Beam-beam simulations and analysis of lattice nonlinearity using PTC

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Outline

➤ Introduction
  ● FCC-ee design lattices provided by K. Oide

➤ Beam-beam simulations
  ● BBWS: beam-beam + simple linear map
  ● SAD: beam-beam + design lattice
  ● At present, only consider average turn-by-turn radiation damping/excitation lumped at one point, no quadrupole radiation and no “saw-tooth” effects in orbit

➤ Analysis of lattice nonlinearity (LN)
  ● Lattice translation: SAD => Bmad => PTC [Straightforward]
  ● Resonance driving terms (RDTs) calculations using PTC

➤ Summary
1. Interplay of BB and latt. nonlin.

➢ The idea
  ● Method demonstrated in D. Zhou et al, TUPE016, IPAC13
  ● One-turn map:

\[ M = M_{\text{RAD}} \circ M_{\text{BB}} \circ M_{0} \]

  ● \( M_{0} \) can be simple matrix or IP-to-IP realistic map from a design lattice
  ● Interplay of BB, lattice and other issues reflected in luminosity, DA, beam tail, particle loss, etc.
  ● Separated simulation/analysis of BB and LN help understand the mechanisms of their interplay
# 1. Parameters for simulations (half ring)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Z</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (km)</td>
<td>49990.9</td>
<td>49990.9</td>
</tr>
<tr>
<td>E (GeV)</td>
<td>45.6</td>
<td>175</td>
</tr>
<tr>
<td>Number of IPs</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$N_b$</td>
<td>90300</td>
<td>78</td>
</tr>
<tr>
<td>$N_p(10^{11})$</td>
<td>0.33</td>
<td>1.7</td>
</tr>
<tr>
<td>Full crossing angle (rad)</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$\varepsilon_x$ (nm)</td>
<td>0.09</td>
<td>1.3</td>
</tr>
<tr>
<td>$\varepsilon_y$ (pm)</td>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>$\beta_x^*$ (m) [optional]</td>
<td>1 [0.5]</td>
<td>1 [0.5]</td>
</tr>
<tr>
<td>$\beta_y^*$ (mm) [optional]</td>
<td>2 [1]</td>
<td>2 [1]</td>
</tr>
<tr>
<td>$\sigma_z$ (mm)$^{SR}$</td>
<td>2.7</td>
<td>2.1</td>
</tr>
<tr>
<td>$\sigma_\delta(10^{-3})^{SR}$</td>
<td>0.37</td>
<td>1.4</td>
</tr>
<tr>
<td>Betatron tune $\nu_x/\nu_y$</td>
<td>.55/.57</td>
<td>.54/.57</td>
</tr>
<tr>
<td>Synch. tune $\nu_s$</td>
<td>0.0075</td>
<td>0.0375</td>
</tr>
<tr>
<td>Damping rate/turn ($10^{-2}$) [x/y/z]</td>
<td>0.019/0.019/0.038</td>
<td>1.1/1.1/2.2</td>
</tr>
<tr>
<td>Lum./IP($10^{34}$cm$^{-2}$s$^{-1}$)</td>
<td>68</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Ref. F. Zimmermann, FCC-ee design meeting, Dec. 9, 2015
2. BB simulations: Lum.: t

- $\beta_x^* = 1\text{m}$, $\beta_y^* = 2\text{mm}$
- Lattice ver. FCCee_t_65_26_1_2
2. BB simulations: Lum.: $t$

- $\beta_x^* = 0.5\text{m}, \beta_y^* = 1\text{mm}$
- Lattice ver. FCCee_t_65_26

![Graphs showing luminescence vs. $I_{\text{bunch}}(e^+)$ and $I_{\text{bunch}}(e^-)$ for different beam configurations.](image)
2. BB simulations: Lum.: Z

- $\beta_x^* = 1m$, $\beta_y^* = 2mm$ [Ver. FCCee_z_65_36]
- Lattice ver. FCCee_z_65_36
2. BB simulations: Particle loss:

- Particle losses in tracking
  - Threshold observed
  - Nominal $N_p=1.7E11$
  - Loss rate depend on $\beta^*_{x,y}$
  - Slipping out of RF bucket?
  - Improper setting of simulations?

Mismatch in transverse beam sizes,
No crab waist for the strong beam

![Graphs showing particle loss over turns with different $N_p$ values and $\beta^*$ parameters.](image-url)
3. RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with \( n \) elements, one can normalize the one turn map \( \mathcal{M}_{1\to n} \) as [Ref. L. Yang et al., Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

\[
\mathcal{M}_{1\to n} = A_1^{-1} e^{i\mathcal{R}_{1\to n}} A_1,
\]

with \( \mathcal{R} \): rotation, \( e^{i\mathcal{R}} \): nonlinear Lie map, \( A_1 \): normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), \( A_i \) in \( x \) plane at the \( i \)th element can be approximated in perturbation theory as

\[
A_i x = \sqrt{\beta_{x,i}} x + \eta_{x,i} \delta,
\]

\[
A_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i} \delta.
\]
3. RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables \((J, \phi)\) one can write

\[ h_{x}^{\pm} \equiv \sqrt{2J_{x}} e^{\pm i\phi_{x}} = X \mp iP_{x}, \]

\[ \mathcal{R}_{i\rightarrow j} h_{x}^{\pm} = \mathcal{R}_{i\rightarrow j} \sqrt{2J_{x}} e^{\pm i\phi_{x}} = e^{\pm i\mu_{i\rightarrow j,x}} h_{x}^{\pm}, \]

where \(\mu_{i\rightarrow j,x}\) is the phase advance of \(i \rightarrow j\). Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of \(h_{abcde}\) as

\[ h = \sum h_{abcde} h_{x}^{+a} h_{x}^{-b} h_{y}^{+c} h_{y}^{-d} \delta^{e}. \]

Each \(h_{abcde}\) (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.
3. RDTs

➤ RDTs indicate lattice nonlinearity

<table>
<thead>
<tr>
<th>$h_{abcde}$</th>
<th>Driving effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{11001}, h_{00111}$</td>
<td>Linear chromaticity $\zeta_x, \zeta_y$</td>
</tr>
<tr>
<td>$h_{21000}, h_{12000}</td>
<td></td>
</tr>
<tr>
<td>$h_{30000}, h_{03000}</td>
<td></td>
</tr>
<tr>
<td>$h_{10020}, h_{01200}</td>
<td></td>
</tr>
<tr>
<td>$h_{20010}, h_{02100}</td>
<td></td>
</tr>
<tr>
<td>$h_{00210}, h_{00120}</td>
<td></td>
</tr>
<tr>
<td>$h_{22000}, h_{00220}, h_{11110}$</td>
<td>$d \nu_x/dJ_x, d \nu_y/dJ_y, d \nu_{x,y}/dJ_{y,x}$</td>
</tr>
<tr>
<td>$h_{40000}, h_{04000}</td>
<td></td>
</tr>
<tr>
<td>$h_{31000}, h_{13000}</td>
<td></td>
</tr>
<tr>
<td>$h_{00310}, h_{00130}</td>
<td></td>
</tr>
<tr>
<td>$h_{20020}, h_{02200}</td>
<td></td>
</tr>
<tr>
<td>$h_{30010}, h_{03100}</td>
<td></td>
</tr>
<tr>
<td>$h_{10030}, h_{01300}</td>
<td></td>
</tr>
</tbody>
</table>

**Table**: Low-order driving terms.
3. RDTs: PTC calculation

PTC applied to SuperKEKB: an example

- $2v_x - v_y \left[ (J_x)(J_y)^{1/2} \right]$ resonance
- 3D fields near IP [Solenoid and FF quad fringe fields] generate lots of low-order nonlinearities, hard to be compensated using arc multipoles [global correction]
- Simplified lattices show less nonlinearity

![Graphs showing different scenarios](image)

(a) $\text{IP} \to s = 10\text{m}$

(b) $\text{s} = 3000\text{m} \to \text{IP}$

(c) Whole ring

Figure: $|h_{20010}|$ accumulated along the ring.
3. RDTs: PTC calculation

PTC applied to FCC-ee t lattice: an example

- $2v_x - v_y \ [(J_x)(J_y)^{1/2}]$ resonance for latt. ver. FCCee_t_65_26
- In general, no significant 3rd resonances in FCC-ee lattices
3. RDTs: PTC calculation

➤ PTC applied to FCC-ee t lattice: an example
  - 4th order RDTs for latt. ver. FCCee_t_65_26
  - Residual 4th order RDTs exist, and depend on lattice design/optimization

\[
dv_{x,y} / dJ_{y,x}
\]

\[
4v_x
\]
4. Summary

➤ Beam-beam simulations
  • Small loss [order of a few percent] of luminosity due to BB+Lattice
    [No limit in luminosity performance, and should be controllable via optics optimization]
  • Particle loss in SAD simulations [to be understood]

➤ Lattice analysis using PTC
  • PTC is ready for RDTs calculations
  • Identify sources of nonlinearity
  • Identify dominant nonlinear terms in one-turn-map
  • Evaluate lattice designs and optimizations

➤ To do list
  • Understand macro-particle losses in beam-beam simulations with lattices
    • Simulations for FCC-ee Z lattices with $\beta_x^*=0.5\text{m}$, $\beta_y^*=1\text{mm}$
    • Simulations with quadrupole/distributed radiation and “saw-tooth” effects