Moulting Black Holes

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and MS: 103.nnnn
molt†，《英》moult/mɔːlt/【動】 | 自 | （鳥が）羽毛が生え替る，
（動物が）毛[角]が抜け[生え]替る，脱皮する // Snakes 〜．ヘビは脱皮する．
一 | 他 | （羽毛・殻など）を脱ぐ，落す．
一 | 【名】 ☐ ☐ 抜け替り；その時期．
molt·er 【名】 ☐ 羽[毛，角]が生え替る時期の鳥[虫，動物]．
Introduction
No hair theorem

Black holes have no “hair”

Given charges $M, J, Q$

Unique BH solution
“Moulting” black holes

- AdS BH can spit out excessive charges to “hair” and increase entropy
  - Einstein-Maxwell has BH with charged condensate [Gubser]
  - AdS$_5$ BH with R-charge [Bhattacharyya+Minwalla+Papadodimas]
  - “Enigmatic” BHs in 4d N=2 sugra / MSW [Denef+Moore]

— but not under very good control
AdS$_3$/CFT$_2$: ideal arena

- D1-D5 system
  - Bulk: $\text{AdS}_3 \times S^3 \times T^4$ or $\text{AdS}_3 \times S^3 \times K3$
  - Boundary: D1-D5 CFT

- Pivotal role in micro. understanding of BH in string theory
  - Strominger-Vafa: the first microscopic explanation of $S_{BH}$
  - BMPV black hole

- Under good control 😊
  \[ J_L \neq 0, J_R = 0 \]
What we do

- **Strategy:** find max entropy config for fixed $M, J$

- **Results**
  - CFT side: found a novel phase
  - Gravity side: 2-center solution: ‘hairy BH’ and BR
  - But $S_{CFT} > S_{grav}$

- **Implications**
  - Some states lift but some don’t?
  - A new index?
Outline

- Introduction ✓
- Boundary CFT side
- Bulk gravity side
- Discussion
- Conclusion
Boundary CFT side of the story
The AdS$_3$/CFT$_2$ correspondence

- **bulk**
  - Type IIB sugra
  - $AdS_3 \times S^3 \times M_4$
    - with $M_4 = T^4$ or K3
  - Strong coupling

- **boundary**
  - 2d $\mathcal{N} = (4,4)$ SCFT
    - (D1-D5 CFT)
    - in the Ramond sector
  - Sigma model on $\text{Sym}^N M_4$
    - with $N \equiv N_1 N_5$
  - Weak (zero) coupling
Some facts about D1-D5 CFT

- **Charges:** $L_0, \bar{L}_0$; $SU(2)_L \times SU(2)_R$ R-charges $J_L, J_R$
- **Supersymmetric states:** $\bar{L}_0 = 0$
  - $L_0 = P$: momentum; $J_L, J_R$: angular momenta

"Standard lore" phase diagram

![Phase Diagram](image.png)

- $S = 2\pi \sqrt{NP - \frac{J_L^2}{4}}$
- BMPV black hole
- Cardy formula
- gas of sugra particles
- unitarity bound

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States of D1-D5 CFT

- **Building blocks:** “effective strings”
  - Has length $n$
  - Carries “base” spin $(j_L, j_R)_{\text{base}} = (\pm 1, \pm 1)$
  - Excitations on it can carry momentum $p$ and angular momenta $j_L$

This has entropy $S = 2\pi \sqrt{np - j_L^2/4}$

- **States of the CFT is constructed by combining eff. strings**

\[ \sum_i n_i = N \]

charges add up
Example: BMPV ensemble

- A single string carries all charges \(N, P, J_L\)
- Base spin ignorable

\[
S_{\text{CFT}} = 2\pi \sqrt{NP - J_L^2/4} = S_{\text{BMPV}}
\]
The question

- What’s the max entropy states, for given $L_0, J_L$?
  - Better for $P$ to be carried by a long string
  - Making the long string carry $J_L$ reduces entropy
  - It may be better to make some of $J_L$ carried by ‘short’ strings

\[ \Delta P = P \]
\[ \Delta J_L = J_L - l \]
\[ \Delta J_R = 0 \]
\[ \Delta P = 0 \]
\[ \Delta J_L = l \]
\[ \Delta J_R = \text{any} \]
Maximizing entropy

\[ S(l) = 2\pi \sqrt{(N - l)P - (J_L - l)^2 / 4} \]

maximize for \( l \)

For \( 2P \leq J_L \leq N + P \), max is

\[ S(l_{\text{max}}) = 2\pi \sqrt{P(N + P - J_L)} \equiv S_{\text{CFT}}, \]

\[ l_{\text{max}} = J_L - 2P \]

- Always larger than \( S_{\text{BMPV}} \)
- Comes in an \( SU(2)_R \) multiplet with \( J_R = J_L - 2P \)
The new CFT phase diagram

old

new

BMPV black hole

gas of sugra particles

unitarity bound

BMPV phase

"Enigmatic" phase

Applies for both $T^4$ & K3

\[ P = \frac{J_L}{2} \]

\[ P = \frac{J_L^2}{4N} \]

\[ P = J_L - N \]

\[ P = \frac{N}{5} \]
Numerical check

- Compare with $T^4$ partition function
  - Can’t use Cardy (which is valid only for $P \gg N$)
  - Use DMVV formula and evaluate by computer

\[ S \text{ versus } J_L \text{ for } k = \frac{P}{N} = \frac{1}{5} \]

Confirmed!
Degeneracy (entropy) ≠ index (elliptic genus)

\[ Z = \text{Tr}_{\text{BPS}}[q^{L_0}y^{J_L}] \quad \chi = \text{Tr}[(-1)^{J_L+J_R}q^{L_0}y^{J_L}] \]

- Absolute # of states
- Not protected
- Not absolute # of states
- Protected

For enigmatic phase, entropy ≠ K3 elliptic genus

- Elliptic genus always gives BMPV entropy: \( \chi \sim \exp(2\pi \sqrt{NP - J_L^2 / 4}) \)
eden outside the Cardy regime \( P \gg N \) [Castro-Murthy]

- New phase not captured by elliptic genus
Summary — CFT side

- Found an “new phase” in non-Cardy regime
- Exists even outside the “BMPV parabola”
- Dominates over BMPV when coexists
- Comes in an $SU(2)_R$ multiplet with $J_R = J_L - 2P$
- Not captured by elliptic genus
  - Gets lifted at strong coupling?
  - Zero coupling artifact?
Bulk sugra side of the story
The goal

- We found CFT phase with more entropy than BMPV
- What’s the bulk dual configuration?

→ Find max entropy configuration given $N, P, J_L$
Supersymmetric sugra solutions

- Assume two isometries \( \rightarrow \) reduces to 4d N=2 sugra
- Most general timelike susy solutions in 4D known
  [Bates-Denef] [Gauntlett-Gutowski] [Bena-Warner]

Solution determined given harmonic functions on \( \mathbb{R}^3 \)

\( V, K^I, L_I, M \) (\( I = 1,2,3 \))
Ansatz

CFT states came in an $SU(2)_R$ multiplet with $J_R = J_L - 2P$

$J_R \leftrightarrow 4d$ angular momentum

2-center configs.

\[ V = \frac{1 + T}{r} - \frac{T}{\Sigma} \quad L_i = -\frac{Q_i}{4r} - \frac{Q'_i}{4\Sigma} \]

\[ K^I = -\frac{q^I}{2r} - \frac{q''^I}{2\Sigma} \quad M = m_0 + \frac{m}{r} + \frac{m'}{\Sigma} \]

\[ r = |\vec{x}|, \quad \Sigma = |\vec{x} - \vec{a}| \]

Parameters constrained to give desired total charges

Play with parameters and maximize BH entropy
Result: max-entropy 2-center configs.

\[ S_{\text{BMPV+hair}} = 2\pi \left( \sqrt{N} - \sqrt{J_L - P} \right) \sqrt{P} \]
\[ S_{\text{BR}} = 2\pi \left( \sqrt{N} - \sqrt{N - P} \right) \sqrt{P - J_L + N} \]

\[ < S_{\text{CFT}} \]
Bulk phase diagram

- 2-center configs very barely dominates over BMPV
Summary — gravity side

- Found 2-center configs for $P \sim J_L \sim N$
- Exists even outside the “BMPV parabola”
- Exists in the same region as CFT enigmatic phase does
- Dominates over BMPV partly in coexisting region
- Entropy strictly smaller than CFT enigmatic phase
Discussion
Discussion

- Found configs whose entropy is larger than BMPV
  - BMPV is unstable, in spite of susy
- BH configs exist even outside the BMPV parabola!
  - Elliptic genus doesn’t capture this BH

Cf. Matching of CFT and sugra elliptic genera [de Boer]

It was believed that there are no BHs in the gray region, but there are!
Partial lifting (1)

**Standard lore**

- don’t contribute to index
- contribute to index
- protected
- pairwise lift

CFT $\rightarrow 0$

Gravity $\rightarrow \infty$

$\lambda$
Partial lifting (2)

The reality

Protected for dynamical reasons, not susy?
Conclusion
Conclusion

- Studied phases of D1-D5 system for given $N, P, J_L$

- Found new phases both in CFT & gravity
  - *Controlled* example of “hairy” BH

- New phases dominate over BMPV

- Some states survive strong coupling, although not protected by susy!
  - *Dynamical protection? A new index??*

- Further investigation needed
Thanks!