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Sections indicated by an asterisk (*) lie somewhat out of the main line of presentation, and may be omitted in a first reading.

Preface

This self-contained text provides a comprehensive introduction to modern astrophysics and cosmology, at a level suitable for advanced undergraduates and graduate students. The physical and mathematical concepts essential for a thorough understanding of the classical theory of fields are carefully developed. Some relevant aspects of thermodynamics, statistical physics and elementary particle physics are reviewed. Considerable emphasis is placed on the description of experimental techniques and results.

Cosmology is the study of the large-scale structure of the universe, where “the universe” means all that exists in a physical sense. An ancient discipline that draws on almost every branch of physics,¹ cosmology has at last become a quantitative science where theories may be tested against observational data with an accuracy unimaginable only a decade or so ago. The empirical basis for our understanding of stellar and galactic structure and evolution, described in this book, has paved the way for the development of modern observational cosmology. The beginning of the new era in theoretical cosmology can be associated with the development of the gauge theories of weak, electromagnetic and strong interactions. Indeed, the subjects of elementary particles, astrophysics and cosmology have become inextricably interconnected, which has led to great advances in our understanding of the universe.

The foundation of modern cosmology is general relativity, a theory that has been tested and confirmed in a wide variety of terrestrial and extraterrestrial observations. For instance, the recent detection of the acoustic peaks in the angular power spectrum of the cosmic microwave background radiation is a great success for general-relativistic perturbation theory. General relativity is also central to the explanation of such frontier astrophysical phenomena as gravitational collapse, black holes, neutron stars, gravitational waves, active galactic nuclei, gravitational lensing, etc.

In this book, Einstein’s general theory of relativity is expounded using pri-

¹ For example, the observed temperature fluctuations in the cosmic microwave background radiation (CMB) depend on general relativity, fluid dynamics and the interaction of light with matter.

marily the “classical” tensor calculus, since this is easier to grasp than the language of differential forms and the exterior calculus of Cartan. However, to bridge the “mathematical culture gap” that exists between the old-fashioned tensor calculus and the modern differential geometry (now widely used in general relativity and other branches of physics), Cartan’s calculus is developed from first principles and applied to a few selected problems. Readers will also learn how the basic concepts of the ordinary tensor calculus can be introduced in an elegant, coordinate-free manner.

An overview of the well-established theoretical ideas and some of the most important experimental results in cosmology are presented in the first chapter. The physical and mathematical concepts at the heart of Einstein’s theories of special and general relativity are expounded in the next chapter. Those concepts are employed in the third chapter to develop the Friedmann–Lemaître cosmological model, the theory of gravitational lensing and general-relativistic perturbation theory. A number of different methods for establishing the extragalactic distance scale are also described. The fundamental principles of stellar structure and evolution are presented in the fourth chapter. In the final chapter, the Schwarzschild metric is derived from Einstein’s field equations, various experimental tests of general relativity are described, and an important equation for relativistic stellar models is obtained. The physics of black holes and experimental evidence for their existence are also discussed at length, as well as the subjects of gravitational radiation, post-Newtonian approximation, gravitomagnetism and spin precession in a gravitational field. This chapter includes an introduction to symmetries (isometries) of the metric, and a brief discussion of the Weyl tensor and conformal mapping. Each chapter is largely self-contained, and could form the basis for a textbook.

The theory of cosmic inflation offers an attractive resolution of some of the great puzzles in cosmology: why the universe is “flat” and uniform at large scales, and how the observed density variations arose. However, the inflationary paradigm is founded on the physics of quantum fields in curved space-time, a theoretical framework that is inconsistent at a fundamental level. For this reason, only the basic ideas behind inflationary cosmology are described (see Section 3.13).

Acknowledgements

For valuable comments on certain aspects of special and general relativity, astrophysics and cosmology, in particular with regard to their own work, I would like to thank John Bahcall, Matthias Bartelmann, Werner Benger, Jim Condon, Daniel Eisenstein, Francis Everitt, Laura Ferrarese, Andrea Ghez, Karl Glazebrook, Christopher Hirata, Hideo Kodama, Ofer Lahav, Takuya

Matsuda, Bohdan Paczynski, Jim Peebles, William Percival, Saul Perlmutter, Michael Perryman, Fritz Rohrlich, Subir Sarkar, Peter Schneider, George Smoot, Norbert Straumann, Max Tegmark, Kip Thorne, John Tonry, Robert Vessot, Joachim Wambsganss and Dan Watson.

For permission to reprint various plots, images and illustrations I am grateful to the individuals and groups mentioned in the figure captions.

I wish to express my special gratitude to Matthias Bartelmann, Etienne Forest, Keisuke Fujii and Andreas Müller for their interest and help.

Many thanks are also due to my commissioning editor Christoph von Friedeburg for his enthusiastic support, and the production team at Wiley-VCH for their help in preparing the manuscript for publication.

Above all, I thank my wife Vesna and children Tara, Una and Dejan for their love, encouragement and forbearance, without which this book could never have been completed.

Tsukuba,
January 2007

Radoje Belusevic