PADN- 731 (PADN731Rfpattern.doc)
Begun to write since May 12nd, 2010.
※注：20090526時点においては、未だ個人見解レベルの検討書です。詳しくはERL-Grへ。

Particle Accelerator Development Note

RF pattern
～ ＦＦＡＧのＲＦ周波数パターン ～

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要約

ＲＦ周波数パターンをどのようにもっていくか、という観点はあまり議論されていない。これは電磁石のパターンにＲＦが合わせるようにもっていくことが常とされ、電気回路上も特に問題がないからである。理論においてもこの発想が基盤となっているので、シンクロトロン振動の原理説明においても「差」の議論のみで終わっている。しかしながら、加速シミュレーションを行う上では、差ではなく、絶対値が必要となる。ここでは、ＲＦのパターンと粒子エネルギー、運動量の関係を確認する。ＦＦＡＧのように径方向への軌道変移が大きいものに小呈をあてて議論する。

従来のシンクロトロン振動での位相偏差の議論

$$p = q\, r\, B$$

$$\frac{\Delta p}{p} = \frac{\Delta r}{r} + \frac{\Delta B}{B}$$

$$B = B_0 \left(\frac{r}{r_0}\right)^k \quad \text{for FFAG accelerators} \qquad \frac{\Delta B}{B} = k\frac{\Delta r}{r} \qquad \frac{\Delta p}{p} = (1+k)\frac{\Delta r}{r}$$

$$\upsilon\, T = C$$

$$\frac{\Delta \upsilon}{\upsilon} + \frac{\Delta T}{T} = \frac{\Delta C}{C}$$

$$p = m_0\, c\, \beta\, \gamma$$

$$\frac{\Delta p}{p} = \frac{\Delta \beta}{\beta} + \frac{\Delta \gamma}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\frac{\Delta\gamma}{\gamma} = \frac{\Delta\left(\frac{1}{\sqrt{1-\beta^2}}\right)}{\gamma} = \frac{\Delta\beta}{\gamma}\frac{-\frac{1}{2}(-2\beta)}{(1-\beta^2)^{3/2}} = \frac{\Delta\beta}{\beta}\beta^2\gamma^2 = \frac{\Delta\beta}{\beta}\frac{\beta^2}{1-\beta^2}$$

$$\frac{\Delta p}{p} = \frac{\Delta\beta}{\beta} + \frac{\Delta\beta}{\beta}\frac{\beta^2}{1-\beta^2}\gamma^2 = \frac{\Delta\beta}{\beta}\left(1+\frac{\beta^2}{1-\beta^2}\right) = \frac{\Delta\beta}{\beta}\left(\frac{1}{1-\beta^2}\right) = \frac{\Delta\beta}{\beta}\gamma^2$$

$$= \gamma^2\frac{\Delta\upsilon}{\upsilon}$$

$$\frac{1}{\gamma^2}\frac{\Delta p}{p} + \frac{\Delta T}{T} = \frac{\Delta C}{C} - \frac{\Delta r}{r} = \frac{1}{1+k}\frac{\Delta p}{p}$$

$$\frac{\Delta T}{T} = \left(\frac{1}{1+k} - \frac{1}{\gamma^2}\right)\frac{\Delta p}{p}$$

包含した評価式。
磁束密度の径方向指数を取り込んだ形。
まず、turn by turn での粒子エネルギー増加量 eV を一定とした場合を示す。

Reference Relations

$$\frac{p}{p_{inj}} = \frac{B\rho}{B\rho_{inj}} = \left(\frac{r}{r_{inj}}\right)^{k} \frac{r}{r_{inj}} \cong \left(\frac{C}{C_{inj}}\right)^{k+1} \Rightarrow C = C_{inj}\left(\frac{p}{p_{inj}}\right)^{1/(1+k)}$$

$$T_{ref} = \frac{C}{c\,\beta} = \frac{C_{inj}\left(\frac{p}{p_{inj}}\right)^{1/(1+k)}}{c\,\beta}$$

$$p = m_0\,c\,\beta\,\gamma = m_0\,c\sqrt{1-\frac{1}{\gamma^2}}\,\gamma = m_0\,c\sqrt{\gamma^2-1}$$

$$T = \frac{C}{c\,\beta} = \frac{C_{inj}\left(\frac{\sqrt{\gamma^2-1}}{\sqrt{\gamma_{inj.}^{\,2}-1}}\right)^{1/(1+k)}}{c\sqrt{1-\frac{1}{\gamma^2}}} = \frac{C_{inj}\left(\frac{\gamma^2-1}{\gamma_{inj.}^{\,2}-1}\right)^{1/2(1+k)}\gamma}{c\sqrt{\gamma^2-1}}$$

$$= \frac{C_{inj}}{c\,(\gamma_{inj.}^{\,2}-1)^{1/2(1+k)}}\,\gamma\,(\gamma^2-1)^{1/2(1+k)-1/2}$$

$$= \frac{C_{inj}\,\gamma}{c\,(\gamma_{inj.}^{\,2}-1)^{1/2(1+k)}\,(\gamma^2-1)^{k/2(1+k)}}$$

$$f = \frac{c\,(\gamma_{inj.}^{\,2}-1)^{1/2(1+k)}\,(\gamma^2-1)^{k/2(1+k)}}{C_{inj}\,\gamma}$$

$$t_{n+1} = t_n + T$$

$$\gamma_{n+1} = \gamma_n + \frac{e\,V}{m_0\,c^2}$$

In the case that V is constant,

$$\frac{\partial t}{\partial N} = T = \frac{C_{inj}\ \gamma}{c\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}\ (\gamma^{2} - 1)^{k/2(1+k)}}$$

$$\frac{\partial \gamma}{\partial N} = \frac{e\ V}{m_0\ c^2}$$

$$\frac{\partial \gamma}{\partial t} = \frac{\partial N}{\partial t}\ \frac{\partial \gamma}{\partial N} = \frac{e\ V}{m_0\ c^2\ \{\frac{C_{inj}\ \gamma}{c\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}\ (\gamma^{2} - 1)^{k/2(1+k)}}\}}$$

$$= \frac{e\ V\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}}{m_0\ c\ C_{inj}}\ \frac{(\gamma^{2} - 1)^{k/2(1+k)}}{\gamma}$$

$$\frac{\gamma}{(\gamma^{2} - 1)^{k/2(1+k)}}\ \frac{\partial \gamma}{\partial t} = \frac{e\ V\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}}{m_0\ c\ C_{inj}}$$

$$(\gamma^{2} - 1)^{-k/2(1+k)}\ \frac{\partial[\gamma^2 - 1]}{\partial t} = \frac{2\ e\ V\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}}{m_0\ c\ C_{inj}}$$

$$(\gamma^2 - 1)^{(2+k)/2(1+k)} - (\gamma_{inj.}{}^{2} - 1)^{(2+k)/2(1+k)} = \frac{(2 + k)\ e\ V\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}}{(1 + k)\ m_0\ c\ C_{inj}}\ t$$

$$(\gamma^2 - 1)^{(2+k)/2(1+k)} = (\gamma_{inj.}{}^{2} - 1)^{(2+k)/2(1+k)} + \frac{(2 + k)\ e\ V\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}}{(1 + k)\ m_0\ c\ C_{inj}}\ t$$

$$= (\gamma_{inj.}{}^{2} - 1)^{(2+k)/2(1+k)}\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}$$

$$\gamma^2 - 1 = (\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}$$

$$\gamma = \sqrt{1 + (\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}}$$

$$T = \frac{C_{inj}\ \gamma}{c\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}\ (\gamma^{2} - 1)^{k/2(1+k)}}$$

$$= \frac{C_{inj}\ \sqrt{1 + (\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}}}{c\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}\ \{(\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}\}^{k/2(1+k)}}$$

$$= \frac{C_{inj}\ \sqrt{1 + (\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}}}{c\ (\gamma_{inj.}{}^{2} - 1)^{1/2(1+k)}\ (\gamma_{inj.}{}^{2} - 1)^{k/2(1+k)}\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{k/(2+k)}}$$

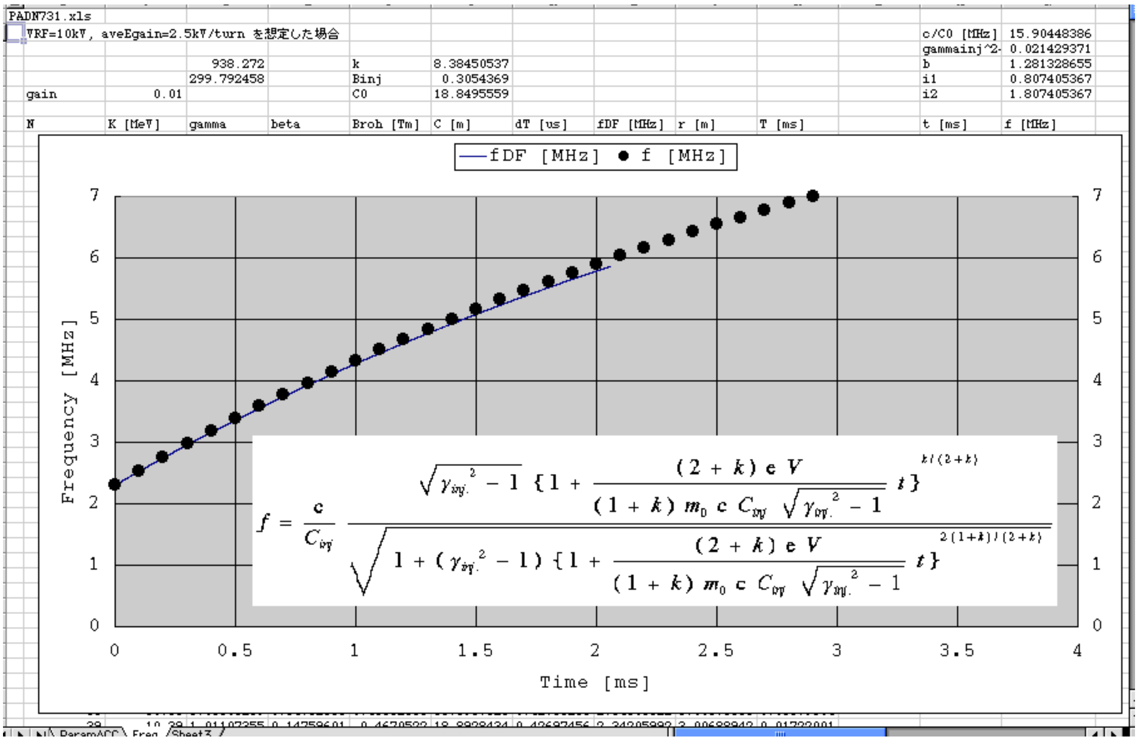
$$= \frac{C_{inj}\ \sqrt{1 + (\gamma_{inj.}{}^{2} - 1)\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{2(1+k)/(2+k)}}}{c\ \sqrt{\gamma_{inj.}{}^{2} - 1}\ \{1 + \frac{(2 + k)\ e\ V}{(1 + k)\ m_0\ c\ C_{inj}\ \sqrt{\gamma_{inj.}{}^{2} - 1}}\ t\}^{k/(2+k)}}$$

$$f = \frac{c}{C_{inj}} \frac{\sqrt{\gamma_{inj.}^{2} - 1}\ \left\{1 + \frac{(2+k)\, e\, V}{(1+k)\, m_0\, c\, C_{inj} \sqrt{\gamma_{inj.}^{2} - 1}}\, t\right\}^{k/(2+k)}}{\sqrt{1 + (\gamma_{inj.}^{2} - 1)\left\{1 + \frac{(2+k)\, e\, V}{(1+k)\, m_0\, c\, C_{inj} \sqrt{\gamma_{inj.}^{2} - 1}}\, t\right\}^{2(1+k)/(2+k)}}}$$

下記の加速器パラメーターで計算する。

PADN731.xls		Injection	Exctraction		Reference
Particle	[-]	p	p		p
Mass of particle	[GeV]	0.938272	0.938272		0.938272
Velocity of Light	[Gm/s]	0.299792458	0.299792458		0.29979246
Kinetic Energy	[GeV]	0.01	0.1		0.5
Gamma	[-]	1.01065789	1.106578902		1.53289451
Beta	[-]	0.144844011	0.428195502		0.75790872
Broh	[Tm]	0.458155354	1.482970597		3.63611181
Vertical beam size index	[%]	100	56		35
Area ratio shared for Bending magnets	[%]	50	50		50.5
Bending Radius	[m]	1.5	1.7		3.03
Averaged Radius of Accelerator	[m]	3	3.4		6
Circumference	[m]	18.850	21.363		37.6991118
Magnetic Flux Density	[T]	0.3054	0.8723		1.2000369
k-value	[-]			8.385	
Gap height, assumed h=30 at 100MeV	[mm]	85.7	30		40
Excitation current	[kAt]	21	21		38
	[GeV]				
Transit time	[ns]	434	166		166
Frequency	[MHz]	2.304	6.009		6.02707354

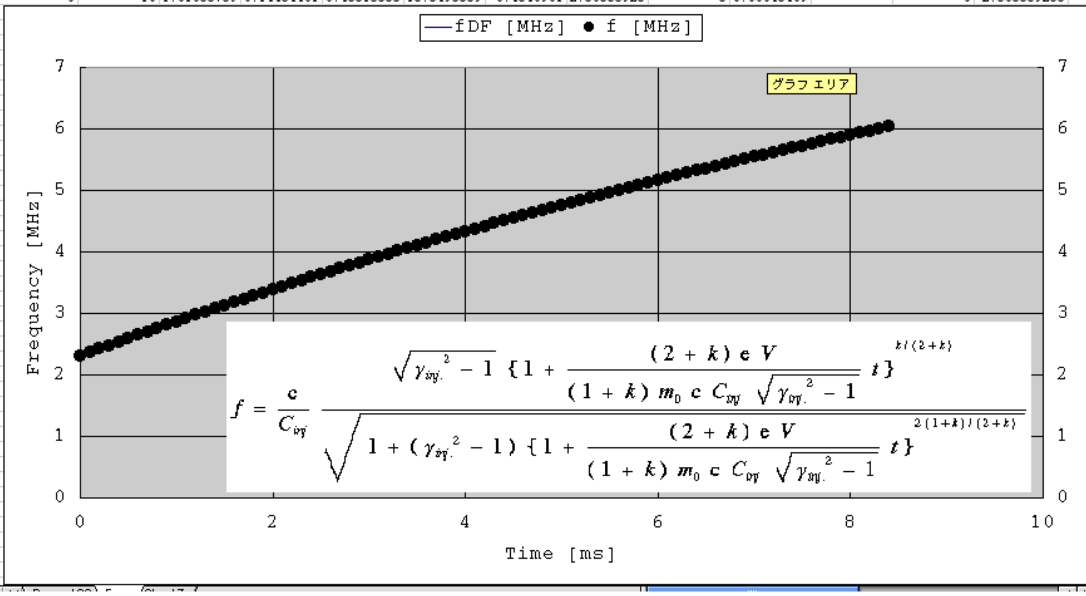
10kV 加速の場合、



2.5 kV 加速の場合

PADN731b.xls

	B	C	D	E	F	G	H	I	J	K	L	M	N
	PADN731b.xls												
	VRF=10kV, aveEgain=2.5kV/turn を想定した場合											c/C0 [MHz]	15.90448386
												gammainj^2-	0.021429371
			938.272		k	8.38450537						b	0.320332164
			299.792458		Binj	0.3054369						i1	0.807405367
	gain	0.0025			C0	18.8495559						i2	1.807405367
	N	K [MeV]	gamma	beta	Broh [Tm]	C [m]	dT [us]	fDF [MHz]	r [m]	T [ms]		t [ms]	f [MHz]
	0	10	1.01065789	0.14484401	0.45815535	18.8495559	0.4340901	2.30366923	3	0.00043409		0	2.303669233



式をまとめると

$$f = \frac{c}{C_{inj}} \frac{\sqrt{\gamma_{inj}^2 - 1}\ \{1 + \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}}\,t\}^{k/(2+k)}}{\sqrt{1 + (\gamma_{inj}^2 - 1)\,\{1 + \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}}\,t\}^{2(1+k)/(2+k)}}}$$

$$\gamma = \sqrt{1 + (\gamma_{inj}^2 - 1)\,\{1 + \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}}\,t\}^{2(1+k)/(2+k)}}$$

$$p = m_0\,c\,\sqrt{(\gamma_{inj}^2 - 1)\,\{1 + \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}}\,t\}^{2(1+k)/(2+k)}}$$

$$C = C_{inj}\,\{1 + \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}}\,t\}^{1/(2+k)}$$

となり、下記のように簡略化される。

For simplicity ,

$$a \equiv \sqrt{\gamma_{inj}^2 - 1}\ , \quad b \equiv \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,\sqrt{\gamma_{inj}^2 - 1}} = \frac{(2+k)\,e\,V}{(1+k)\,m_0\,c\,C_{inj}\,a}$$

$$f = \frac{c}{C_{inj}} \frac{a\,(1 + b\,t)^{k/(2+k)}}{\sqrt{1 + a^2\,(1 + b\,t)^{2(1+k)/(2+k)}}}$$

$$\gamma = \sqrt{1 + a^2\,(1 + b\,t)^{2(1+k)/(2+k)}}$$

$$p = m_0\,c\,a\,(1 + b\,t)^{(1+k)/(2+k)}$$

$$C = C_{inj}\,(1 + b\,t)^{1/(2+k)}$$