# COMET Technical Note No. 11 Beam optics calculation of AC dipole section at proton beam line by TRANSPORT 

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An AC dipole section will be used as an external extinction device in COMET experiment. An optics calculation of the AC dipole section for a proton beam line was done with TRANSPORT. In this calculation, an aperture of two AC dipoles was considered and quadrupole magnets were optimized so that a proton beam can be transmitted through the dipoles with a little loss assuming a emittance of $\epsilon_{x}=5 \pi \mathrm{~mm} \cdot \mathrm{mrad}$ and $\epsilon_{y}=10 \pi$ $\mathrm{mm} \cdot \mathrm{mrad}$. A deflection of the proton beam by the AC dipole field was also calculated. The result shows a good separation of the proton beam in empty bucket from main bunch by the AC dipole system.

In COMET experiment, 8 GeV proton beam at a main ring (MR) of J-PARC will be used to produce pions. In the MR, the proton bunches are distributed every two buckets and are extracted by a slow extraction method. The proton beam will be transported from the MR to a pion production target in an experimental hall. This transport line is referred to a proton beam line. In COMET experiment, it is important to improve the beam-extinction-level by two orders. For this purpose, an AC dipole system will be installed into the proton beam line as an external extinction device.
The AC dipole system consists of two AC dipoles. They are operated with sinusoidal coil current and the phase difference between the two AC dipoles is $\pi$. A schematic layout of optical components at the AC dipole section is shown in Fig. 1. Where, $\mathrm{Q}_{n}$ and $\mathrm{B}_{n}$ indicate quadrupole magnets and dipole magnets (AC dipoles), respectively. A center of the proton


FIG. 1: Schematic view of AC dipole section
bunch in longitudinal direction are transmitted the AC dipoles when the first AC dipole field is zero. On the other hand, protons arrived at the AC dipole in different timing are swept. The sweeping angle depends on the phase of $B$ field when the protons arrived at the AC dipole. Here, it is assumed that the bends occur in vertical ( $y-$ ) direction. The protons deflected by a large angle are stopped at the collimator. Protons deflected by a small angle are transmitted through the collimator, and then they are swept in opposite direction at the second AC dipole. Since the second AC dipole generate compensate field the deflection at the first AC dipole, the proton beam moves to an original beam axis.

The $B$ field at the first AC dipole is given by,

$$
\begin{align*}
B(t) & =B_{0} \sin (2 \pi f t),  \tag{1}\\
B_{0} & =600 \quad \text { Gauss, }  \tag{2}\\
T & =1.3 \quad \mu \mathrm{sec},  \tag{3}\\
f & =\frac{1}{2 T}=0.385 \quad \mathrm{MHz},  \tag{4}\\
\tau & =100 \quad \text { nsec, },  \tag{5}\\
B(\tau / 2) & =72.3 \quad \text { Gauss, } \tag{6}
\end{align*}
$$

here, $T$ and $\tau$ are a space between the beam pulse and bunch width, respectively (Fig. 2). It is assumed that the center of a main bunch in longitudinal direction is transmitted through the AC dipole at $t=0$.
The apertures of the two AC dipoles are $\pm 5 \mathrm{~mm}$ in $x$-direction and $\pm 25 \mathrm{~mm}$ in $y$-direction for each. The length of the AC dipoles are 2 m . To transmit the proton beam through the aperture, beam should form a waist at the AC dipoles in $x$-direction, on the other hand, the beam should be parallel in $y$-direction. And then, at the collimator the beam should be focused in $y$-direction (Fig. 1).


FIG. 2: Relationship between beam bunch and AC field. In this calculation, it is assumed that bunch width, $\tau=100$ nsec and space between the beam pulse, $T=1.3 \mu \mathrm{sec}$.

By using TRANSPORT, proton beam optics was calculated at the AC dipole section so that it meet the above focusing conditions. In this calculation, the optics elements are aligned in symmetry with respect to a center plane between $\mathrm{Q}_{4}$ and $\mathrm{Q}_{5}$ as shown in Fig.1. The collimator is positioned at downstream side from the symmetry plane. This collimator layout make smaller beam size at the second AC dipole in $y$-direction than a layout where the collimator is aligned at the symmetry plane (Fig. 3). Momentum of the proton beam is $8.889 \mathrm{GeV} / \mathrm{c}$ and momentum spread is $0.003(0.3 \%)$. This momentum


FIG. 3: $y$ vs accumulated length. Left: The collimator is installed at the symmetry plane. Right: The collimator is installed at downstream side from the symmetry plane by 0.5 m .
spread is involved into bending angles at the AC dipoles in this calculation. Length of the drift space and optics elements are fixed, and these values are shown in Table I.
Field gradients of the quadrupoles, $\mathrm{Q}_{1}, \mathrm{Q}_{2}, \mathrm{Q}_{3}$, and $\mathrm{Q}_{4}$ are adjusted by TRANSPORT under constrains as follows.

$$
\begin{align*}
& R_{11}=0.0 \quad \text { at } \mathrm{B}_{1},  \tag{7}\\
& R_{44}=0.0 \quad \text { at } \mathrm{B}_{1},  \tag{8}\\
& R_{21}=0.0 \quad \text { at the collimator center, }  \tag{9}\\
& R_{34}=0.0 \quad \text { at the collimator center. } \tag{10}
\end{align*}
$$

Here, $R_{i j}$ are components of transfer matrix. These constrains mean that the beam are transmitted with parallel to point (point to parallel) from initial position to the first AC dipole center and point to parallel (parallel to point) from the first AC dipole center to the collimator center in $x(y)$-direction.

The initial beam profiles were optimized under the beam emittance of $\epsilon_{x}=5 \pi \mathrm{~mm}$ mrad and $\epsilon_{y}=10 \pi \mathrm{~mm}$ mrad so that the beam can be transmitted through the AC-dipole apertures. The optimized initial beam parameters were $\delta x=0.02 \mathrm{~m}$, $x^{\prime}=0.00025 \mathrm{rad}, \delta y=0.003 \mathrm{~m}$, and $y^{\prime}=0.00333 \mathrm{rad}$.

The optics calculations were done for three different magnetic fields of the AC dipole, $B_{\mathrm{AD}}=0,72.3$, and 600 Gauss . A magnetic field at $B_{2}$ was set to inverse value of that of $B_{1}$ in these calculations.

| Position |  |
| :---: | :---: |
| $L_{1}$ | 1.5 m |
| $L_{2}$ | 0.5 m |
| $L_{3}$ | 1.0 m |
| $L_{4}$ | 1.0 m |
| $L_{5}$ | 0.5 m |
| $L_{6}$ | 1.5 m |
| $L_{7}$ | 0.5 m |
| $L_{Q}$ | 1.5 m |
| $L_{A}$ | 2.0 m |
| $L_{C}$ | 1.0 m |

TABLE I: Length of drift spaces and optics elements.

Calculated beam profile, position of beam center, and beam ellipse are shown in Fig. 4, 5, and 6, respectively. In Figs. 4, and $6, x(y)$ means a distance from a beam center, $x_{c}\left(y_{c}\right)$. The values of positions of beam center, $x_{c}, y_{c}$, and beam sizes, $\delta x, \delta y$, at several positions are shown in table II. The solid lines at $x=0.005 \mathrm{~m}(y=0.025 \mathrm{~m})$ in upper (lower) figures in Fig. 4 indicate pole width of the $A C$ dipoles $\left(B_{1}\right.$ and $\left.B_{2}\right)$. It is found that the profiles of proton beam at the two AC dipole are smaller than the pole width of the AC dipoles in $x$ - and $y$-direction at $B_{\mathrm{AD}}=0$ Gauss. Bending angle by the AC dipole was 4.05 mrad at $B_{\mathrm{AD}}=600$ Gauss in this TRANSPORT calculation. This bending angle is same as an analytical calculation as follows.

$$
\begin{align*}
\theta & =\frac{0.300 B[\mathrm{~T}] L[\mathrm{~m}]}{p[\mathrm{GeV} / c]}  \tag{11}\\
& =0.00405 \mathrm{rad} \tag{12}
\end{align*}
$$

Dispersion at the AC dipole is so small for proton beam with momentum spread of $0.3 \%$ that there are no difference in the beam envelopes between these three B fields.

In Fig. 7, beam ellipses, $y-y_{c}$ vs $y^{\prime}$ at the collimator are shown with several AC dipole fields. In this figure, black, red, and green circles indicate $\mathrm{B}_{\mathrm{AD}}=0,72.3$, and 600 Gauss, respectively. When $B_{\mathrm{AD}}$ of 72.3 Gauss is applied at AC dipoles, the deflection of the beam center at the collimator is 3.2 mm while the half beam width is 3.4 mm . On the other hand, when $B_{\mathrm{AD}}$ of 600 Gauss is applied at AC dipoles, the deflection is 26 mm at the collimator while the half beam width is 3.4 mm (Fig.7). So if the aperture limit of the collimator is $\pm 7 \mathrm{~mm}$ in $y$-direction, we can stop protons in empty bucket while protons which are distributed on main bunch in length of 100 nsec will be passed through the aperture of collimator.

At $B_{\mathrm{AD}}=72.3$ Gauss the deflection of beam at the second AC dipole is 7.25 mm and the half beam size is 18.93 mm . An outer side of beam will pass out of the aperture of the second AC dipole by 1 mm in $y$-direction. This condition may be improved by adjusting optics system of Q magnets which are located at downstream side of the collimator. However at present, it is unknown whether this adjustment is important or not because 1 mm is very small comparing to full width of beam size $(=38 \mathrm{~mm})$. So, in this time, further adjustment of the optics is not done.


FIG. 4: Upper (Lower): $x(y)$ vs accumulated length. Left: $B_{\mathrm{AD}}=0$ Gauss, center: $B_{\mathrm{AD}}=72.3$ Gauss, right: $B_{\mathrm{AD}}=600$ Gauss.


FIG. 5: Beam center vs accumulated length. Left: $B_{\mathrm{AD}}=0$ Gauss, center: $B_{\mathrm{AD}}=72.3$ Gauss, right: $B_{\mathrm{AD}}=600$ Gauss. Upper: x direction, lower: y direction.


FIG. 6: Left (Right): Beam ellipses in $x(y)-x^{\prime}\left(y^{\prime}\right)$ phase space.

Field gradients for each Q magnet in this calculation are
Q1: 63.6038
[kG/m],
Q2 : - 66.0482
[kG/m],
Q3 : - 62.8334
[kG/m],
[kG/m].
(16)

This field gradient is so small that pole tips of the magnet is not magnetically saturated.


FIG. 7: Beam ellipse in $\left(y+y_{c}\right)-y^{\prime}$ at the collimator.
$\left.\begin{array}{|l|ll|c|c|c|}\hline \mathrm{L}[\mathrm{m}] & \delta x[\mathrm{~m}] & \delta y[\mathrm{~m}] & x_{c}[\mathrm{~m}] & \begin{array}{c}y_{c}[\mathrm{~m}] \\ B_{\mathrm{AD}}=0 \mathrm{Gauss}\end{array} & \begin{array}{c}y_{c}[\mathrm{~m}] \\ B_{\mathrm{AD}}=72.3 \mathrm{Gauss}\end{array}\end{array} \begin{array}{c}y_{c}[\mathrm{~m}] \\ B_{\mathrm{AD}}=600 \mathrm{Gauss}\end{array}\right]$

TABLE II: Calculated beam size and center position for several $B_{\mathrm{AD}} \mathrm{s} . \delta x, \delta y$ and $x_{c}$ were same for all $B_{\mathrm{AD}} \mathrm{s}$.

