W_L W_L scattering in Higgsless models: Identifying better effective theories

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Composite Higgs/Higgsless models and related topics toward LHC era
Outline

1. Introduction
2. Four-Site Higgsless Model
3. Comparison between continuum and three-site Higgsless models
4. Summary
**Introduction**

Three-Site Higgsless model itself is a perfectly consistent model based on a 4-D gauged non-linear $\sigma$ theory

Here, I consider

- how well the Three-Site Higgsless model performs as a low energy effective model of 5D Higgsless models
- and which modifications can make it more representative
“Continuum limit” of the three-site model

Continuum limit

Deconstruction with many sites

Three site model
“Continuum limit” of the three-site model

Continuum limit

Deconstruction with many sites

Three site model

SU(2) × SU(2)
“Continuum limit” of the three-site model

Continuum limit

Deconstruction with many sites

Three site model

SU(2) × SU(2)

ψL0
ψL1
ψR1

uR2
dR2

fermion coupling to U(1) looks like non-local in the continuum limit...
Minimal four-site Higgsless model

Continuum limit

Deconstruction with many sites

Deconstruction with few sites

$SU(2) \times SU(2) \times U(1)$
Minimal four-site Higgsless model

Continuum limit

Deconstruction with many sites

Deconstruction with few sites

SU(2) × SU(2) × U(1)

ψ_{L0} ψ_{L1} ψ_{R1} u_{R2} d_{R2}

local in the continuum
Minimal four-site Higgsless model

\[ g \quad f \quad \Sigma_1 \quad g' \quad f' \quad \Sigma_2 \quad \tilde{g} \]

\[ \uparrow \text{site - 0} \quad \uparrow \text{site - 1} \quad \uparrow \text{site - 2} \quad \uparrow \text{site - 3} \]

SU(2) \_0 \quad SU(2) \_1 \quad U(1) \_2 \quad U(1) \_3
## Minimal four-site Higgsless model

The model is represented by a network of sites, each associated with a specific symmetry group:

- **SU(2)₀** at site -0
- **SU(2)₁** at site -1
- **U(1)₂** at site -2
- **U(1)₃** at site -3

### Table

<table>
<thead>
<tr>
<th>Group</th>
<th>$Q_{L₀}$, $L_{L₀}$</th>
<th>$Q_{L₁}$, $Q_{R₁}$, $L_{L₁}$, $L_{R₁}$</th>
<th>$u_{R₂}$, $d_{R₂}$, $ν_{R₂}$, $ℓ_{R₂}$</th>
<th>$Σ₁$</th>
<th>$Σ₂$</th>
<th>$σ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2)₀</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SU(2)₁</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>U(1)₂</td>
<td>$Y_{SM}$</td>
<td>0</td>
<td>$Y_{SM}$</td>
<td>$\left( \begin{array}{c} +\frac{1}{2} \ -\frac{1}{2} \end{array} \right)$</td>
<td>$q \left( = \frac{1}{6} \right)$</td>
<td></td>
</tr>
<tr>
<td>U(1)₃</td>
<td>0</td>
<td>$\frac{1}{6}$ (Q); $-\frac{1}{2}$ (L)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\tilde{q} \left( = q = \frac{1}{6} \right)$</td>
</tr>
</tbody>
</table>
**Minimal four-site Higgsless model**

\[
\begin{align*}
\mathcal{L} &= \frac{f^2}{4} \text{Tr}(D_\mu \Sigma_1)(D^\mu \Sigma_1) + \frac{f^2}{4} \text{Tr}(D_\mu \Sigma_2)(D^\mu \Sigma_2) + \frac{f^2}{8} \text{Tr}(D_\mu \sigma)(D^\mu \sigma) \\
D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 + ig W_{0\mu}^a \frac{\tau^a}{2} \Sigma_1 - i\tilde{g} \Sigma_1 W_{1\mu}^a \frac{\tau^a}{2} \\
D_\mu \Sigma_2 &= \partial_\mu \Sigma_2 + i\tilde{g} W_{1\mu}^a \frac{\tau^a}{2} \Sigma_2 - ig' \Sigma_2 B_{2\mu} \frac{\tau^3}{2} \\
D_\mu \sigma &= \partial_\mu \sigma + ig' q B_{2\mu} \sigma - i\tilde{g}' \tilde{q} \sigma B_{3\mu},
\end{align*}
\]

**Lagrangian for the link fields**
Minimal four-site Higgsless model

\[ \mathcal{L}_{\text{quark}} = M \left[ \varepsilon_L \bar{Q}_L \Sigma_1 \sigma Q_R + \bar{Q}_R Q_L + \bar{Q}_L \Sigma_2 \left( \varepsilon_{uR} \sigma^* \left( u_{R2} \right) \right) + \varepsilon_{dR} \right] \]  

\[ \mathcal{L}_{\text{lepton}} = M \left[ \varepsilon_L \bar{L}_L \Sigma_1 (\sigma^*)^3 L_R + \bar{L}_R L_L + \bar{L}_L \Sigma_2 \left( \varepsilon_{\nu R} \sigma^3 \left( \nu_{R2} \right) \right) + \varepsilon_{\ell R} \right] \]
Minimal four-site Higgsless model

Charged gauge sector: Unchanged
Minimal four-site Higgsless model

Neutral gauge sector: An extra $U(1)$

\[
\mathcal{M}_Z^2 = \frac{\tilde{g}^2 v^2}{2} \begin{pmatrix}
  x^2 & -x & 0 & 0 \\
  -x & 2 & -xt & 0 \\
  0 & -xt & x^2 t^2 \left(1 + q^2 \left[\frac{f'}{f}\right]^2\right) & -xt u q^2 \left[\frac{f'}{f}\right]^2 \\
  0 & 0 & -xt u q^2 \left[\frac{f'}{f}\right]^2 & u^2 q^2 \left[\frac{f'}{f}\right]^2
\end{pmatrix}
\]
Minimal four-site Higgsless model

\[
\begin{align*}
\frac{1}{e^2} &= \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g''^2} + \frac{1}{\tilde{g}'^2} = \frac{1}{g^2 s^2} \left[ 1 + s^2 \left( 1 + \frac{1}{u^2} \right) x^2 \right] \\
M_Z^2 &= \frac{g^2 v^2}{4} \left[ (1 + t^2) - \left\{ \frac{(1 - t^2)^2}{4} + \frac{t^4}{u^2} \right\} x^2 + O(x^4) \right] \\
\left( \frac{\tilde{g}'}{\tilde{g}} \equiv u \right)
\end{align*}
\]
Z-standard weak mixing angle

\[ s_Z^2 c_Z^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_Z^2} \]

\[ = s^2 c^2 \left[ 1 + (c^2 - s^2) \left\{ \left( 1 - \frac{1}{4c^2} \right) - \frac{t^2}{u^2} \right\} x^2 + O(x^4) \right] \]

\[ s_Z^2 = s^2 + \Delta, \quad c_Z^2 = c^2 - \Delta, \]

\[ \Delta = \left\{ s^2 \left( c^2 - \frac{1}{4} \right) - \frac{s^4}{u^2} \right\} x^2 + O(x^4). \]
The expressions of $Z$-wavefunctions at site 0 and 1 in terms of physical quantities reproduce the three-site expression at next to leading order

\begin{align*}
v^0_Z &= c \left[ 1 + c^2 \left\{ -\frac{1 + 2t^2 - 3t^4}{8} + \frac{t^4}{2u^2} \right\} x^2 + O(x^4) \right], \\
&= c_Z \left[ 1 + \frac{\Delta}{2c^2} \right] \left[ 1 + c^2 \left\{ -\frac{1 + 2t^2 - 3t^4}{8} + \frac{t^4}{2u^2} \right\} x^2 + O(x^4) \right], \\
&= c_Z \left[ 1 - c^2 \frac{1 - t^2 - 2t^4}{8} x^2 + O(x^4) \right].
\end{align*}

\begin{align*}
v^1_Z &= c_Z \frac{1 - s_Z^2/c_Z^2}{2} x - \frac{2 - 9c^2 + 6c^4}{16c^3} x^3 + O(x^5).
\end{align*}
Expressions of several multi-gauge boson couplings unchanged

\[
g_{zw\bar{w}} = g(v_{W}^0)^2(v_{Z}^0) + \tilde{g}(v_{W}^1)^2(v_{Z}^1) = e \frac{c_{Z}}{s_{Z}} \left( 1 + \frac{1}{8c_{Z}^2} x^2 + O(x^4) \right),
\]

\[
g_{zw\bar{w}'} = g(v_{W}^0)(v_{W}^0)(v_{Z}^0) + \tilde{g}(v_{W}^1)(v_{W}^1)(v_{Z}^1) = -\frac{ex}{4s_{Z}c_{Z}} \left( 1 + \frac{s_{Z}^2}{4c_{Z}^2} x^2 + O(x^4) \right),
\]

\[
g_{\bar{w}ww} = g^2(v_{W}^0)^4 + \tilde{g}^2(v_{W}^1)^4 = \frac{e^2}{s_{Z}^2} \left( 1 + \frac{5}{16} x^2 + O(x^4) \right).
\]
Expressions of several multi-gauge boson couplings unchanged

Ideal delocalization of fermion works almost in the same way as in the case of three-site model
Expressions of several multi-gauge boson couplings unchanged

Ideal delocalization of fermion works almost in the same way as in the case of three-site model

Constraints/Signatures unchanged
Z’ and Z”

The existence of Z” cannot be captured by the three-site model?
Z’ and Z”

The existence of Z” cannot be captured by the three-site model?

Let’s see
Z’ and Z” in extra dimensional models are almost degenerate

\[ u^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \]

a choice in four-site model which reproduce almost degenerate Z’ and Z”
\( Z' \) and \( Z'' \) in four-site model

\[
\left( u^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \right)
\]

\( Z' \) (\( Z'' \)) mass

\[
M_{Z',Z''}^2 = \tilde{g}^2 v^2 \left[ 1 + \frac{1}{8u^2} \left( u^2 + t^2 (4 + u^2) \mp w \right) x^2 + O(x^4) \right]
\]

\[
w \equiv \sqrt{u^4 + 2t^2 u^2 (-4 + u^2) + t^4 (4 + u^2)^2}
\]
\[ Z' \text{ and } Z'' \text{ in four-site model} \quad \left( u^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \right) \]

**Z' (Z'') coupling to WW**

\[
g Z' W W (Z'' W W) = g(v_W^0)^2 v_{Z'}(Z'') + \tilde{g}(v_W^1)^2 v_{Z'}(Z'')
\]

\[
= \frac{e}{2s_Z} \frac{\sqrt{2} t^2 u}{\sqrt{w\{w \pm (u^2 + t^2[-4 + u^2])\}}} x + O(x^3)
\]

Looks quite different from the three-site model...

\[
(g Z' W W)_{\text{three-site}} = \frac{e}{4s_Z} x + O(x^3)
\]
\[ Z' \text{ and } Z'' \text{ in four-site model} \quad \left( u^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \right) \]

\textbf{Z' (Z'')} coupling to WW

\[
g_{Z'WW}(Z''WW) = g (v^0_W)^2 v^0_{Z'(Z'')} + \tilde{g} (v^1_W)^2 v^1_{Z'(Z'')} \\
= \frac{e}{2s_Z} \frac{\sqrt{2} t^2 u}{\sqrt{w\{w \pm (u^2 + t^2 [-4 + u^2])\}}} x + O(x^3) 
\]

But, there is a relation between three couplings:

\[
(g^2_{Z'WW} + g^2_{Z''WW})_{\text{four-site}} = (g^2_{Z'WW})_{\text{three-site}} = \frac{e^2 x^2}{16s_Z^2} [1 + O(x^2)]
\]
\(Z' \text{ and } Z'' \text{ in four-site model} \quad \left( u^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \right) \)

**Z' (Z'') coupling to WW**

\[
g_{Z'WW(Z''WW)} = g(v_W^0)^2 v_{Z'}^0 + \tilde{g}(v_W^1)^2 v_{Z'}^1
\]

\[
= \frac{e}{2 s_Z} \frac{\sqrt{2} t^2 u}{\sqrt{w \left\{ w \pm (u^2 + t^2 [-4 + u^2]) \right\}} } x + O(x^3)
\]

**But, there is a relation between three couplings:**

\[
\left( g_{Z'WW}^2 + g_{Z''WW}^2 \right)_{\text{four-site}} = \left( g_{Z'WW}^2 \right)_{\text{three-site}} = \frac{e^2 x^2}{16 s_Z^2} \left[ 1 + O(x^2) \right]
\]

The pair of degenerate Z' and Z'' bosons in four-site model jointly couple to W-boson pairs like the single Z' boson of the three-site model.
Four-Site model \[
\left( \frac{u^2 q^2 \left( \frac{f'}{f} \right)^2}{2} = 2 \right)
\]
(SU(2) × SU(2) × U(1) 5D model)

and

Three-Site model

(SU(2) × SU(2) 5D model)

are phenomenologically quite similar

Three-Site model can be used as an effective model not only for SU(2) × SU(2) 5D model but also for SU(2) × SU(2) × U(1) 5D model
Four-Site model \( \left( w^2 q^2 \left( \frac{f'}{f} \right)^2 = 2 \right) \)
( \( SU(2) \times SU(2) \times U(1) \) 5D model )

and

Three-Site model
( \( SU(2) \times SU(2) \) 5D model )

are phenomenologically quite similar

We concentrate on the comparison between

Three-Site model and \( SU(2) \times SU(2) \) 5D model
Comparison between continuum and three-site Higgsless models

- $SU(2) \times SU(2)$ 5D model with a flat extra dimension
- $SU(2) \times SU(2)$ 5D model with a warped extra dimension
- Three-site Higgsless model
$SU(2)_L \times SU(2)_R$ Higgsless Model with a flat extra dimension

$$0 \leq z \leq \pi R$$

$$S_{5D} = \int_0^{\pi R} dz \int d^4x \frac{1}{g_5^2} \left[ -\frac{1}{4} W_{\mu\nu}^{L\alpha} W_{\alpha\beta}^{L\alpha} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{L\alpha} W_{\nu z}^{L\alpha} \eta^{\mu\nu} \right]$$

$$+ \frac{1}{g_5^2} \left[ -\frac{1}{4} W_{\mu\nu}^{Ra} W_{\alpha\beta}^{Ra} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Ra} W_{\nu z}^{Ra} \eta^{\mu\nu} \right]$$

We introduce brane localized kinetic terms for $SU(2)_L$ and $U(1)_Y$ at $z = 0$ to achieve $M_W/M_{W'} \ll 1$

$$S_{z=0} = \int_0^{\pi R} dz \int d^4x \delta(z-\epsilon) \left[ -\frac{1}{4g_0^2} W_{\mu\nu}^{L\alpha} W_{\alpha\beta}^{L\alpha} \eta^{\mu\alpha} \eta^{\nu\beta} - \frac{1}{4g_Y^2} W_{\mu\nu}^{R3} W_{\alpha\beta}^{R3} \eta^{\mu\alpha} \eta^{\nu\beta} \right]$$

$(\epsilon \to 0^+)$

Four free parameters : $R$, $g_5$, $g_0$, $g_Y$
$SU(2)_L \times SU(2)_R$ Higgsless Model with a warped extra dimension

\[ R \left( \equiv R' e^{-b/2} \right) \leq z \leq R' \]

\[ S_{5D} = \int_R^{R'} dz \frac{R}{z} \int d^4x \frac{1}{g_5^2} \left[ -\frac{1}{4} W^{La}_{\mu\nu} W^{La}_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W^{La}_{\mu z} W^{La}_{\nu z} \eta^{\mu\nu} \right] + \frac{1}{g_5^2} \left[ -\frac{1}{4} W^{Ra}_{\mu\nu} W^{Ra}_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W^{Ra}_{\mu z} W^{Ra}_{\nu z} \eta^{\mu\nu} \right] \]

We introduce brane localized kinetic terms for $U(1)_Y$ at $z = R$ in order to arrange non-trivial weak mixing angle ($M_W/M_{W'} \ll 1$ is achieved by taking $b \gg 1$)

\[ S_{z=0} = \int_R^{R'} dz \int d^4x \delta(z - R - \epsilon) \left[ -\frac{1}{4 g_Y^2} W^{R3}_{\mu\nu} W^{R3}_{\alpha\beta} \eta^{\mu\alpha} \eta^{\nu\beta} \right] \]

($\epsilon \to 0^+$)

Four free parameters: $R'$, $b$, $g_5$, $g_Y$
EW gauge sector of the three site model also has four free parameters

We choose four free parameters in each model to fix the values of four physical quantities (three EW quantities + $M_{W'}$)

We calculate other quantities and compare the results in each model to see how reasonable the three site Higgsless model is as a low energy effective theory of each 5D Higgsless model

We focus on triple-gauge-boson couplings
Triple-gauge-boson couplings of SM gauge bosons


\[
\mathcal{L}_{TGV} = -ie \frac{c_Z}{s_Z} [1 + \Delta \kappa_Z] W^+ W^- Z^{\mu\nu} - ie [1 + \Delta \kappa_\gamma] W^+ W^- A^{\mu\nu}
\]

\[
- ie \frac{c_Z}{s_Z} \left[ 1 + \Delta g^Z_1 \right] (W^{+\mu\nu} W^- - W^{-\mu\nu} W^+) Z \nu
\]

\[
- ie (W^{+\mu\nu} W^- - W^{-\mu\nu} W^+) A \nu ,
\]

\[\Delta \kappa_\gamma = 0, \quad \Delta \kappa_Z = \Delta g^Z_1\]

<table>
<thead>
<tr>
<th>\Delta g^Z_1</th>
<th>\text{Three Site}</th>
<th>\text{5D Flat}</th>
<th>\text{5D Warped}</th>
</tr>
</thead>
<tbody>
<tr>
<td>\frac{1}{2c^2}</td>
<td>\frac{M_W^2}{M_{W'}^2}</td>
<td>\frac{x^2}{12c^2}</td>
<td>\frac{M_W^2}{M_{W'}^2}</td>
</tr>
<tr>
<td>\frac{1}{2c^2}</td>
<td>\frac{M_W^2}{M_{W'}^2}</td>
<td>\frac{3x^2}{16c^2}</td>
<td>\frac{M_W^2}{M_{W'}^2}</td>
</tr>
</tbody>
</table>

\[
(x_1 \simeq 2.4048)
\]

\[
\frac{\Delta g^Z_1}{\Delta g^Z_1 |_{\text{three-site}}} \simeq 0.61, \quad \frac{\Delta g^Z_1}{\Delta g^Z_1 |_{\text{flat-5D}}} \simeq 0.46
\]

Difference appears only in the next to leading order term
Triple-gauge-boson couplings which involve a heavy gauge boson

<table>
<thead>
<tr>
<th></th>
<th>Three Site</th>
<th>5D Flat</th>
<th>5D Warped</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{Z'WW}$</td>
<td>$-\frac{1}{2s} \left( \frac{M_W}{M_{W'}} \right)$</td>
<td>$-\frac{4\sqrt{2}}{\pi^2} \frac{e}{s} \left( \frac{M_W}{M_{W'}} \right)$</td>
<td>$-0.36 \left( \frac{M_W}{M_{W'}} \right)$</td>
</tr>
<tr>
<td>$g_{ZW'W}$</td>
<td>$-\frac{1}{2sc} \left( \frac{M_W}{M_{W'}} \right)$</td>
<td>$-\frac{4\sqrt{2}}{\pi^2} \frac{e}{sc} \left( \frac{M_W}{M_{W'}} \right)$</td>
<td>$-0.36\frac{1}{c} \left( \frac{M_W}{M_{W'}} \right)$</td>
</tr>
</tbody>
</table>

- Values of $g_{Z'WW}$, $g_{ZW'W}$ are suppressed by a factor of $M_W/M_{W'}$.
- Values of $g_{Z'WW}$, $g_{ZW'W}$ in flat and warped 5D Higgsless models are almost the same:
  \[
  \frac{g_{Z'WW}|_{\text{warped-5D}}}{g_{Z'WW}|_{\text{flat-5D}}} \approx \frac{g_{ZW'W}|_{\text{warped-5D}}}{g_{ZW'W}|_{\text{flat-5D}}} \approx 1
  \]
- Values of $g_{Z'WW}$, $g_{ZW'W}$ in the three site model are only about 13% smaller than those in 5D Higgsless models:
  \[
  \frac{g_{Z'WW}|_{\text{three-site}}}{g_{Z'WW}|_{\text{warped-5D}}} \approx \frac{g_{ZW'W}|_{\text{three-site}}}{g_{ZW'W}|_{\text{warped-5D}}} \approx 0.87
  \]
Why do $gzw'w$ and $gzw^w$ take similar values in different models?
Question

Why do $gzw\bar{w}$ and $gzw'w$ take similar values in different models?

- Sum rules
- Lowest KK mode dominance
Sum rules for the cancelation of $E^4$ term in $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$
\sum_i g_{Z(i)WW}^2 = g_{WWWW} - g_{ZWWW}^2 - g_{\gamma WW}^2
$$

Sum rules for the cancelation of $E^2$ term in $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$
3 \sum_i g_{Z(i)WW}^2 M_{Z(i)}^2 = 4g_{WWWW} M_W^2 - 3g_{ZWWW}^2 M_Z^2
$$

Note: In the case of deconstructed models, additional contributions from NG boson scattering appear

(i = 1 term of LHS)/RHS

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<th>5D Flat</th>
<th>5D Warped</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{Z'WW}^2 / g_{WWWW} - g_{ZWWW}^2 - g_{\gamma WW}^2$</td>
<td>1</td>
<td>$\frac{960}{\pi^6} \approx 0.996$</td>
<td>0.992</td>
</tr>
<tr>
<td>$\frac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WWWW} M_W^2 - 3g_{ZWWW}^2 M_Z^2}$</td>
<td>$\frac{3}{4}$</td>
<td>$\frac{96}{\pi^4} \approx 0.986$</td>
<td>0.986</td>
</tr>
</tbody>
</table>

* Remaining 1/4 comes from the NG-boson scattering amplitude
  Chivukula, Simmons He, Kurachi, Tanabashi, PRD 78: 095003, 2008
HLS generalization

\[ \mathcal{L} = -\frac{v^2}{4} \text{tr} \left[ \left( (D_\mu U_1)^\dagger U_1 - (D_\mu U_2)U_2^\dagger \right)^2 \right] \]

\[ -\frac{v^2}{4} a \text{tr} \left[ \left( (D_\mu U_1)^\dagger U_1 + (D_\mu U_2)U_2^\dagger \right)^2 \right] \]

\[ = \frac{v^2}{4} (1 + a) \text{tr} \left[ (D_\mu U_1)^\dagger (D_\mu U_1) \right] \]

\[ + \frac{v^2}{4} (1 + a) \text{tr} \left[ (D_\mu U_2)^\dagger (D_\mu U_2) \right] \]

\[ + \frac{v^2}{2} (1 - a) \text{tr} \left[ (D_\mu U_1)^\dagger U_1 (D_\mu U_2)U_2^\dagger \right] \]

a=1 reproduce the three-site model

HLS : Hidden Local Symmetry
Bando, Kugo, Uehara, Yamawaki, Yanagida, Phys. Rev. Lett. 54 (1985) 1215
Charged gauge sector

\[ M^2_W = \frac{\tilde{g}^2 v^2}{4} \begin{pmatrix} x^2 (1 + a) & -2xa \\ -2xa & 4a \end{pmatrix} \]

\[ M^2_{W'} = \tilde{g}^2 v'^2 a \begin{pmatrix} 1 + \frac{x^2}{4} + \frac{x^4}{16a} + \ldots \end{pmatrix} \]

\[ x^2 = 4a \left( \frac{M_W}{M_{W'}} \right)^2 + 8a^2 \left( \frac{M_W}{M_{W'}} \right)^4 + 4a^2(5a + 2) \left( \frac{M_W}{M_{W'}} \right)^6 + \ldots \]

\[ W^\mu = v_0^W W_0^\mu + v_1^W W_1^\mu \]

\[ v_0^W = 1 - \frac{x^2}{8} + \frac{(3a - 8)x^4}{128a} + \ldots \]

\[ v_1^W = \frac{x}{2} - \frac{(a - 2)x^3}{16a} + \ldots \]

\[ W'^\mu = v_0^{W'} W_0^\mu + v_1^{W'} W_1^\mu \]

\[ v_0^{W'} = -\frac{x}{2} - \frac{2 - a}{16a} x^3 + \ldots \]

\[ v_1^{W'} = 1 - \frac{x^2}{8} + \ldots \]
Neutral gauge sector

\[
M_Z^2 = \frac{\tilde{g}^2 v^2}{4} \left( \begin{array}{ccc}
    x^2(1 + a) & -2xa & -tx^2(1 - a) \\
    -2xa & 4a & -2txa \\
    -tx^2(1 - a) & -2txa & t^2 x^2(1 + a)
\end{array} \right)
\]

Photon

\[
A^\mu = \frac{e}{g} W_0^\mu + \frac{e}{\tilde{g}} W_1^\mu + \frac{e}{g'} B^\mu
\]

\[
\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2}
\]
Neutral gauge sector

\[
M_Z^2 = \frac{g^2 v^2}{4} \left( \begin{array}{ccc}
 x^2(1 + a) & -2xa & -tx^2(1 - a) \\
 -2xa & 4a & -2txa \\
 -tx^2(1 - a) & -2txa & t^2 x^2(1 + a)
\end{array} \right)
\]

Z boson

\[
M_Z^2 = \frac{g^2 v^2}{4c^2} \left[ 1 - \frac{x^2}{4} \frac{(c^2 - s^2)^2}{c^2} + \frac{(a - 1)(s^2 - c^2)^2 x^4}{32c^4 a} + \cdots \right]
\]

\[
Z^\mu = v_Z^0 W_0^\mu + v_Z^1 W_1^\mu + v_Z^2 B^\mu
\]

\[
v_Z^0 = c - \frac{c^3(1 + 2t^2 - 3t^4)}{8} x^2 + \cdots,
\]

\[
v_Z^1 = \frac{c(1 - t^2)}{2} x + \frac{c^3(1 - t^2) \left( 2(1 + t^2)^2 - a(1 + 6t^2 + t^4) \right)}{16a} x^3 + \cdots,
\]

\[
v_Z^2 = -s - \frac{sc^2(3 - 2t^2 - t^4)}{8} x^2 \cdots.
\]
Neutral gauge sector

\[ M^2_Z = \frac{\tilde{g}^2 v^2}{4} \left( \begin{array}{ccc}
  x^2(1 + a) & -2xa & -tx^2(1 - a) \\
  -2xa & 4a & -2txa \\
  -tx^2(1 - a) & -2txa & t^2x^2(1 + a)
\end{array} \right) \]

Z’ boson

\[ M^2_{Z'} = \tilde{g}^2 v^2 a \left[ 1 + \frac{x^2}{4c^2} + \frac{x^4(1 - t^2)^2}{16a} + \ldots \right] \]

\[ Z'^\mu = v^0_{Z'}W_0^\mu + v^1_{Z'}W_1^\mu + v^2_{Z'}B^\mu \]

\[ v^0_{Z'} = -\frac{x}{2} + \frac{-2a(1 - t^2) + a^2(1 + t^2)}{16a^2}x^3 + \ldots, \]

\[ v^1_{Z'} = 1 - \frac{(1 + t^2)}{8}x^2 + \ldots, \]

\[ v^2_{Z'} = -\frac{t}{2}x + \frac{t(2(1 - t^2) + a(1 + t^2))}{16a}x^3 + \ldots. \]
Multi-gauge-boson couplings

\[ g_{WWZ} = \frac{ec_Z}{s_Z} \left[ 1 + \frac{1}{8c^2} x^2 + O(x^4) \right] = \frac{ec_Z}{s_Z} \left[ 1 + \frac{a}{2c^2} \left( \frac{M_W}{M'_W} \right)^2 \right] , \]

\[ g_{WWW} = \frac{e^2}{s_Z^2} \left[ 1 + \frac{5}{16} x^2 + O(x^4) \right] = \frac{e^2}{s_Z^2} \left[ 1 + \frac{5a}{4} \left( \frac{M_W}{M'_W} \right)^2 \right] . \]

Ideal fermion delocalization

\[ \psi_L = \cos \varepsilon_L \psi_{L0} + \sin \varepsilon_L \psi_{L1} \]

\[ \varepsilon_L^2 \to \left[ \frac{x^2}{2} + \frac{x^4}{8a} + \frac{(1 - a)x^6}{32a^2} + \ldots \right] \]

\[ g_{L}^W = \frac{e}{\sqrt{1 - \frac{M_W^2}{M_Z^2}}} \left[ 1 + O(x^4) \right] \]

\[ g_{3L}^Z = \frac{eM_W}{M_Z \sqrt{1 - \frac{M_W^2}{M_Z^2}}} \left[ 1 + O(x^4) \right] \]

\[ g_{Y_L}^Z = -\frac{eM_Z}{M_W \sqrt{1 - \frac{M_W^2}{M_Z^2}}} \left[ 1 + O(x^4) \right] \]
**Z’WW, ZW’W couplings and sum rules**

\[
g_{Z'WW} = g v_Z^0 (v_W^0)^2 + \tilde{g} v_{Z'}^1 (v_W^1)^2 = -\frac{\sqrt{a}}{2} \frac{e}{s_Z} \left( \frac{M_W}{M_{W'}} \right) + \cdots.
\]

\[
g_{ZW'W} = g v_Z^0 v_W^0, v_W^0 + \tilde{g} v_{Z'}^1 v_W^1, v_W^1 = -\frac{\sqrt{a}}{2} \frac{e}{s_Z c_Z} \left( \frac{M_W}{M_{W'}} \right) + \cdots.
\]

<table>
<thead>
<tr>
<th>( \frac{g_{Z'WW}^2}{g_{W'WW}^2 - g_{ZW'WW}^2 - g_{Z'WW}^2} )</th>
<th>5D 2x2 Flat</th>
<th>5D 2x2 Warped</th>
<th>Three-site</th>
<th>HLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{960}{\pi^6} \sim 0.996 )</td>
<td>0.992</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \frac{3 g_{Z'WW}^2 M_{Z'}^2}{4 g_{W'WW} M_W^2 - 3 g_{ZW'WW} M_W^2} )</th>
<th>5D 2x2 Flat</th>
<th>5D 2x2 Warped</th>
<th>Three-site</th>
<th>HLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{96}{\pi^4} \sim 0.986 )</td>
<td>0.986</td>
<td>3/4</td>
<td>( \frac{3}{4} a )</td>
<td></td>
</tr>
</tbody>
</table>
Z'WW, ZW'W couplings and sum rules

\[ g_{Z'WW} = g v_{Z'}_0 (v_W^0)^2 + \tilde{g} v_{Z'}_1 (v_W^1)^2 = -\frac{\sqrt{a}}{2} \frac{e}{s_Z} \left( \frac{M_W}{M_{W'}} \right) + \cdots. \]

\[ g_{ZW'W} = g v_{Z}^0 v_W^0, v_W^0 + \tilde{g} v_{Z'} v_W^1, v_W^1 = -\frac{\sqrt{a}}{2} \frac{e}{s_Z c_Z} \left( \frac{M_W}{M_{W'}} \right) + \cdots. \]

<table>
<thead>
<tr>
<th>[ \frac{g_{Z'WW}^2}{g_{WW} - g_{ZWW}^2 - g_{Z'WW}^2} ]</th>
<th>5D 2x2 Flat</th>
<th>5D 2x2 Warped</th>
<th>Three-site</th>
<th>HLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{960}{\pi^6} \approx 0.996 ]</td>
<td>0.992</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| \[ \frac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WW} M_W^2 - 3g_{ZWW}^2 M_{W}^2} \] | \[ \frac{96}{\pi^4} \approx 0.986 \] | 0.986 | 3/4 | \[ \frac{3}{4} a \] |

\( a = \frac{4}{3} \) provide a better approximate description of the continuum models than the three-site model.
Does $a=4/3$ have any meaning?
Does $a=4/3$ have any meaning?

$$g_{\pi\pi\pi\pi} = \frac{4 - 3a}{12v^2}$$

Contact 4-pi interaction vanishes
$T_0^0$ in the hidden local symmetry model with various $a$

\[ g, g' = 0, \quad M_Y = 500 \text{ GeV} \] are assumed.
Summary

Four-site model is more appropriate to describe the delocalized fermions, however, its phenomenologies are well approximated by simpler three-site model.

We have considered how well the three-site Higgsless model performs as a general representative of Higgsless models.

Our comparisons have employed sum rules relating the masses and couplings of the gauge field KK modes.

We have studied several modification which have the potential to improve upon its performance.