Higgs Self-Coupling in $\gamma\gamma$ Collisions

R. BELUSEVIC$^a$ and G. JIKIA$^b$

$^a$KEK, Oho 1-1, Tsukuba-shi, Ibaraki-ken 305-0801, Japan

$^b$Albert–Ludwigs–Universität Freiburg, Fakultät für Mathematik und Physik
Hermann–Herder Str. 3a, D-79104 Freiburg, Germany
Abstract: To establish the Higgs mechanism experimentally, one has to determine the Higgs self-interaction potential responsible for the electroweak symmetry breaking. This requires a measurement of the trilinear and quadrilinear self-couplings of the Higgs particle, as predicted by the Standard Model (SM). We propose that the trilinear Higgs self-coupling be measured in the process $\gamma\gamma \to HH$ just above the kinematic threshold of $2M_H$, where $M_H$ is the Higgs mass. Our calculation shows that the statistical sensitivity of the cross-section $\sigma_{\gamma\gamma \to HH}$ to the Higgs self-coupling is maximal near the $2M_H$ threshold for $M_H$ between 115 and 150 GeV, and is larger than the statistical sensitivities of $\sigma_{e^+e^- \to ZHH}$ and $\sigma_{e^+e^- \to \nu\bar{\nu}HH}$ to this coupling for $2E_e \leq 700$ GeV.

1 Introduction

In order to provide a mechanism for the generation of particle masses in the Standard Model (SM) without violating its gauge invariance, a complex scalar SU(2) doublet $\Phi$ with four real fields and hypercharge $Y = 1$ is introduced. The dynamics of the field $\Phi$ is described by the Lagrangian

$$L_\Phi = (D_\mu \Phi)\dagger(D^{\mu}\Phi) - \mu^2 \Phi\dagger\Phi - \lambda(\Phi\dagger\Phi)^2,$$  \hspace{1cm} (1)

where $(D_\mu \Phi)\dagger(D^{\mu}\Phi)$ is the kinetic-energy term and $\mu^2 \Phi\dagger\Phi + \lambda(\Phi\dagger\Phi)^2$ is the Higgs self-interaction potential. This potential gives rise to terms involving only the physical Higgs field $H$:

$$V_H = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4.$$  \hspace{1cm} (2)

Here $v \equiv \sqrt{-\mu/\lambda} = 246$ GeV is the vacuum expectation value of the scalar field $\Phi$. We see that the Higgs mass $M_H = \sqrt{2\lambda v}$ is related to the quadrilinear self-coupling strength $\lambda$. The trilinear self-coupling of the Higgs field and the self-coupling among four Higgs fields are given, respectively, by

$$\lambda_{hhh} \equiv \lambda v = \frac{M_H^2}{2v}, \hspace{1cm} \lambda_{hhhh} \equiv \frac{\lambda}{4} = \frac{M_A^2}{8v^2}. \hspace{1cm} (3)$$

Evidently, the Higgs self-couplings are uniquely defined by the mass of the Higgs boson, which represents a free parameter of the model.

All of the couplings of the Higgs boson to fermions and gauge bosons are completely determined in terms of coupling constants and fermion masses. Higgs production and decay processes can be computed in the SM unambiguously in terms of the Higgs mass alone. Since Higgs-boson coupling to fermions and gauge bosons is proportional to the particle masses, the Higgs boson will be produced in association with heavy particles, and will decay into the heaviest particles that are kinematically accessible.

The minimal supersymmetric extension of the Standard Model (MSSM) introduces two SU(2) doublets of complex Higgs fields, whose neutral components have vacuum expectation values $v_1$ and $v_2$. In this model, spontaneous electroweak symmetry breaking results in five physical Higgs-boson states: two neutral scalar fields $h^0$ and $H^0$, a pseudoscalar $A^0$ and two charged bosons $H^\pm$. This extended Higgs system can be described at tree level by two parameters: the ratio $\tan \beta \equiv v_2/v_1$, and a mass parameter, which is generally identified with the mass of the pseudoscalar boson $A^0$, $M_A$. While there is a bound of about 140 GeV on the mass of the lightest CP-even neutral Higgs boson $h^0$ [1, 2], the masses of the $H^0$, $A^0$ and $H^\pm$ bosons may be as large as 1 TeV.

The trilinear self-coupling of the lightest MSSM Higgs boson at tree level is given by

$$\lambda_{hhh} = \frac{M_A^2}{2v} \cos 2\alpha \sin(\beta + \alpha), \hspace{1cm} \text{where} \hspace{1cm} \tan 2\alpha = \tan 2\beta \frac{M_Z^2 + M_H^2}{M_Z^2 - M_H^2}. \hspace{1cm} (4)$$
We see that for arbitrary values of the MSSM input parameters $\tan \beta$ and $M_\Lambda$, the value of the $h^0$ self-coupling differs from that of the SM Higgs boson. However, in the so-called ‘decoupling limit’ $M_A^2 \sim M_{h^0}^2 \sim M_{H^\pm}^2 \gg v^2/2$, the trilinear and quadrilinear self-couplings of the lightest $CP$-even neutral Higgs boson $h^0$ approach the SM value.

In the non-supersymmetric two-Higgs-doublet model (the simplest extension of the SM), large one-loop effects can occur. For charged Higgs bosons with masses of about 400 GeV, the decay widths of $h^0 \to \gamma\gamma$, $h^0 \to \gamma Z$ and $h^0 \to b\bar{b}$ may differ from the SM values by as much as 10%–25% \cite{3}. In this model, the non-decoupling effects of the additional heavier Higgs bosons in loops can produce $\mathcal{O}(100\%)$ deviations of the effective $h^0h^0h^0$ self-coupling from the SM prediction, even if the Higgs couplings to gauge bosons and fermions are almost SM-like \cite{4}.

The precision electroweak data obtained over the past sixteen years consists of over one thousand individual measurements. Many of these measurements may be combined to provide a global test of consistency with the Standard Model. The best constraint on $M_H$ is obtained by making a global fit to these data, which yields $M_H = 113^{+62}_{-42}$ GeV \cite{5} (see also \cite{6}). The available electroweak data, therefore, strongly suggest that the most likely mass for the SM Higgs boson is just above the limit of 114.4 GeV set by direct searches at the LEP $e^+e^-$ collider\cite{7}. In contrast to any anomalous couplings of the gauge bosons, an anomalous self-coupling of the Higgs particle would contribute to electroweak observables only at two-loop and higher orders, and is therefore practically unconstrained by these precision measurements \cite{8}.

Since photons couple directly to all fundamental fields carrying the electromagnetic charge (leptons, quarks, $W$ bosons, supersymmetric particles), $\gamma\gamma$ collisions provide a comprehensive means of exploring virtually every aspect of the SM and its extensions. In $\gamma\gamma$ collisions, the Higgs boson is produced as a single resonance in a state of definite $CP$. This is perhaps the most important advantage over $e^+e^-$ annihilations, where this $s$-channel process is highly suppressed. For the Higgs-boson mass in the range 115–200 GeV, the effective cross-section for $\gamma\gamma \to H$ is larger than that for Higgs production in $e^+e^-$ annihilations. In this mass range, the process $e^+e^- \to ZH$ requires considerably higher centre-of-mass (CM) energies than $\gamma\gamma \to H$. The reaction $\gamma\gamma \to H$ proceeds through a loop diagram and receives contributions from all particles with mass and charge, and is therefore a powerful probe of new physics beyond the SM. Moreover, we find that the sensitivity of the cross-section $\sigma_{\gamma\gamma \to HH}$ to the trilinear Higgs self-coupling is maximal near the $2M_H$ threshold for $M_H$ between 115 and 150 GeV, and is larger than the sensitivities of $\sigma_{e^+e^- \to ZHH}$ and $\sigma_{e^+e^- \to \nu\bar{\nu}HH}$ to this coupling for $2E_{\text{cm}} \lesssim 700$ GeV. By combining data from $e^+e^-$ and $\gamma\gamma$ collisions, the total decay width of the Higgs boson can be determined in a model-independent way with a precision of about 10% (see \cite{9} and references therein).

2 Higgs-pair production in $\gamma\gamma$ and $e^+e^-$ collisions

The production of a pair of SM Higgs bosons in photon-photon collisions, $\gamma\gamma \to HH$, which is related to the Higgs-boson decay into two photons, is due to $W$-boson and top-quark box and triangle loop diagrams. The total cross-section for $\gamma\gamma \to HH$ in polarized photon-photon collisions, calculated at the leading one-loop order \cite{10} as a function of the $\gamma\gamma$ centre-of-mass energy and for $M_H$ between 115 and 150 GeV, is shown in Fig. 1a. The cross-section calculated for equal photon helicities, $\sigma_{\gamma\gamma \to HH}(J_z = 0)$, rises sharply above the $2M_H$ threshold for different values of $M_H$, and has a peak value of about 0.4 fb at a $\gamma\gamma$ centre-of-mass energy of 400 GeV. In contrast, the cross-section for opposite photon helicities, $\sigma_{\gamma\gamma \to HH}(J_z = 2)$, rises more slowly with energy, because a pair of Higgs bosons is produced in a state with orbital angular momentum of at least 2h.

The cross-sections for equal photon helicities are of special interest, since only the $J_z = 0$ amplitudes contain contributions with trilinear Higgs self-coupling. By adding to the SM Higgs potential $V(\Phi^+\Phi)$ a gauge-invariant dimension-6 operator $(\Phi^+\Phi)^3$, one introduces a gauge-invariant
anomalous trilinear Higgs coupling $\delta\kappa$ [10]. For the reaction $\gamma\gamma \rightarrow HH$, the only effect of such a coupling in the unitary gauge would be to replace the trilinear $HHH$ coupling of the SM in Eq. 3 by an anomalous Higgs self-coupling

$$\bar{\lambda}_{HHH} = (1 + \delta\kappa)\lambda_{HHH}. \quad (5)$$

The dimensionless anomalous coupling $\delta\kappa$ is normalized so that $\delta\kappa = -1$ exactly cancels the SM $HHH$ coupling. The cross-sections $\sigma_{\gamma\gamma \rightarrow HH}$ for five values of $\delta\kappa$ are shown in Fig. 1b.

In an experiment to measure the trilinear Higgs self-coupling, the contribution from $\gamma\gamma \rightarrow HH$ for opposite photon helicities represents an irreducible background. However, this background is suppressed if one chooses a $\gamma\gamma$ CM energy somewhat below 400 GeV.

We envisage that an $e^-e^-$ linac and a terawatt laser system will be used to produce Compton-scattered $\gamma$-ray beams for a photon collider. Both the energy spectrum and polarization of the backscattered photons depend strongly on the polarizations of the incident electrons and photons. A longitudinal electron-beam polarization of 90% and a 100% circular polarization of laser photons are assumed throughout.

To ascertain the potential of $\gamma\gamma$ colliders for measuring an anomalous Higgs self-coupling, one must take into account the fact that the photons are not monochromatic [11]. At a fixed $e^-e^-$ CM energy $\sqrt{s}$, the cross-section for Higgs-pair production in polarized $\gamma\gamma$ collisions is given by

$$\sigma_{\gamma\gamma \rightarrow HH} = \int_{4M_h^2/s}^{s_m^2} d\tau \frac{dL_{\gamma\gamma}}{d\tau} \left[ \frac{1}{2} \left( 1 + \langle \xi_2^{(1)} \xi_2^{(2)} \rangle \right) \sigma_{++}(\hat{s}) + \frac{1}{2} \left( 1 - \langle \xi_2^{(1)} \xi_2^{(2)} \rangle \right) \sigma_{+-}(\hat{s}) \right] \quad (6)$$

Figure 1: (a) The total $\gamma\gamma \rightarrow HH$ cross-section as a function of the $\gamma\gamma$ centre-of-mass energy. Contributions for equal ($J_z = 0$) and opposite ($J_z = 2$) photon helicities are shown separately. (b) The cross-sections for $HH$ production in $\gamma\gamma$ collisions for $M_H = 120$ GeV and anomalous trilinear Higgs self-couplings $\delta\kappa = 0, \pm 1, \pm 0.3$. 

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where \( \hat{s}/s = \tau \) and
\[
\frac{dL_{\gamma\gamma}}{d\tau} = \int_{\tau/y_m}^{y_m} \frac{dy}{y} f_\gamma(x, y) f_\gamma(x, \tau/y),
\]
\[
\sqrt{\tau} = z_{\gamma\gamma} \equiv \frac{W_{\gamma\gamma}}{\sqrt{s}}, \quad 0 \leq y \equiv \frac{E_\gamma}{E_e} \leq y_m = \frac{x}{x+1}, \quad x \equiv \frac{4E_e\omega_0}{m_e^2}.
\] (7)

Here, \( E_e \) is the energy of each incident electron beam, \( \omega_0 \) is the laser photon energy, \( E_\gamma \) is the energy of the backscattered photon, \( W_{\gamma\gamma} \) is the \( \gamma\gamma \) CM energy, \( f_\gamma(x, y) \) is the photon momentum distribution function and \( \xi_2^{(1,2)} \) are mean photon helicities [11]. In Eq. 6, \( \hat{\sigma}_{++}(\hat{s}) \) and \( \hat{\sigma}_{+-}(\hat{s}) \) are the cross-sections for Higgs-pair production, calculated assuming monochromatic photons with total helicities \( J_z = 0 \) and \( J_z = 2 \), respectively. As usual, the dimensionless parameter \( x \) has been set to 4.8 (\( y_m = 0.82 \)) to avoid undesirable backgrounds. We choose a configuration that maximizes the \( \gamma\gamma \) luminosity for \( J_z = 0 \) in the high-energy part of the photon spectrum [12]:
\[
L_{\gamma\gamma} = \int_{\tau=(0.8 y_m)^2}^{y_m^2} d\tau \frac{dL_{\gamma\gamma}}{d\tau} \approx (1/3) L_{e^-e^-}. \] (8)

Figure 2: For the process \( \gamma\gamma \rightarrow HH \), the number of standard deviations from the SM-predicted event rates, defined by Eq. (9), is plotted as a function of the \( e^-e^- \) centre-of-mass energy assuming a \( \gamma\gamma \) luminosity \( L_{\gamma\gamma} = 300 \text{ fb}^{-1} \) and a combined efficiency \( \varepsilon = 1.0 \) or 0.5.

In terms of standard deviations, the discrepancy between the SM prediction for event rates and that for zero HHH coupling is defined by
\[
\#STD = \frac{|\sigma(\delta\kappa = 0) - \sigma(\delta\kappa = -1)|}{\sqrt{\sigma(\delta\kappa = 0)}} \sqrt{L_{\gamma\gamma}}.
\] (9)
This relative sensitivity to the Higgs self-coupling is plotted in Fig. 2 using the cross-sections of Eq. (6), a $\gamma\gamma$ luminosity of 300 fb$^{-1}$ (see Eq. (8)), ideal detection efficiency and no backgrounds. For $M_H = 120$ GeV, a maximum sensitivity of about 5$\sigma$ (9$\sigma$) is attained at an $e^-e^-$ CM energy of 310 (370) GeV. Note that the abscissa in Fig. 2 shows the $e^-e^-$ CM energies; for instance, $E_{e^-e^-} = 310$ GeV corresponds to a maximum $\gamma\gamma$ CM energy of $y_m E_{e^-e^-} \approx 254$ GeV, which is just 14 GeV above the HH-threshold. Of course, the ideal sensitivities shown in Fig. 2 will be reduced considerably by realistic detection and reconstruction efficiencies and the presence of backgrounds. For example, with a combined efficiency of 0.5, the vertical scale of Fig. 2 would be multiplied by 0.7. Nevertheless, Fig. 2 shows that, in photon-photon collisions, the optimum $e^-e^-$ CM energy for measuring the trilinear Higgs self-coupling is rather low — between 300 and 400 GeV — for a Higgs-boson mass below 130 GeV.

It is well known that hadron colliders are not well suited for measuring the self-coupling of the Higgs boson if $M_H \leq 140$ GeV [13]. The potential of a future $e^+e^-$ collider for determining the HHH coupling has therefore been closely examined [14, 15, 16, 17]. The trilinear Higgs boson self-coupling can be measured either in the double Higgs-strahlung process $e^+e^- \rightarrow ZHH$ or in the $W$-fusion reaction $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu HH$.

The total cross-section for pair production of 120-GeV Higgs bosons in $e^+e^-$ collisions, calculated for unpolarized beams, is presented in Fig 3a for anomalous trilinear Higgs self-couplings $\delta\kappa = 0$ or $-1$. If the electron beam is 100% polarized, the double Higgs-strahlung cross-section will stay approximately the same, while the $W$-fusion cross-section will be twice as large. From Fig 3a, we infer that the SM double Higgs-strahlung cross-section exceeds 0.1 fb at 400 GeV for $M_H = 120$ GeV, and reaches a broad maximum of about 0.2 fb at a CM energy of 550 GeV. The SM cross-section for $W$-fusion stays below 0.1 fb for CM energies up to 1 TeV.

For both processes, the number of standard deviations from the SM prediction for event rates, defined analogously to Eq. (9), is shown in Fig. 3b for $M_H = 120$ or 150 GeV and an $e^+e^-$ luminosity of 1000 fb$^{-1}$. We again assume a combined efficiency $\varepsilon = 1.0$ or 0.5; the polarization of the electron beam is 90%. For $M_H = 120$ GeV, a maximum ideal sensitivity of about 6$\sigma$ is achieved in the double Higgs-strahlung process at a CM energy of 500 GeV. This is significantly higher than the optimal CM energy in $\gamma\gamma$ collisions.

3 Backgrounds

We present an order-of-magnitude estimate of the most important backgrounds to the process $\gamma\gamma \rightarrow HH$. The dominant background is the $W$-boson pair production $\gamma\gamma \rightarrow W^+W^-$, with a total cross-section of 70 pb at 300 GeV. By imposing the invariant mass cut

$$|M(q\bar{q}) - M_H| < 5 \text{ GeV}, \quad (10)$$

the $W^+W^-$ background can be reduced by about four orders of magnitude. To suppress this background further, one could rely on the fact that the predominant decay mode of the SM Higgs boson with $M_H = 115$ to 130 GeV is into a pair of $b$ quarks, with a SM branching ratio that decreases from 73% to 53% with increasing Higgs-boson mass. (This is the dominant decay mode as well of the MSSM $h^0$ boson for various values of the MSSM parameters — in particular, for $\tan \beta > 1$.) In order to select the $HH \rightarrow b\bar{b}b\bar{b}$ events, we require that at least three jets be identified as originating from $b$-quarks. If we assume that the standard method used for tagging $b$-hadrons at the LEP $e^+e^-$ collider [18] can also be used at a photon collider, then the sample tagged as $b$-quark would have the following flavour composition: 4.3% light quarks, 10.4% $c$-quarks and 85.4% $b$-quarks [18]. The requirement that at least three jets originating from $W^\pm$ decays be identified as $b$-jets would suppress $\gamma\gamma \rightarrow W^+W^-$ by another three orders of magnitude, to a level well below the HH signal.
Figure 3: (a) The total cross-sections for $e^+e^- \rightarrow ZHH$ and $e^+e^- \rightarrow \nu_\ell \bar{\nu}_\ell HH$ as functions of the $e^+e^-$ centre-of-mass energy for $M_H = 120$ GeV and anomalous trilinear Higgs self-couplings $\delta \kappa = 0$ or $-1$.

(b) For HH production in $e^+e^-$ collisions, the number of standard deviations from the SM prediction for event rates is plotted as a function of the $e^+e^-$ centre-of-mass energy assuming an $e^+e^-$ luminosity $L_{e^+e^-} = 1000$ fb$^{-1}$ and a combined efficiency $\varepsilon = 1.0$ or 0.5.

The next most significant background is the non-resonant production of four heavy quarks in photon-photon collisions. The total cross-sections for $b\bar{b}b\bar{b}$, $b\bar{c}c\bar{c}$ and $c\bar{c}c\bar{c}$ production, shown in Table 1, are quite large and do not decrease with energy. For instance, the cross-section for the production of four $c$-quarks is even larger than that for $W^+W^-$ production. Since the cross-sections calculated for two-photon helicities $J_z = 0$ and $J_z = 2$ have similar magnitudes, the polarization of the photon beams does not lead to a reduction in the four-quark background. However, the $b$ and $c$ quarks are produced mostly in the forward or backward direction, and hence a simple cut on the polar angle $|\cos \theta_{q,\bar{q}}| < 0.9$ suppresses these backgrounds by at least two orders of magnitude. In contrast, near the HH production threshold, the angular distribution of $b$-jets originating from Higgs-boson decays is isotropic, and the efficiency of this angular cut is about 80%. The residual cross-sections for quark production are still much larger than the cross-section for double Higgs-boson production, and so the invariant-mass cut (10) should be imposed also. As shown in Table 1, after these two cuts, the cross-section for $b\bar{b}b\bar{b}$ production is already an order of magnitude smaller than $\sigma_{\gamma\gamma \rightarrow HH}$, and the cross-sections for $b\bar{c}c\bar{c}$ and $c\bar{c}c\bar{c}$ production are of the same order as $\sigma_{\gamma\gamma \rightarrow HH}$. The additional requirement that at least three jets be identified as $b$-jets would suppress these cross-sections well below that of the signal. After the invariant-mass cut (10), the polar angular cut and the $b$-tagging requirement, the reconstruction efficiency for the HH final state is roughly 50%.

Other potential background sources are $\gamma\gamma \rightarrow b\bar{b}Z$, $\gamma\gamma \rightarrow c\bar{c}Z$, $\gamma\gamma \rightarrow q\bar{q}'W$, $\gamma\gamma \rightarrow W^+W^-Z$ and $\gamma\gamma \rightarrow ZZ$ processes. We believe that appropriate invariant-mass and angular cuts, combined
Table 1: The cross-sections for the production of four heavy quarks in unpolarized $\gamma\gamma$ collisions for $E_{\gamma\gamma} = 250$ and 300 GeV.

| $E_{\gamma\gamma}$ = 250 (300) GeV | $\sigma_{\text{tot}}$ (fb) | $|\cos \theta_{q\bar{q}}| < 0.9$ (115 GeV $< M_{q\bar{q}} < 125$ GeV) |
|-----------------------------------|--------------------------|----------------------------------|
| $M_H = 120$ GeV                   |                          |                                  |
| $\gamma\gamma \rightarrow b\bar{b}b\bar{b}$ | 360 (380) | 5.0 (3.9) (0.015 (0.015)) |
| $\gamma\gamma \rightarrow b\bar{b}c\bar{c}$ | 9400 (9800) | 66 (52) (0.13 (0.16)) |
| $\gamma\gamma \rightarrow c\bar{c}c\bar{c}$ | 81000 (83000) | 150 (120) (0.24 (0.26)) |

with a $b$-jet tagging requirement, would suppress these backgrounds to a manageable level.

4 Conclusions

We have studied the potential of a photon collider to probe the trilinear self-coupling of the Higgs boson in the process $\gamma\gamma \rightarrow HH$. Our calculation shows that the sensitivity of the cross-section $\sigma_{\gamma\gamma \rightarrow HH}$ to the Higgs self-coupling is maximal near the kinematic threshold for Higgs-pair production for $M_H$ between 115 and 150 GeV, and is larger than the sensitivities of $\sigma_{e^+e^- \rightarrow ZHH}$ and $\sigma_{e^+e^- \rightarrow \nu\bar{\nu}HH}$ to this coupling for $2E_e \leq 700$ GeV, even if the integrated luminosity in $\gamma\gamma$ collisions is only one third of that in $e^+e^-$ annihilations. Since the cross-section $\sigma_{\gamma\gamma \rightarrow HH}$ does not exceed 0.4 fb, it is essential to attain the highest possible luminosity, rather than energy, in order to measure the trilinear Higgs self-coupling. We find that appropriate angular and invariant-mass cuts and a $b$-tagging requirement, which result in a Higgs-pair reconstruction efficiency of about 50%, would suppress the dominant $W$-pair and four-quark backgrounds well below the HH signal. For such a reconstruction efficiency, a centre-of-mass energy $E_{ee} = 350$ GeV and $M_H = 120$ GeV, an integrated $\gamma\gamma$ luminosity $L_{\gamma\gamma} \approx 450$ fb$^{-1}$ would be needed to exclude a zero trilinear Higgs-boson self-coupling at the $5\sigma$ level (statistical uncertainty only). An even higher luminosity is required for an accurate measurement of this coupling.

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References


