Beam-beam simulations and nonlinear lattice analysis for FCC-ee

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Acknowledgements:
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Outline

➤ Lattice
  ● Lattice version: FCCee_t_82_by2_1a_nosol_DS_2 by K. Oide
    ● Error seeds in vertical offsets of S{DF} to generate vertical emittance

➤ Analysis of lattice nonlinearity (LN)
  ● FMA
  ● Resonance driving terms (RDTs) calculations using PTC

➤ Beam-beam simulations
  ● SAD: beam-beam + lattice

➤ Summary
1. FMA

- Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2
  - Bare lattice: $\beta_x^*=1\text{m}, \beta_y^*=2\text{mm}$
  - Start point for tracking: FRF
  - Tracking: SAD + NAFF, 1024 turns

\[ \delta=0: \]

\[ \delta=2\sigma_p: \]
1. FMA

Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

- Bare lattice: $\beta_x^* = 1\text{m}$, $\beta_y^* = 2\text{mm}$
- Start point for tracking: IP
- Tracking: SAD + NAFF, 1024 turns

$\delta=0$: Footprint in tune space is hard to obtain, indicating that starting tracking from IP is not good.

$\delta=2\sigma_p$: 
1. FMA

- **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**
  - Bare lattice + Error seed #25
  - Start point for tracking: FRF
  - Tracking: SAD + NAFF, 1024 turns

\[
\delta = 0:
\]

\[
\delta = 2\sigma_p:
\]
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2
- Bare lattice + Error seed #3
- Start point for tracking: FRF
- Tracking: SAD + NAFF, 1024 turns

\[ \delta = 0: \]

\[ \delta = 2\sigma_p: \]
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2
  ● Here error seeds in vertical offsets of S{DF}^* are used to generate vertical emittance
  ● But optics corrections might also be necessary, such as local coupling at IP, dispersion function...
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

- Compare with D. Shatilov’s results [Refer to talk at 36th FCC-ee optics design meeting]: Aperture almost agrees, but some discrepancy in footprint in tune space.
- Hamiltonian (maps) for each type of element
- Integration algorithm and Number of slices

SAD:

Acceleraticum by P. Piminov:

Refer to D. Shatilov, 36th FCC-ee optics design meeting
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

- Compare with D. Shatilov’s results [Refer to talk at 36th FCC-ee optics design meeting]: Loss of DA is smaller in SAD.
- DA also depends on tracking turns: 1024 in SAD and 2048 in thin slices tracking
- Since damping turns are the order of 100, likely we can conclude that coupling does not cause much loss in DA (?)
2. RDTs

- Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with $n$ elements, one can normalize the one turn map $\mathcal{M}_{1\rightarrow n}$ as [Ref. L. Yang et al., Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\rightarrow n} = A_{1}^{-1} e^{i\mathcal{R}_{1\rightarrow n}} A_{1},$$

with $\mathcal{R}$: rotation, $e^{i\mathcal{R}}$: nonlinear Lie map, $A_{1}$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $A_{i}$ in $x$ plane at the $i$th element can be approximated in perturbation theory as

$$A_{i}x = \sqrt{\beta_{x,i}} x + \eta_{x,i} \delta,$$

$$A_{i}p_{x} = \frac{-\alpha_{x,i} x + p_{x}}{\sqrt{\beta_{x,i}}} + \eta_{x,i}^\prime \delta.$$
2. RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables \((J, \phi)\) one can write

\[
h^\pm_x \equiv \sqrt{J_x} e^{\pm i \phi_x} = \frac{X \mp i P_x}{\sqrt{2}},
\]

\[
R_{i \rightarrow j} h^\pm_x = R_{i \rightarrow j} \sqrt{J_x} e^{\pm i \phi_x} = e^{\pm i \mu_{i \rightarrow j, x}} h^\pm_x,
\]

where \(\mu_{i \rightarrow j, x}\) is the phase advance of \(i \rightarrow j\). Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of \(h_{abcde}\) as

\[
h = \sum h_{abcde} h^+_x h^-_x h^+_y h^-_y \delta^e.
\]

Each \(h_{abcde}\) (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.
2. RDTs

➤ RDTs indicate lattice nonlinearity

The effective Hamiltonian corresponding to chromaticity is

\[ h_c = \sum h_{1100e} h_x^{-1} h_x^{-1} \delta^e + \sum h_{0011e} h_y^{-1} h_y^{-1} \delta^e, \]

\[ h_c = J_x \sum h_{1100e} \delta^e + J_y \sum h_{0011e} \delta^e. \]

Then the tunes are calculated as

\[ \nu_x = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_x} = -\frac{1}{2\pi} \sum h_{1100e} \delta^e, \]

\[ \nu_y = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_y} = -\frac{1}{2\pi} \sum h_{0011e} \delta^e. \]

Therefore the RDTs of \( h_{1100e} \) and \( h_{0011e} \) correspond to linear and high-order chromaticity.
2. RDTs

- **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**
  - No significant changes in RDTs up to 4th order due to errors in vertical offsets of $S\{DF\}^*$. 5th and higher-order RDTs may dominate the nonlinear dynamics.
  - It might be interesting to study lattice nonlinearity after optics corrections with more errors in magnets.

$2\nu_y [(J_x)(J_y)]$

![Graphs and diagrams showing RDTs and tune variations for different seeds and orders.](image)

Small amplitude in bare lattice

Green: 5th order
Yellow: 6th order
2. RDTs

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

- Possible interplay of beam-beam resonances and RDTs in lattice
- Emittance growth due to beam-beam resonances [K. Ohmi has a code to analyse it]

Q: Will beam-beam break the cancellation condition?

D. Shatilov
36th meeting
3. BB simulations

- FCCee_t_82_by2_1a_nosol_DS_2
  - Significant lum. loss due to lattice nonlinearity [to be understood]
  - With errors: Local coupling and dispersion at IP not corrected; Use radiation damping/excitation matrices

![Graph showing specific lum. vs. product of bunch currents](image1)

![Graph showing lum./IP vs. product of bunch currents](image2)
3. BB simulations

➤ Particle losses in tracking

- Nominal $N_p = 1.7E11$, $\beta_x^* = 1m$, $\beta_y^* = 2mm$
- Beamstrahlung effect with finite DA cause particle losses
- Loss rate depend on $\beta^*_{x,y}$, and DA
- Lifetime seems acceptable from previous studies

Refer to D. Zhou, FCC week 2016
3. BB simulations

➤ Particle losses in tracking

● Alternative estimate: weak-strong simulation [K. Ohmi]

Lifetime given by weak-strong simulation

- In equilibrium, particles escape a boundary is the same number as damping from the boundary. [M. Sands, SLAC-R-121 (1970)]

\[
\frac{dN}{dt} = f(J_i) \frac{dJ_i}{dt} \quad \frac{dJ_i}{dt} = \frac{-2J_i}{\tau_i}
\]

\[
\tau_e = \frac{N}{\frac{dN}{dt}} = \frac{t_i}{2J_{i,max}f(J_{i,max})}
\]

f(J): equilibrium beam distribution. For example f(J)=exp(-J/\epsilon) for Gaussian. f(J)=N(J)/N_0 in the last slide.

Refer to K. Ohmi, 37th FCC-ee optics design meeting
4. Summary

➤ FMA
  • Compared with D. Shatilov’s results, discrepancies seem understandable

➤ RDTs
  • 5th and high-order RDTs from errors in vertical offsets of $S\{DF\}^*$
  • Possible to study interplay of RDTs and beam-beam resonances

➤ Luminosity and lifetime
  • Lum. loss for new lattice to be understood
  • Need optics correction with errors in vertical offsets of $S\{DF\}^*$