\nu}^a \rightarrow 0 \ (g \rightarrow 0) \]

Singularity with divergent Riemann curvature is excluded quantum mechanically

\( \rightarrow \) toward resolution of information loss problem!
Consequence of AF 3

In very early universe, fluctuations of conformal mode become dominant

→ Exact Conformal Symmetry

Initial fluctuations are scalar-like and scale-invariant &
Tensor mode is small

Agreement with the observation
Inflation induced by quantum gravity

\[ Z = \int [dg \cdots]_g \exp(iI) \]
\[ = \int [d\phi dh \cdots]_g \exp(iS(\phi) + iI) \]

Jacobian = Wess-Zumino action

Dynamics of conformal mode is induced from the measure

\[ S(\phi) = -\frac{b_1}{(4\pi)^2} \int d^4x \sqrt{-\hat{g}} \left\{ 2\phi \tilde{\Delta}_4 \phi + \left( G_4 - \frac{2}{3} \nabla^2 \hat{R} \right) \phi \right\} + O(\phi^3) \]

Kinetic term

Conformal Field Theory (CFT) at \( t_r^2 \to 0 \)

\( t_r^2 \) measures a deviation from CFT

(exact) \( 0 < t_r^2 < \infty \) (broken)

Planck mass $m_{\text{pl}}$ $\gg$ Dynamical scale $\Lambda_{\text{QG}}$

Inflation starts at the Planck scale and ends at the dynamical scale.

Wess-Zumino action
\[
\begin{align*}
\left. b_1 B(\tau) \left( \dddot{H} + 7 \dot{H}^2 + 4 \dot{H}^2 + 18 H^2 \dot{H} + 6 H^4 \right) - 3\pi m_{\text{pl}}^2 \left( \dot{H} + 2 H^2 \right) \right) &= 0 \\
\left. b_1 B(\tau) \left( 2H \dddot{H} - \dot{H}^2 + 6H^2 \dot{H} + 3H^4 \right) - 3\pi m_{\text{pl}}^2 H^2 + 8\pi^2 \rho \right) &= 0
\end{align*}
\]

Einstein action

\[
B(l_{\tau}^2) = 1 - u_1 l_{\tau}^2(\rho) + \cdots = \frac{1}{1 + u_1 l_{\tau}^2(\tau)} (p \to 1/\tau)
\]

Big Bang $(t_{\tau} = \infty)$:
extra degrees of freedom in higher-derivative gravitational fields shift to matter fields $\rho$.

\[
H = \dot{a}(\tau)/a(\tau) = \dot{\phi}(\tau)
\]
Evolutional scenario

Number of e-foldings

\[ \mathcal{N}_e = \log \frac{a(\tau_\Lambda)}{a(\tau_{pl})} \sim \frac{m_{pl}}{\Lambda_{QG}} \]

\[ \Lambda_{QG} \simeq 10^{17} \text{GeV} \]

correlation length:
\[ \xi = \frac{1}{\Lambda_{QG}} \]

Planck length at Planck time grows up to the Hubble distance today

Planck length

\[ a \sim \tau^{1/2} \]

\[ a \sim \tau^{2/3} \]

scale

\[ 10^{17} \text{GeV} \]

\[ \frac{2.7K}{10^{29}} \]

K.H., Minamizaki, Sugamoto

Calculation of CMB Multipoles

The evolution of scalar curvature fluctuation

\[ \frac{\delta R}{R} \bigg|_{\tau_{\text{pl}}} \rightarrow \frac{\delta R}{R} \bigg|_{\tau_-} \rightarrow \frac{\delta \rho}{\rho} \bigg|_{\tau_+} \rightarrow \frac{\delta T}{T} \bigg|_{\text{today}} \]

Solve evolution equations on the inflation

Big Bang

Solve cosmological perturbation theory (CMBFAST)

gives the initial conditions of CMBFAST

Simple estimation of the amplitude

\[ \frac{\delta R}{R} \sim \frac{E^2}{12m_{\text{pl}}^2} \]

At the big bang \( E \sim \Lambda_{\text{QG}} \Rightarrow \frac{\delta R}{R} \bigg|_{\tau_+} \sim 10^{-5} \)

de Sitter curvature

Linear perturbation is applicable for \( m_{\text{pl}} < E < \Lambda_{\text{QG}} \)
Scalar perturbations

Gauge invariant variables: $\Phi, \Psi, (\delta \rho, v)$

Scalar equation

Constraint equation

Initially $\Phi = \Psi$

Finally $\Phi = -\Psi$

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Spectrum of quantum gravity (2-pt. function)

Initial QG spectrum at Planck time
= CFT spectrum (scale invariant)

\[ P_S(k) = A \left( \frac{k}{m} \right)^{n_s-1} \]

Scalar spectral index

\[ n_s = 5 - 8 \frac{1 - \sqrt{1 - 2/b_1}}{1 - \sqrt{1 - 4/b_1}} = 1 + 2/b_1 + 4/b_1^2 + O(1/b_1^3) \quad b_1 > 4 \]

Coeff. of Wess-Zumino action
(dependent on matter contents)

Size of fluctuation we consider
is Planck length at Planck time

➔ at the transition point, the size is much more extended than the correlation length \( \xi \)
➔ not disturbed by the dynamics of transition

We can see the Planck scale phenomena directly!

\[ m = a(\tau_{pl}) m_{pl} \]
comoving Planck const.

\[ m = 0.05 \text{ Mpc}^{-1} \]
➔ \( a(\tau_{pl}) \sim 10^{-59} \)
consistent with the evolitional scenario
Scalar fluctuation (Bardeen potential) gradually decreases during inflation

Primordial scalar spectrum (red tilt for \( k > m \))

Initial conditions for CMBFAST

Primordial tensor spectrum

Tensor fluctuation is preserved to be small
CMB Multipoles

\[ P(k) \rightarrow P(k) \frac{k^2}{k^2 + u\lambda^2} \]

\[ \lambda = a(\tau_{\text{pl}}) \Lambda_{\text{QG}} \]

- For the region \( k > 2m \)
  - non-linear effects (CFT) become effective.
  - (in progress)

# cosmological parameters adjusted properly

\( \Omega_b = 0.045 \)
\( \Omega_{\text{cdm}} = 0.22 \)
\( \Omega_{\text{vac}} = 0.735 \)
\( \tau_e = 0.1 \)

WMAP-TT version 2.0 (March 2006)

- \( b_1 = 15, m = 0.05, u = 0.0, h = 0.77, r = 0.7 \)
- \( b_1 = 15, m = 0.05, u = 0.1, h = 0.77, r = 0.3 \)
- \( b_1 = 20, m = 0.05, u = 0.0, h = 0.75, r = 0.5 \)
- \( b_1 = 20, m = 0.05, u = 0.1, h = 0.75, r = 0.3 \)
Summary

- **Asymptotic freedom of traceless tensor mode**
  - indicates the existence of novel dynamical scale: \[ \Lambda_{QG} \approx 10^{17} \text{GeV} \]
  - space-time phase transition (=big bang) at this scale.

- **Repulsive force in quantum gravity**
  - induces inflation.
  - number of e-foldings is given by \[ N_e \approx m_{pl}/\Lambda_{QG} \]

- **Quantum gravity spectrum**
  - given by conformal field theory (=non-perturbative formulation of quantum gravity).
  - scalar fluctuation decreases during inflation and the amplitude at the big bang is estimated as \[ \delta R/R \sim \Lambda_{QG}^2/12m_{pl}^2 \]
  - CMB spectrum is consistent with WMAP.
Quantum gravity is a real physical target in 21 century

Electron mass $m_e = 0.5 \text{ MeV (QED)}$

QCD mass scale $\Lambda_{QCD} \approx 300 \text{MeV}$

Proton mass $m_p = 1 \text{ GeV}$

Weak boson mass $m_W = 80 \text{ GeV (EW theory)}$

X boson mass $m_X = 10^{16} \text{ GeV (GUT)}$

New scale $\Lambda_{QG} \approx 10^{17} \text{GeV (Quantum Gravity)}$

Planck mass $m_{pl} = 10^{19} \text{ GeV}$
Appendix
$\frac{4}{\pi} \left( \frac{l(l+1)}{l+1} \right)^{1/2} \frac{C}{C_{\text{obs}}} \left( \frac{C}{C_{\text{obs}}} \right)^{1/2}$

$W_{\text{MAP-TT}}$ version 2.0 (March 2006)
$W_{\text{MAP-TE}}$ version 2.0 (March 2006)
$W_{\text{MAP-EE}}$ version 2.0 (March 2006)
$W_{\text{MAP-BB}}$ version 2.0 (March 2006)

$b_1=15, m=0.04, r=0.5, u=0.0$
$\quad b_1=15, m=0.04, r=0.4, u=0.06$
**Tensor and Vector perturbations**

Gauge invariant variables: $h_{ij}^{TT}$, $\gamma_i$

**Tensor equation**

\[
-\frac{2}{t_i^2(\tau)} \left\{ \partial_\eta^4 h_{ij}^{TT} - 2 \partial_\eta^2 \delta^2 h_{ij}^{TT} + \delta^4 h_{ij}^{TT} \right\} \\
+ \frac{b_1}{8\pi^2} B_0(\tau) \left\{ \left( \frac{1}{3} \partial^2_\eta \phi + \frac{4}{3} \partial_\eta \phi \partial_\eta \phi \right) \partial^2_\eta h_{ij}^{TT} + \left( \frac{1}{3} \partial_\eta^3 \phi + \frac{8}{3} \partial_\eta^2 \phi \partial_\eta \phi \right) \partial_\eta h_{ij}^{TT} \\
+ \left( -\frac{7}{3} \partial^2_\eta \phi + \frac{2}{3} \partial_\eta \phi \partial_\eta \phi \right) \partial^2 h_{ij}^{TT} \right\} \\
+ M_P^2 e^{2\phi} \left\{ -\frac{1}{2} \partial_\eta^2 h_{ij}^{TT} - \partial_\eta \phi \partial_\eta h_{ij}^{TT} + \frac{1}{2} \partial^2 h_{ij}^{TT} \right\} = 0.
\]

**Vector equation**

\[
\frac{2}{t_i^2(\tau)} \left\{ \partial_\eta^3 \gamma_i - \partial_\eta \phi^2 \gamma_i \right\} \\
- \frac{b_1}{8\pi^2} B_0(\tau) \left\{ \left( \frac{1}{3} \partial^2_\eta \phi + \frac{4}{3} \partial_\eta \phi \partial_\eta \phi \right) \partial_\eta \gamma_i + \left( \frac{1}{3} \partial_\eta^3 \phi + \frac{8}{3} \partial_\eta^2 \phi \partial_\eta \phi \right) \gamma_i \right\} \\
+ M_P^2 e^{2\phi} \left\{ \frac{1}{2} \partial_\eta \gamma_i + \partial_\eta \phi \gamma_i \right\} = 0.
\]
Running coupling constant \( \beta_t = -\beta_0 t^3_r \)  

[asymptotic freedom]

\[
\mathcal{L}_{\text{eff}} = -\left\{ \frac{1}{t^2_r} - 2\beta_0 \phi + \beta_0 \log \left( \frac{k^2}{\mu^2} \right) + \cdots \right\} C_{\mu\nu\lambda\sigma}^2
\]

\[
= -\frac{1}{t^2_r(p)} C_{\mu\nu\lambda\sigma}^2
\]

where  

\[
t^2_r(p) = \frac{1}{\beta_0 \log(p^2/\Lambda_{QG}^2)},
\]

\( k \) :comoving momentum defined on \( \hat{g}_{\mu\nu} = \eta_{\mu\nu} \)

Physical momentum : \( p = k/a \) with \( a = e^\phi \)

Dynamical scale : \( \Lambda_{QG} = \mu e^{-1/2\beta_0 t^2_r} \)

Conformal mode increasing => running coupling getting large!
**Einstein phase** \((E < \Lambda_{\text{QG}})\)

Low energy effective action (derivative expansion)

\[
I_{\text{low}} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_2 + \mathcal{L}_4 + \cdots \right\}
\]

\(\mathcal{L}_2 = \frac{M_p^2}{2} R + \mathcal{L}_{\text{Matter}}\)

Here, we restrict effective action up to the fourth order, and thus using lowest Einstein’s equation \(M_p^2 R_{\mu\nu} = T_{\mu\nu}^{\text{Matter}}\), a variety of four-derivative actions is reduced, which is merely given by

\[
\mathcal{L}_4 = \frac{\alpha}{(4\pi)^2} R^{\mu\nu} R_{\mu\nu}
\]

with running effect

\[
\alpha(E) = \alpha_0 + \zeta \log \left( \frac{E^2}{\Lambda_{\text{QG}}^2} \right)
\]

\[
\alpha_0 > 0
\]

: phenomenologically determined

\[
\zeta = \frac{N_X}{120} + \frac{N_A}{10} + \cdots > 0
\]

**Higher-derivative terms are irrelevant!**
Wheeler-DeWitt Equations of Conformal Algebra

Conformal algebra and Physical states (on cylinder $R \times S^3$):

\[
[Q_M, Q_N^\dagger] = 2\delta_{MN} H + 2R_{MN}
\]

special conf. transfs.  Hamiltonian  rotation on $S^3$

\[
Q_M |\text{phys}\rangle = H |\text{phys}\rangle = R_{MN} |\text{phys}\rangle = 0
\]

Conformal inv. vacuum = physical state satisfying $Q_M^\dagger |\Omega\rangle = 0$

Physical operators: $e^{\gamma_0 \phi}$ cosmological const.

$\mathcal{R} (\partial \phi) e^{\gamma_2 \phi}$, scalar curvature

\[
\ldots
\]

Conformal charge: $\gamma_n = 2b_1 - 2\sqrt{b_1^2 - (4-n)b_1} = 4 - n + 0(1/b_1)$

$\implies$ scaling behavior of physical operators