Neutrino reactions in the low-y region

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The physics of nonscaling components in the region of low energy transfer (low-y region) is described. The following neutrino-induced processes were considered: resonance production, quasielastic scattering, and coherent meson production off nuclei or nuclear fragments. It is shown that the total exclusive cross section in a certain kinematical domain is energy independent at high energies (above 20 GeV). This fact can, in principle, be used for relative normalization of the neutrino flux.

Neutrino-nucleon scattering processes at low-energy transfer have recently been the subject of renewed interest, both from experimental and theoretical points of view.[1,2] At very high energies, for example, it becomes increasingly difficult to monitor the neutrino flux so that alternative methods for measuring the total cross section are needed. One possibility is to look for measurable quantities which are independent of the primary energy and thus may be used for relative normalization of the cross section at different energies.

A quantity which, at first sight, appears to be energy independent is the deep-inelastic differential cross section $(1/E)d\sigma/dy$ when taken at the limiting value $y=0$ $(y=\nu/E)$. In the QCD framework this cross section reads

$$\frac{d\sigma^{\nu,\bar\nu}}{dy}=\frac{G^2\cos^2\theta_c ME}{\pi}\int_0^1 dx\left[(1-y)F_2(x,Q^2)+\frac{y^2}{2}2xF_1(x,Q^2)\pm y\left[1-\frac{y}{2}\right]xF_3(x,Q^2)\right]. \quad (1)$$

Extrapolating to $y=0$ and neglecting the slow y variation implicit in the Q^2 dependence of the structure functions $(Q^2=2MExy)$ one finds that

$$\left.\frac{1}{E}\frac{d\sigma^{\nu,\bar\nu}}{dy}\right|_{y=0}=\frac{G^2\cos^2\theta_C M}{\pi}\int_0^1 dx\,F_2(x,Q_0^2)=\text{const}\;, \quad (2)$$

where charge symmetry was invoked in assuming $F_2^{\nu}(x,Q_0^2)=F_2^{\bar\nu}(x,Q_0^2)$ at some fixed Q_0^2.

However, this extrapolation is not devoid of problems. A decrease in y toward zero reduces the phase space, thus excluding more and more inelastic channels from the final hadronic state. At exactly $y=0$ the elastic channel is the only one which survives. Moreover, as $y\to 0$, a kinematical region is passed which is in fact not deep inelastic, as may be recalled from Fig. 1. For $y\le y_0$, with y_0 of the order of 2 GeV/E, perturbative QCD breaks down and the whole formalism has to be abandoned.

On the other hand, since very low y values necessarily correspond to low Q^2 and W values (cf. Fig. 1), the total cross section in this region must be entirely due to a few exclusive reactions whose contributions are theoretically

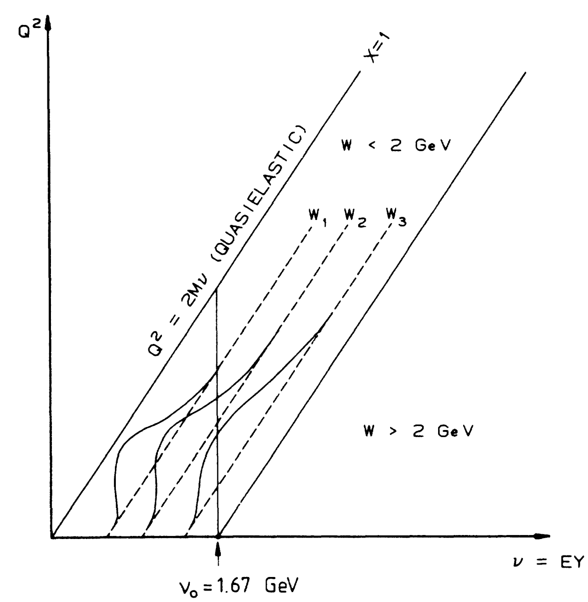

FIG. 1. Kinematical map: Q^2 versus $\nu=Ey$. The quasielastic boundary line separates the allowed from the forbidden domain. Resonances of fixed W populate the diagonal strip. The shapes of the double-differential cross sections $d^2\sigma/dQ^2d\nu$ (curved lines) are sketched along the lines of constant W (dotted). A cut at $\nu=\nu_0=1.67$ GeV (or corresponding y value) ensures that only resonances (and nonresonant background with $W\le 2$ GeV, if any) contribute to the left of it.

calculable. Therefore, our main goal is to present theoretical estimates for those reactions dominating the total cross section at very low y and to discuss their energy dependence in some detail. For convenience let us discuss matters in terms of the variable ν, which is, likewise, directly accessible to calorimeter experiments ($\nu = E_{\text{hadr}}$).

The first reaction which we consider that contributes to the cross section at low-y values is the quasielastic neutrino (antineutrino) nucleon scattering. Its differential cross section is usually expressed as[3]

$$\frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{dQ^2} = \frac{G^2\cos^2\theta_C M^2}{8\pi E^2}\left[A(Q^2) \pm B(Q^2)\frac{s-u}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right], \tag{3}$$

where G and θ_C denote the Fermi coupling constant and Cabibbo angle, respectively, and M is the nucleon mass. The kinematical variables are defined as usual:

$$Q^2 = 2E(E_\mu - P_\mu \cos\theta_{\mu\nu}) ,$$
$$(s-u) = 4EM - Q^2 - m_\mu^2 .$$

A, B, and C are known functions of Q^2 which contain the quasielastic vector and axial-vector form factors $F_V(Q^2)$, $F_M(Q^2)$, and $F_A(Q^2)$, respectively. While $F_V(Q^2=0) \equiv g_V = 1$ and $F_M(0) = \mu_p - \mu_n$ by virtue of the conserved-vector-current (CVC) hypothesis, the axial-vector form factor attains the value $g_A = -1.25$ at $Q^2=0$. It then follows from Eq. (3) that

$$\left.\frac{d\sigma_{\text{elast}}^{\nu}}{dQ^2}\right|_{Q^2=0} = \left.\frac{d\sigma_{\text{elast}}^{\bar{\nu}}}{dQ^2}\right|_{Q^2=0} = \frac{G^2\cos^2\theta_C}{2\pi}\left[1 + \left|\frac{g_A}{g_V}\right|^2\right] \tag{4}$$

is independent of the incident neutrino (antineutrino) energy. At sufficiently high energies the term with $(s-u)^2 \propto E^2$ in Eq. (3) is the dominant one and we find

$$\frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{dQ^2} \xrightarrow{E \gg M} \frac{2G^2\cos^2\theta_C}{\pi} C(Q^2) = \frac{G^2\cos^2\theta_C}{2\pi}\left[F_A^2(Q^2) + F_V^2(Q^2) + \frac{Q^2}{4M^2}F_M^2(Q^2)\right] \tag{5}$$

which also does not explicitly depend on E. It may, likewise, be written in the form

$$\frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{d\nu} = \frac{1}{E}\frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{dy} \xrightarrow{E \gg M} \frac{G^2\cos^2\theta_C M}{\pi}\left[F_A^2(\nu) + F_V^2(\nu) + \frac{\nu}{2M}F_M^2(\nu)\right], \tag{6}$$

since $Q^2 = 2M\nu = 2MEy$ for elastic reactions. The form factors are conventionally parametrized in the dipole form $F(Q^2) \propto 1/(1+m^2/Q^2)^2$, with the empirical mass parameters[4]

$$m_V = 0.84 \text{ GeV}/c^2, \quad m_A = 1.00 \text{ GeV}/c^2 .$$

Their rapid decrease with Q^2 guarantees both convergence and constancy of the integral $\sigma_{\text{elast}}^{\nu,\bar{\nu}} = \int_0^\infty dQ^2 (d\sigma_{\text{elast}}^{\nu,\bar{\nu}}/dQ^2)$ with energy at high energies, as may be easily inferred from Eqs. (5) or (6). Moreover, writing

$$\sigma_{\text{elast}}^{\nu,\bar{\nu}} = \int_0^{\nu_0} d\nu \frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{d\nu} + \int_{\nu_0}^{\infty} d\nu \frac{d\sigma_{\text{elast}}^{\nu,\bar{\nu}}}{d\nu} \tag{7}$$

the second integral is negligibly small compared to the first one for $\nu_0 > 1.5$ GeV, as can be seen from Fig. 5.

The next process to be considered is the resonance production characterized by an invariant hadronic energy $W \le W_0 = 2$ GeV. Since $\nu = (W^2 - M^2 + Q^2)/2M$, only final states with invariant energy W below W_0 are kinematically allowed, if ν is restricted to the region $\nu \le \nu_0 = (W_0^2 - M^2)/2M = 1.67$ GeV. Resonance excitation below $W_0 = 2$ GeV has recently been studied in connection with neutrino-induced single-pion production.[5] There are 18 well-established pion-nucleon resonances of $N(I=\frac{1}{2})$ and $\Delta(I=\frac{3}{2})$ types in the range between the one-pion threshold and $W=2$ GeV contributing to the πN cross section. Production of strange baryon resonances is Cabibbo suppressed and therefore negligible at the 5% level. Using the semirelativistic quark model of Feynman, Kislinger, and Ravndal[6] for resonance excitation, experimental masses, and partial widths,[7] together with the usual parametrization for the resonance transition form factors (for details see Refs. 8–10), the experimental data on single-pion production were nicely reproduced without invoking additional nonresonant background under the resonances (see Fig. 2). The overall

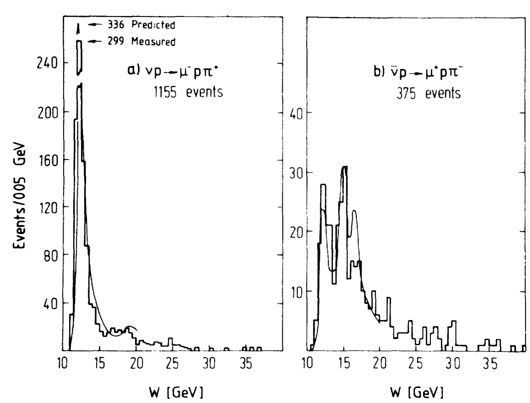

FIG. 2. W distribution for reactions (a) $\nu p \to \mu^- p \pi^+$ and (b) $\nu p \to \mu^+ p \pi^-$. The theoretical curves obtained from quark model calculations (Ref. 5) are compared with Big European Bubble Chamber (BEBC) data (Ref. 12). Theoretical predictions in (b) include nonresonant background which, however, can be disposed of without spoiling the overall agreement, if the transition form factors of higher resonances are chosen appropriately (for details see Ref. 8).

TABLE I. Neutrino-induced single-pion production cross sections at two different neutrino energies. Theoretical predictions based on the Feynman-Kislinger-Ravndal quark model for resonance excitation (Ref. 6) are compared with experimental results (quoted from Ref. 5).

Reaction channel	$\sigma^{\text{theor}}/10^{-40}$ cm^2 $E=2$	$E=20$	$\sigma^{\text{expt}}/10^{-40}$ cm^2 $E=2$	$E=20$	$\sigma^{\text{expt}}/\sigma^{\text{theor}}$ $E=2$	$E=20$
$\nu p \to \mu p \pi^+$	62.6	68	60±7	68±7	0.96±0.09	1.00±0.10
$\bar{\nu} n \to \bar{\mu} n \pi^-$	22.6	62.8	19±6	54±7	0.84±0.27	0.86±0.11
$\bar{\nu} p \to \bar{\mu} p \pi^-$	26.4	30	43±10	30±4	1.63±0.38	1.00±0.13
				36±6		1.20±0.20
$\nu p \to \nu p \pi^0$	10.5		11±4		1.05±0.38	
			Average		1.12±.29	1.02±.11

discrepancy is not worse than 10% (see Table I which relies on Ref. 5).

In our model calculation we have summed up all 18 resonances taking into account all possible decay channels (represented by the total resonance widths), not only decays into πN final states. We refer, however, to single-pion production at $W(\pi N) \leq 2$ GeV for judging the accuracy of the procedure: Since single-pion production at $W(\pi N) \leq 2$ GeV (neglecting nonresonant background, as mentioned above) differs from resonance production only through the replacement of $\Gamma(\text{Res}\to\text{all})$ by the measured[7] partial decay widths $\Gamma(\text{Res}\to\pi N)$, the level of accuracy must be the same.

The energy independence of the resonance cross section above a certain energy (for $E \geq 20$ GeV) is evident from Figs. 3 and 4. This saturation property of the cross section rests on the rapid decrease of the excitation form factors with increasing Q^2. For simplicity let us illustrate this by looking at one resonance of fixed mass M_R and zero width. Generalization to the realistic case of several overlapping resonances is straightforward and will not influence the arguments appreciably. The relevant differential cross section reads[5,9]

$$\frac{d\sigma_{\text{res}}^{\nu,\bar{\nu}}}{dQ^2} = \frac{G^2\cos^2\theta_C}{4\pi}\left[2uv\,|f_0|^2 + \left[\frac{Q^2}{\mathbf{q}^2}\right](u^2|f_\pm|^2 + v^2|f_\mp|^2)\right] \tag{8}$$

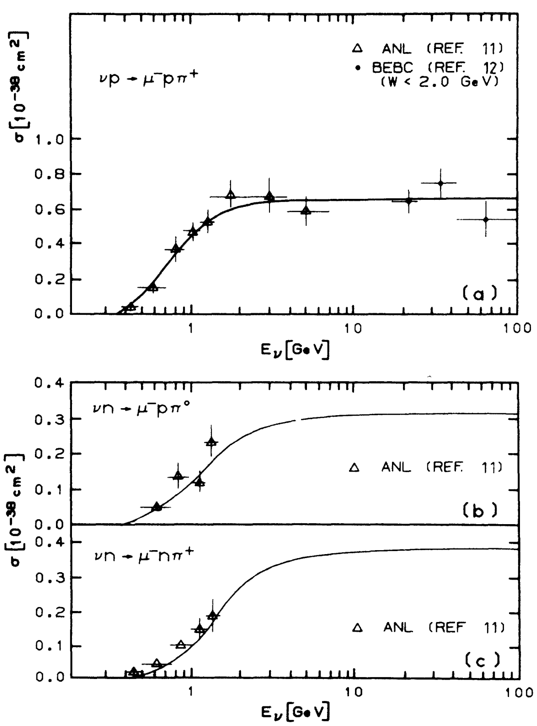

FIG. 3. Energy dependence of neutrino-induced single-pion production, as quoted from Ref. 5, for three different reaction channels (a)–(c) and the whole resonance region $M + m_\pi \leq W \leq 2$ GeV.

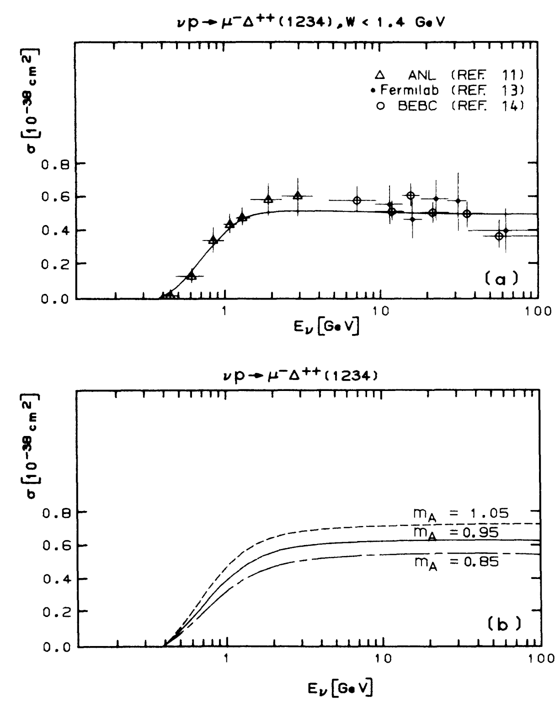

FIG. 4. Energy dependence of Δ^{++} excitation alone ($W \leq 1.4$ GeV). (a) Comparison of the model prediction ($m_A = 1.0$ GeV) with data. (b) Variation of the Δ cross section with axial-vector form-factor mass parameter m_A.

with

$$|f_\pm|^2=\tfrac{1}{2}(|f_{\pm 1}|^2+|f_{\pm 3}|^2)\,,$$
$$|f_0|^2=\tfrac{1}{2}(|f_{0+}|^2+|f_{0-}|^2)\,,$$

where $f_{\pm 1,\pm 3,0\pm}$ denote the helicity matrix elements for resonance production which depend on $Q^2=2M\nu-M_R^2+M^2$ or ν (or Ey), but not explicitly on E. The kinematical quantities u,v are given by

$$u,v=1-\frac{\nu}{2E}\pm\left[\frac{\nu^2+Q^2}{4E^2}\right]^2\rightarrow 1 \tag{9}$$

for large E and fixed ν. Likewise, $Q^2/|\mathbf{q}^2|=Q^2M_R^2/(\nu^2+Q^2)M^2$ depends on ν alone. Thus, at high energies one ends up with

$$\frac{d\sigma_{\rm res}^{\nu,\bar\nu}}{d\nu}\xrightarrow{E\gg M_R}\frac{G^2\cos^2\theta_C M}{\pi}\left[|f_0|^2+\left[\frac{Q^2}{\mathbf{q}^2}\right]\frac{1}{2}(|f_\pm|^2+|f_\mp|^2)\right]=\frac{G^2\cos^2\theta_C M}{\pi}f(\nu) \tag{10}$$

independent of the incident energy E.

A third class of processes contributing to the inclusive neutrino cross section at low-y values is the coherent single meson production off nuclei, in particular the coherent pion production. This cross section is relatively small. We thus refrain from considering higher mesons which will give still smaller contributions. Coherent pion production off nuclei (with nucleon number A) is described by[15]

$$\frac{d\sigma_{\rm coh}^{\nu,\bar\nu}}{d\nu}=\frac{G^2\cos^2\theta_C M}{2\pi^2}f_\pi^2A^2\left[1-\frac{\nu}{E}\right]\frac{1}{16\pi}[\sigma_{\rm tot}^{\pi N}(\nu)]^2\frac{1}{b}\int_0^1 dx\frac{e^{-b|t_{\rm min}(x,\nu)|}}{\left[1+\dfrac{2Mx\nu}{m_A^2}\right]^2}\,. \tag{11}$$

Integration with respect to ν leads to a total cross section which also tends to become constant at high incident energies. This is conceivable since the coherent reaction is still an exclusive process which is bound to saturate at high enough energies. The same holds true for the diffractive meson production off nucleons which contributes mainly at ν values above the resonance region and therefore will be neglected here.

The assumption is that the exclusive neutrino reactions described above are the only ones which can contribute substantially to the inclusive cross section for $\nu\le\nu_0=1.67$ GeV. In the restricted domain $\nu\le 0.5$ GeV, where at most one pion can be generated, we are certain of this, as long as no unconventional process enters the game. For the interval 0.5 GeV $\le\nu\le 1.67$ GeV we cannot claim it so confidently (for instance, some higher resonances beyond 2 GeV may still leak partly into the $W\le 2$ GeV region). The success of the resonance (and coherent model) description of single-pion production, however, shows that at least the dominant part of the inclusive cross section for $\nu\le 1.67$ GeV (corresponding to $W\le 2$ GeV) is correctly taken into account.

We have evaluated the cross sections according to Eqs. (3), (8), and (11). The results are displayed in Fig. 5. Numerical integration of the differential cross sections $d\sigma/d\nu$ for $\nu\le\nu_0=1.67$ GeV gives the following results for the proton, the neutron, and for an average nucleon target, respectively:

$$\begin{aligned}&\sigma^{\rm elast}(n)=0.85\times 10^{-38}\ {\rm cm}^{-2}\,,\\&\sigma^{\rm elast}(p)=0\,,\\&\sigma^{\rm elast}(N)=0.425\times 10^{-38}\ {\rm cm}^2\,,\end{aligned} \tag{12}$$

$$\begin{aligned}&\sigma^{\rm res}(n)=0.75\times 10^{-38}\ {\rm cm}^2\,,\\&\sigma^{\rm res}(p)=0.74\times 10^{-38}\ {\rm cm}^2\,,\\&\sigma^{\rm res}(N)=0.745\times 10^{-38}\ {\rm cm}^2\,,\end{aligned} \tag{13}$$

$$\sigma^{\rm coh}(N)=0.025\times 10^{-38}\ {\rm cm}^2\,. \tag{14}$$

In passing we note that the resonance cross section (13) is somewhat smaller than the sum of the cross sections given in Table IV of Ref. 5, because the restriction $\nu<\nu_0$ cuts off contributions from the higher-Q^2 domain. Adding up all contributions yields

$$\sigma(N)|_{\nu\le\nu_0=1.67\ {\rm GeV}}=1.20\times 10^{-38}\ {\rm cm}^2\,. \tag{15}$$

This may be written as

$$\sigma=\int_0^{\nu_0}\frac{d\sigma}{d\nu}d\nu=\left\langle\frac{d\sigma}{d\nu}\right\rangle\Delta\nu=\frac{1}{E}\left\langle\frac{d\sigma}{dy}\right\rangle\nu_0\,. \tag{16}$$

Hence the average value of the exclusive cross section at $y\rightarrow 0$ is

$$\frac{1}{E}\left\langle\frac{d\sigma}{dy}\right\rangle\Bigg|_{y\le y_0=1.67\ {\rm GeV}/E}=(0.72\pm 0.08)\times 10^{-38}\ {\rm cm}^2\ {\rm GeV}^{-1}\,, \tag{17}$$

where the error reflects the uncertainty in the resonance model, as shown in Table I, and a small uncertainty of the quasielastic data.[4] The above result is not far from the measured deep-inelastic differential cross section extrapolated to $y=0$ (see Ref. 16)

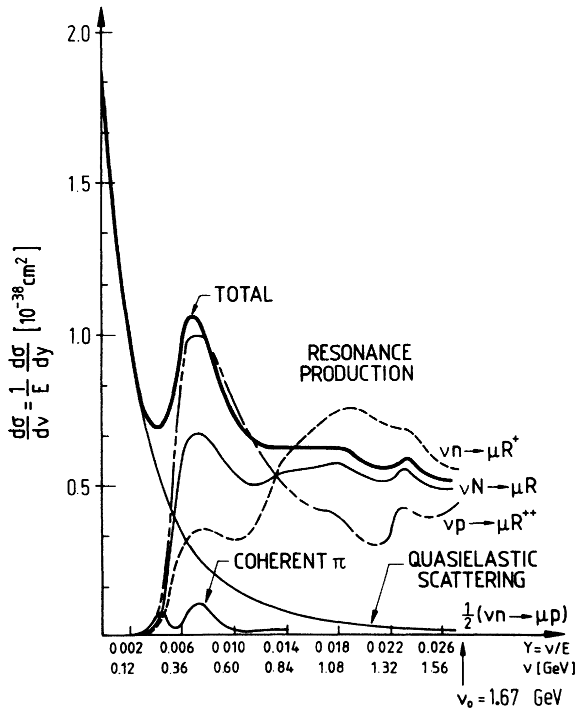

FIG. 5. The differential cross section $d\sigma/d\nu=(1/E)d\sigma/dy$ of the inclusive neutrino-nucleon scattering in the low-y region. The scales on the abscissa refer to y and $\nu=Ey$ with E chosen at 60 GeV. The contributions from elastic, resonant, and coherent processes are indicated separately. The resonance curve is obtained by averaging νn and νp reactions.

$$\left.\frac{1}{E}\frac{d\sigma^{\text{deep inelast}}}{dy}\right|_{y=0} =(0.79\pm0.015)\times10^{-38}\ \text{cm}^2\ \text{GeV}^{-1}\ . \qquad (18)$$

A reasonable agreement between (17) and (18) is expected on duality grounds. The Bloom-Gilman duality[17] requires the average exclusive (resonance) production at low Q^2 (corresponding to low y in our case) to represent the deep-inelastic cross section, rather than to make an additional contribution. There is no obvious reason why this concept should not work in neutrino-nucleon interactions to the same approximate extent as in electron-nucleon interactions. We should also mention that the average cross section (17) is predicted to increase slightly toward $y=0$, where the quasielastic contribution sticks out more prominently, as can be seen from Fig. 5. Whether this is noticeable or not depends on the experimental resolution in this region.

Considering the kinematical domain of very low energy transfer ν or low-y values ($y=\nu/E$), corresponding to an invariant hadronic energy $W<2$ GeV, we have computed the total exclusive cross section due to the quasielastic scattering, resonance excitation, and coherent single-meson production. Each of these processes becomes energy independent at high energies, i.e, beyond $E\gtrsim20$ GeV (except for a small part of the coherent meson production which saturates at still higher E values). Comparing event rates within this limited domain at various energies above $E=20-30$ GeV may thus be used for relative normalization of the neutrino flux. We believe that this work provides a basis for studying possible new physics in this kinematical region.

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[1]R. E. Blair, in *Neutrino '86: Neutrino Physics and Astrophysics,* proceedings of the Twelfth International Conference on Neutrino Physics and Astrophysics, Sendai, 1986, edited by T. Kitagati and H. Yuta (World Scientific, Singapore, 1986), p. 351.
[2]R. Belusevic and J. Smith, Phys. Rev. D **37**, 2419 (1988).
[3]See, e.g., C. H. Llewellyn Smith, Phys. Rep. **3C**, 263 (1972), and references therein.
[4]S. V. Belikov *et al.*, Z. Phys. A **320**, 625 (1985).
[5]D. Rein and L. M. Sehgal, Ann. Phys. (N.Y.) **133**, 79 (1981).
[6]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).
[7]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 1 (1986).
[8]D. Rein, Z. Phys. C **35**, 43 (1987).
[9]F. Ravndal, Nuovo Cimento **A18**, 385 (1973); Lett. Nuovo Cimento **3**, 631 (1972).
[10]F. Ravndal, Phys. Rev. D **4**, 1466 (1971).
[11]G. M. Radecky *et al.*, Phys. Rev. D **25**, 1161 (1982); J. Campbell *et al.*, Phys. Rev. Lett. **30**, 335 (1973).
[12]P. Allen *et al.*, Nucl. Phys. **B264**, 221 (1980).
[13]J. Bell *et al.*, Phys. Rev. Lett. **41**, 1008 (1978); **41**, 1012 (1978).
[14]P. Allen *et al.*, Nucl. Phys. **B176**, 269 (1983).
[15]D. Rein and L. M. Sehgal, Nucl. Phys. **B223**, 29 (1983).
[16]P. Auchincloss, Doctoral thesis, Columbia University 1987, unpublished; compare also F. Sciulli, in *Proceedings of the 1977 International Symposium on Lepton and Photon Interactions at High Energies,* edited by F. Gutbrod (Springer, Hamburg, 1977), p. 239.
[17]E. Bloom and F. Gilman, Phys. Rev. Lett. **25**, 1140 (1970).