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# Transverse wakefields due to asymmetric protrusions into a vacuum chamber

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ABSTRACT

We analyze the effect of a wakefield caused by an asymmetric protrusion inside the accelerator vacuum chamber. The asymmetry leads to a transverse kick on the beam and an increase of the projected transverse beam emittance. Calculations are done for a model rectangular protrusion in a vacuum chamber of rectangular cross-section. Based on our analysis, numerical estimates are given for the SuperKEKB accelerator in KEK, Japan, and TLEP-W proposal at CERN.



## 1. Introduction

Traditionally wakefield calculations are focussed on elements of the vacuum chamber which have a certain degree of symmetry such as an axisymmetry and top-down or right-left symmetry. Such elements have a property that the transverse wakefield vanishes if the beam propagates along the symmetry axis of the system.[1] The wakefields appear only when the beam is offset from the axis, and result in a deflection of the beam which is proportional to the offset. They are the source of transverse instabilities in the beam.

In this paper we attract attention to another type of wakefield that is caused by asymmetric protrusions inside of a vacuum chamber. The protrusions can be parts of elements of the vacuum chamber such as masks, bellows and beam position monitors. The asymmetry leads to a transverse kick *even if the beam is not offset from the axis of the machine*. It does not lead to a beam instability, but results in an increase in projected transverse beam emittance. Given that in modern electron and positron rings the vertical emittance is extremely small, such protrusions can set a limit on the minimally achievable transverse emittance in a given accelerator and lead to a decrease in luminosity of a collider, or a deterioration of the transverse coherence properties in a ring light source.

An effect similar to the one discussed in this paper was earlier studied in Ref. [1], where it was assumed that the transverse kick was due to an offset of the beam orbit from the nominal position. In principle such a distortion can be cured by the proper orbit correction, while the effect described in this paper is not sensitive to a shift in orbit.

The strength of the transverse kick depends on the geometry of the protrusion and the parameters of the beam. In this paper, which is aimed at demonstrating the basic effect, we choose the simple model of a rectangular protrusion in a rectangular vacuum chamber. Even for such a relatively simple geometry the calculation of the wakefield, in general, is too difficult for an analytical treatment, and requires a 3D computer simulation. Only in the limit when the bunch length is sufficiently small can one use the optical model developed in Refs. [2,3] to calculate the wakefield analytically.

This paper is organized as follows. In Section 2 we specify the geometry of the protrusion and discuss the effect of its wakefield on the effective beam emittance. In Section 3 we present the results of the calculations using the optical model, with the details of the calculations given in Appendix A. In Section 4 we present the results of computer simulations with GdfidL [4] and estimate the effect for parameters of SuperKEKB. In Section 5 we give an estimate for TLEP-W accelerator. We conclude this paper by Section 6 with a summary of our results.

## 2. Effect of transverse wake on beam emittance

We first consider a general case where there are localized impedance sources at several locations in the ring. Source $i$ is located at position $s_i$ and generates a transverse (vertical) wake

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[1] We are not considering here the so-called quadrupole wake in non-axisymmetric systems that leads to the differential focusing of the beam even when it propagates on the axis.

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$W_y^{(i)}(z)$, where $z$ is the longitudinal coordinate in the bunch with positive $z$ corresponding to the head of the bunch. We can calculate the vertical deflection angle of the beam due to source $i$ as

$$\theta^{(i)}(z) = \frac{eQW_y^{(i)}(z)}{\gamma mc^2} \tag{1}$$

where $Q$ is the bunch charge and $\gamma mc^2$ is the beam energy. In a ring with a given vertical function $\beta(s)$, where $s$ is the coordinate along the ring circumference, a localized kick (1) would lead to an orbit distortion, in which each slice of the bunch executes a different betatron oscillation. This trajectory of a slice at coordinate $z$ is given by the standard formula for orbit distortion in a ring (see, e.g., [5])

$$\Delta y(s,z) = \frac{\sqrt{\beta(s)}}{2\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\cos(\psi(s)-\psi(s_i)-\pi\nu) \tag{2}$$

where $\psi(s)$ is the betatron phase, $\nu$ is the vertical tune, and the summation goes over all impedance sources. Correspondingly, there is a $z$-dependent slope $\Delta y'(s,z)$ given by

$$\Delta y'(s,z) = \frac{\beta'(s)}{4\sqrt{\beta(s)}\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\cos(\psi(s)-\psi(s_i)-\pi\nu) - \frac{1}{2\sqrt{\beta(s)}\,\sin(\pi\nu)}\sum_i \theta^{(i)}(z)\sqrt{\beta(s_i)}\,\sin(\psi(s)-\psi(s_i)-\pi\nu). \tag{3}$$

Due to this distortion of the beam shape, its emittance increases by some amount $\Delta\epsilon$. Assuming that the increase is small, $\Delta\epsilon$ is given by the following formula:

$$\Delta\epsilon = \frac{1}{2}\left[\frac{1+\alpha^2}{\beta}\langle(\Delta y-\langle\Delta y\rangle)^2\rangle + \beta\langle(\Delta y'-\langle\Delta y'\rangle)^2\rangle + 2\alpha\langle(\Delta y-\langle\Delta y\rangle)(\Delta y'-\langle\Delta y'\rangle)\rangle\right] \tag{4}$$

where the angular brackets indicate averaging with the longitudinal distribution function of the beam and $\alpha(s) = -\beta'(s)/2$.

Eqs. (2)–(4) can be used for practical calculations but are too complicated for general analysis of the magnitude of the effect. In what follows, we consider a simple case of a single localized source of impedance $W_y(z)$ located at coordinate $s_0$ where $d\beta/ds|_{s=s_0} = 0$. Eq. (4) then simplifies to

$$\Delta\epsilon = \frac{1}{2}\left[\frac{1}{\beta_0}\langle(\Delta y_b-\langle\Delta y_b\rangle)^2\rangle + \beta_0\langle(\Delta y_b'-\langle\Delta y_b'\rangle)^2\rangle\right] \tag{5}$$

where $\beta_0 = \beta(s_0)$ and $\Delta y_b(z) = \Delta y(s_0,z)$ and $\Delta y_b'(z) = d\Delta y(s,z)/ds|_{s=s_0}$. Using $d\beta/ds|_{s=s_0} = 0$ from (2) and (3) we obtain

$$\Delta y_b(z) = \frac{\theta(z)}{2}\beta(s_0)\cot(\pi\nu), \qquad \Delta y_b'(z) = \frac{\theta(z)}{2}. \tag{6}$$

Substituting (6) into (5) we obtain

$$\Delta\epsilon = \tfrac{1}{8}\beta_0\theta_{\rm rms}^2\csc(\pi\nu)^2 \tag{7}$$

where the rms angular spread of $\theta$ is given by

$$\theta_{\rm rms} = \frac{e}{\gamma mc^2}\langle(W_y-\langle W_y\rangle)^2\rangle^{1/2}. \tag{8}$$

In the next section we will show how the impedance of a rectangular protrusion can be calculated using the optical model and apply this impedance to evaluating the projected beam emittance increase.

## 3. Geometry of protrusion and transverse wakefields in optical model

We consider a vacuum chamber with rectangular cross-section $a\times b$ and the reference orbit of the beam located at the center of the chamber, at $x = a/2$, $y = b/2$. A rectangular protrusion in the pipe occupies the region $b-h<y<b$ and $|z|<w/2$ with $h<b/2$.

For very short bunches one can use the optical model to calculate the wakefield of the beam. The transverse impedance $Z_y$ according to this model applied to the geometry of Fig. 1 is calculated in Appendix A and is given by Eq. (A.9). According to the optical model $Z_y \propto 1/\omega$, so that the product $\omega Z_y$ is independent of frequency. In this case the transverse wake of the bunch can be expressed through the transverse impedance of a point charge as follows:

$$W_y(z) = \frac{i}{2\pi}\int_{-\infty}^{\infty} Z_y(\omega)\tilde{\lambda}_z(\omega)e^{-i\omega z/c}\,d\omega = (\omega Z_y)\int_z^{\infty}\lambda_z(z')\,dz' \tag{9}$$

where $\tilde{\lambda}_z(\omega)$ is the Fourier transform of the longitudinal distribution $\lambda_z(z)$ ($\lambda_z(z)$ is normalized so that $\int\lambda_z(z)\,dz = 1$). Using (9) and assuming a Gaussian distribution function $\lambda_z(z)$ it is easy to find

$$\theta_{\rm rms} = \frac{Qe(\omega Z_y)}{\sqrt{12}\gamma mc^2}. \tag{10}$$

As is pointed out in Appendix A, for small values of $h$ one can use the following formula for $(\omega Z_y)$:

$$\omega Z_y \approx \frac{8\pi^2 h}{a^2}F\left(\frac{b}{a}\right) \tag{11}$$

with the function $F$ given by Eq. (A.11). Eqs. (10) and (11) allow one to evaluate the strength of the transverse kick for a given size of protrusion and to estimate the emittance increase of the beam. In the next section we will compare the wakefield predicted by the optical model with that obtained numerically using the 3D time domain code GdfidL and finally estimate the emittance growth for the parameters of the SuperKEKB collider.

## 4. SuperKEKB low energy ring

In this section we will estimate the effect of a single protrusion on the beam emittance for the parameters of the low-energy ring

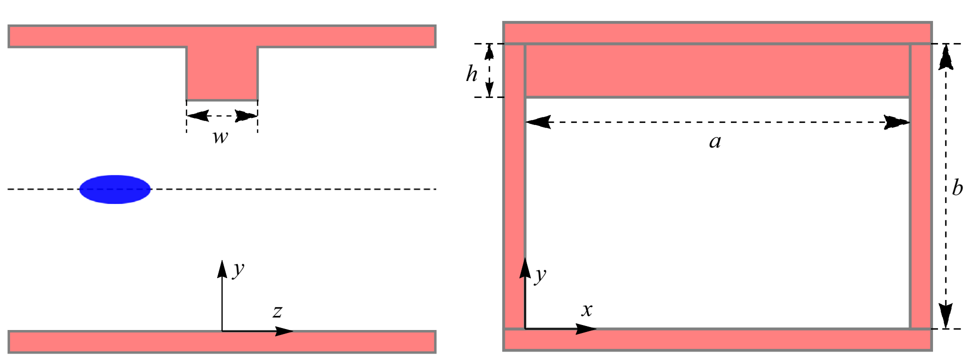


**Fig. 1.** Geometry of a rectangular vacuum chamber with an asymmetric protrusion. The design orbit goes through the center of the $a\times b$ cross-section.

(LER) of SuperKEKB [6]. The parameters of the ring are shown in Table 1.

As was mentioned above, over most of the circumference, the LER has a vacuum chamber which can be approximated by a rectangular cross-section with $a=10$ cm and $b=5$ cm. With an rms bunch length of 6 mm, one cannot know in advance whether the optical model is applicable for this case. We supplemented the optical model calculations with simulations using the computer code GdfidL for a protrusion length $w=1$ cm and three different depths $h=1$, 0.5 and 0.25 cm. The vertical wake $w_y(z)$ was calculated for $\sigma_z=0.5$ mm bunch passing through the center of the vacuum chamber. This wake was then used as a pseudo-Green function for the calculation of the wakes of longer bunches by convolution. The plot of the function $w_y(z)$ and the corresponding impedance are shown in Fig. 2. One can see that at high frequencies $f \gtrsim 20$ GHz the impedance indeed follows the optical model scaling $Z(\omega) \propto 1/\omega$, while at lower frequencies it deviates from the scaling.

The wake of a 6 mm bunch calculated with the help of the pseudo-Green function of Fig. 2 is shown in Fig. 3 for three different values of $h$ of the protrusion.

One can see that for the biggest protrusion with $h=1$ cm the optical model reasonably well approximate the wake over a large part of the beam, while for smaller protrusions it overestimates the wake.

Using the wakes shown in Fig. 3 we calculated the value of $\theta_{\rm rms}$ for each of the three cases. The corresponding emittance growth $\Delta\epsilon$ was found using (7) assuming the beta function at the location of the protrusion $\beta_0=20$ m and taking the fractional tune $[\nu_y]=0.57$ from Table 1. The results are shown in Table 2. One can see from the table that a single 1 cm protrusion increases the nominal vertical emittance of 8.6 pm by a considerable fraction. Decreasing the depth $h$ makes $\Delta\epsilon$ much smaller, however, if there are many such protrusions in the ring, their cumulative effect can still be noticeable.

## 5. TLEP-W accelerator

In this section we consider an example of TLEP-W machine considered in Ref. [7] with the parameters given in Table 3.

**Table 1**
Parameters of SuperKEKB LER.

| Parameter | LER |
|---|---|
| Electron energy (GeV) | 4 |
| Bunch charge (nC) | 14.5 |
| Circumference (m) | 3016 |
| Vertical emittance (pm) | 8.6 |
| Bunch length, rms (mm) | 6 |
| Vertical tune, $\nu_y$ | 44.57 |

Performing the convolution with the wakefield of the 0.5 mm bunch from the previous section we obtain the wake for a bunch of length of 2 mm. The plots of the wakefields are shown in Fig. 4. Again, the largest protrusion shows a satisfactory agreement with the optical model.

Using the wake shown in Fig. 3 we calculated the value of $\theta_{\rm rms}$ for each of the three cases. The corresponding emittance growth $\Delta\epsilon$ was found using (7), assuming the beta function at the location of the protrusion $\beta_0=20$ m and taking $\csc(\pi\nu)=1$. The results are shown in Table 4.

## 6. Conclusions

In this paper we demonstrated how a wakefield of an asymmetric protrusion in a vacuum chamber can lead to the projected emittance growth of a beam in a circular accelerator. We argued that for short bunches the wakefield of the protrusion can be evaluated with the help of the optical model of Refs. [2,3]. Using a simple model of a rectangular protrusion in a rectangular chamber we calculated the emittance growth in LER of SuperKEKB and

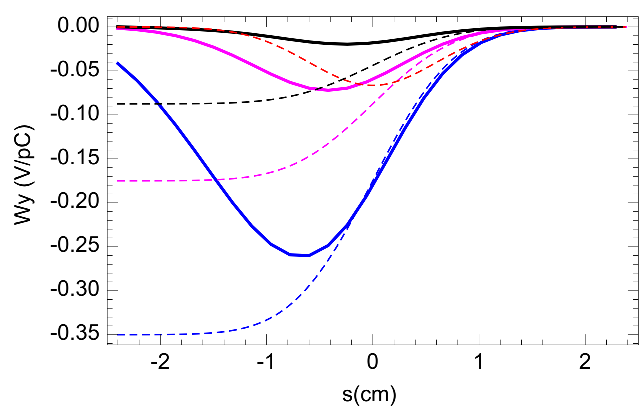


**Fig. 3.** Wakefield of a bunch with $\sigma_z=6$ mm for three values of the protrusion depth $h$: 1 cm (blue), 0.5 cm (magenta) and 0.25 cm (black). The dashed lines of the same color show the corresponding wakes computed with the optical model. The red dashed line shows the profile of the beam. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

**Table 2**
Emittance increase in LER of SuperKEKB.

| Corrugation depth $h$ (cm) | 1 | 0.5 | 0.25 |
|---|---|---|---|
| $\theta_{\rm rms}$ (nrad) | 290 | 77 | 20 |
| $\Delta\epsilon$ (pm) | 0.21 | 0.016 | 0.001 |

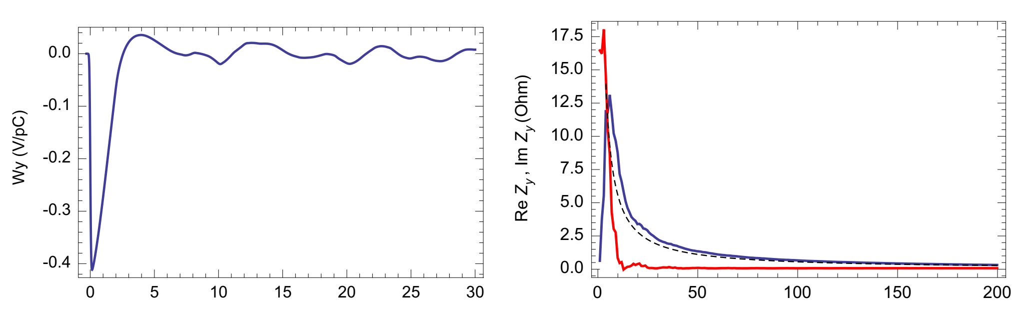


**Fig. 2.** Vertical wake (left) and impedance (right) of a 0.5 mm bunch with $h=1$ cm protrusion. On the right plot the blue line is $\Re Z$, the red line is $\Im Z$ and the dashed black line is the real part of the impedance calculated using Eq. (A.10) (the imaginary part in the optical model is zero). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

**Table 3**
Parameters of TLEP-W.

| Parameter | TLEP-W |
|---|---|
| Electron energy (GeV) | 80 |
| Bunch charge (nC) | 107 |
| Circumference (km) | 80 |
| Vertical emittance (pm) | 20 |
| Bunch length, rms (mm) | 2.2 |

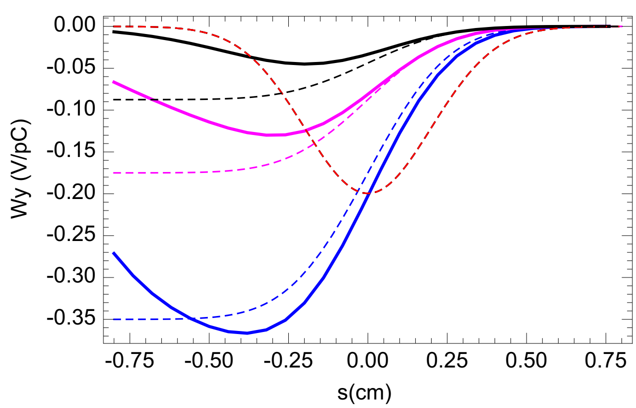


**Fig. 4.** Wakefield of a bunch with rms length of 2 mm for three values of protrusion depth $h$: 1 cm (blue), 0.5 cm (magenta) and 0.25 cm (black). The dashed lines of the same color show the corresponding wakes computed with the optical model. The red dashed line gives the profile of the beam. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

**Table 4**
Emittance increase in FCC.

| Corrugation depth $h$ (cm) | 1 | 0.5 | 0.25 |
|---|---|---|---|
| $\theta_{\rm rms}$ (nrad) | 146 | 53 | 18 |
| $\Delta\epsilon$ (pm) | 0.054 | 0.007 | 0.0008 |

TLEP-W for several representative values of the protrusion height. Our results can be used in the formulation of asymmetry tolerances of the vacuum chamber in future accelerators with record small vertical beam emittances.

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## Appendix A. Calculation of the wakefield in optical model

For short bunches calculation of wakefields and impedances can be carried out using the so-called optical approximation in wakefield theory [2,3]. In this model one considers a leading charge $q$ moving inside the vacuum chamber along an orbit given by $\boldsymbol{r}_1=(x_1,y_1)$, with a trailing charge $q$ moving at distance $s$ behind $(s>0)$ with the trajectory given by $\boldsymbol{r}_2=(x_2,y_2)$; both charges are assumed to move at the speed of light $c$. The longitudinal wake $w_\parallel(\boldsymbol{r}_1,\boldsymbol{r}_2,s)$ is given by

$$w_\parallel(\boldsymbol{r}_1,\boldsymbol{r}_2,s)=-\frac{c}{q}\int_{-\infty}^{\infty}E_{1,z}(\boldsymbol{r}_2,z=ct-s,t)\,dt \tag{A.1}$$

where $E_{1,z}(\boldsymbol{r},z,t)$ is the $z$-component of the electric field generated by the leading particle at the position of the trailing one. The longitudinal impedance is related to the wake by the Fourier transform

$$Z_\parallel(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)=\frac{1}{c}\int_0^{\infty}ds\,w_\parallel(\boldsymbol{r}_1,\boldsymbol{r}_2,s)e^{i\omega s/c}. \tag{A.2}$$

Knowledge of the longitudinal impedance allows one to compute the transverse impedance using the Panofsky–Wenzel theorem. In the general case, the transverse impedance is represented by a vector $\boldsymbol{Z}_\perp$ perpendicular to the particle's orbit, and is given by $\boldsymbol{Z}_\perp=c\nabla_{\boldsymbol{r}_2}Z_\parallel/\omega$. In our problem we are interested in the vertical impedance,

$$Z_y=\frac{c}{\omega}\partial_{y_2}Z_\parallel. \tag{A.3}$$

The longitudinal wake in the optical approximation is $w_l(\boldsymbol{r}_1,\boldsymbol{r}_2,s)=(2\pi)^{-1/2}I(\boldsymbol{r}_1,\boldsymbol{r}_2)\delta(s)$ with

$$I=\int_{S_{\rm p}}\nabla\phi_1(\boldsymbol{r})\cdot\nabla\phi_2(\boldsymbol{r})\,dS \tag{A.4}$$

where $S_{\rm p}$ is the protrusion aperture, that is the area $0<x<a$, $b-h<y<b$. The functions $\phi_1$ and $\phi_2$ are solutions of the equations

$$\nabla^2\phi_1(\boldsymbol{r})=-4\pi\delta(\boldsymbol{r}-\boldsymbol{r}_1),\quad \nabla^2\phi_2(\boldsymbol{r})=-4\pi\delta(\boldsymbol{r}-\boldsymbol{r}_2) \tag{A.5}$$

with boundary conditions $\phi_1=\phi_2=0$ on the walls of the vacuum chamber. The longitudinal impedance corresponding to this wake is $Z_l(\boldsymbol{r}_1,\boldsymbol{r}_2)=I/2\pi c$.

Before solving (A.5) we note that Eq. (A.4) can be simplified if we use the identity $\nabla\cdot(\phi_1\nabla\phi_2)=\nabla\phi_1\cdot\nabla\phi_2+\phi_1\nabla^2\phi_2$ and notice that $\nabla^2\phi_2=0$ in the area of the diaphragm. We then have

$$I=\int_{S_{\rm coll}}\nabla\cdot(\phi_1\nabla\phi_2)\,dS=\int_{\Gamma}\phi_1\boldsymbol{n}\cdot\nabla\phi_2\,dl \tag{A.6}$$

where the integration is performed over the boundary of the diaphragm and $\boldsymbol{n}$ is a unit normal vector (in the outward direction) on the boundary. Note that $\phi_1$ is zero on the wall of the pipe, and what is left in just the integral is the edge of the diaphragm $0<x<a$, $y=b-h$.

The functions $\phi$ in (A.5) are given by the solution of a 2D electrostatic problem $\Delta\phi=-4\pi\delta(x-x_0)\delta(y-y_0)$ with $\phi=0$ on the wall of a pipe with rectangular cross-section $0\le x\le a$, $0\le y\le b$. The solution can be found in [8]

$$\phi=8\sum_{k=1}^{\infty}\frac{1}{k\sinh\frac{k\pi b}{a}}\sinh\frac{k\pi(b-y_0)}{a}\sinh\frac{k\pi y}{a}\sin\frac{k\pi x_0}{a}\sin\frac{k\pi x}{a},\quad \text{for } y<y_0$$

$$\phi=8\sum_{k=1}^{\infty}\frac{1}{k\sinh\frac{k\pi b}{a}}\sinh\frac{k\pi(b-y)}{a}\sinh\frac{k\pi y_0}{a}\sin\frac{k\pi x_0}{a}\sin\frac{k\pi x}{a},\quad \text{for } y>y_0. \tag{A.7}$$

The potential $\phi_1$ is given by (A.7) with $x_0\to x_1$ and $y_0\to y_1$, and for $\phi_2$ we need to replace $x_0$ by $x_2$ and $y_0$ by $y_2$. Vector $\boldsymbol{n}$ in (A.6) is directed along the negative direction $y$, so $\boldsymbol{n}\cdot\nabla\phi_2=-\partial_y\phi_2$. Because the integration in (A.6) goes along the line $y=b-h$ with $y>y_1,y_2$ we use the second line in (A.7) for the potential. Integration over $x$ goes from 0 to $a$ and in the final result we set $x_1=x_2=a/2$ and $b-y=h$, yielding

$$I=32\pi\sum_{k=\rm odd}^{\infty}\frac{1}{k\left(\sinh\frac{k\pi b}{a}\right)^2}\sinh\frac{k\pi h}{a}\sinh\frac{k\pi y_1}{a}\cosh\frac{k\pi h}{a}\sinh\frac{k\pi y_2}{a}. \tag{A.8}$$

We can now calculate the product of the frequency with the transverse impedance, $\omega Z_y$, using (A.8)

$$\omega Z_y = \frac{1}{2\pi} \frac{\partial I}{\partial y_2}\bigg|_{y_1 = y_2 = b/2} = \frac{4\pi}{a} \sum_{k = \mathrm{odd}}^{\infty} \frac{1}{\sinh \frac{k\pi b}{a}} \sinh \frac{2k\pi h}{a}. \qquad \text{(A.9)}$$

For small values of $h$ one can use the Taylor expansion to obtain

$$\omega Z_y \approx \frac{8\pi^2 h}{a^2} F\left(\frac{b}{a}\right) \qquad \text{(A.10)}$$

with

$$F(\xi) = \sum_{k = \mathrm{odd}}^{\infty} \frac{k}{\sinh k\pi\xi}. \qquad \text{(A.11)}$$

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