Probing proton-neutron pairing with Gamow-Teller strengths in two-nucleon configurations

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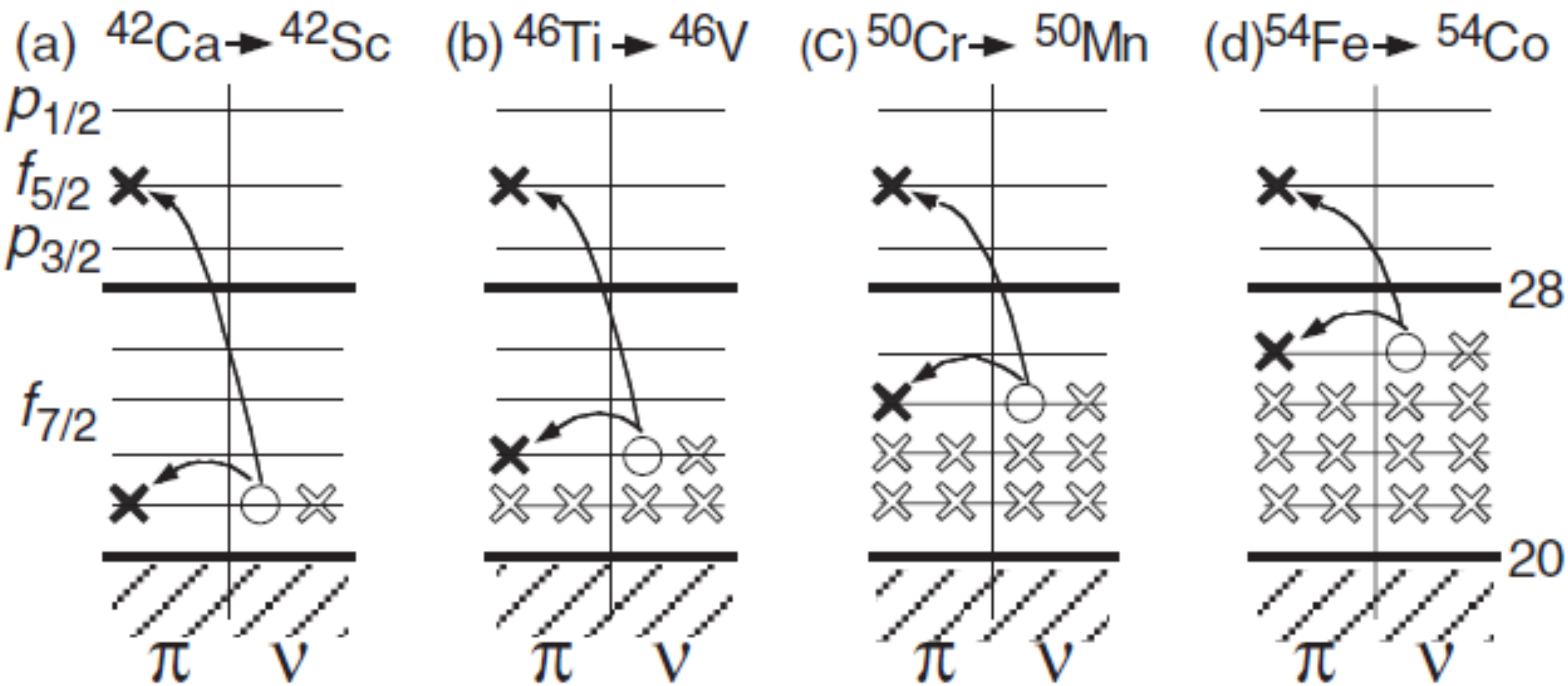
Y. Utsuno and Y. Fujita, arXiv 1701.00869

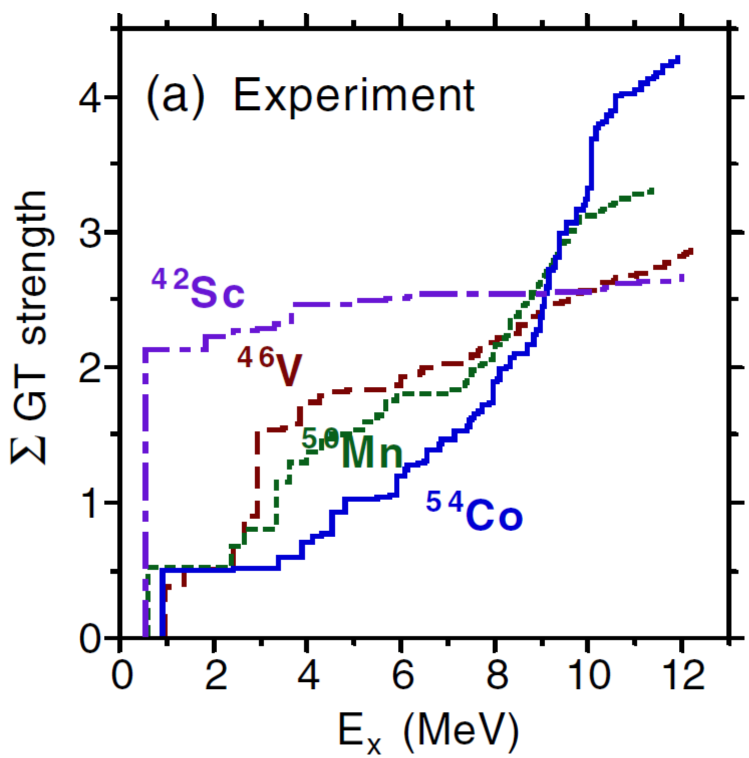
Introduction: Fujita-san's seminar in Jan. 2016

- Systematics of Gamow-Teller distributions for N=Z+2 → N=Z nuclei
  - Using charge exchange reaction (^{3}He, t)

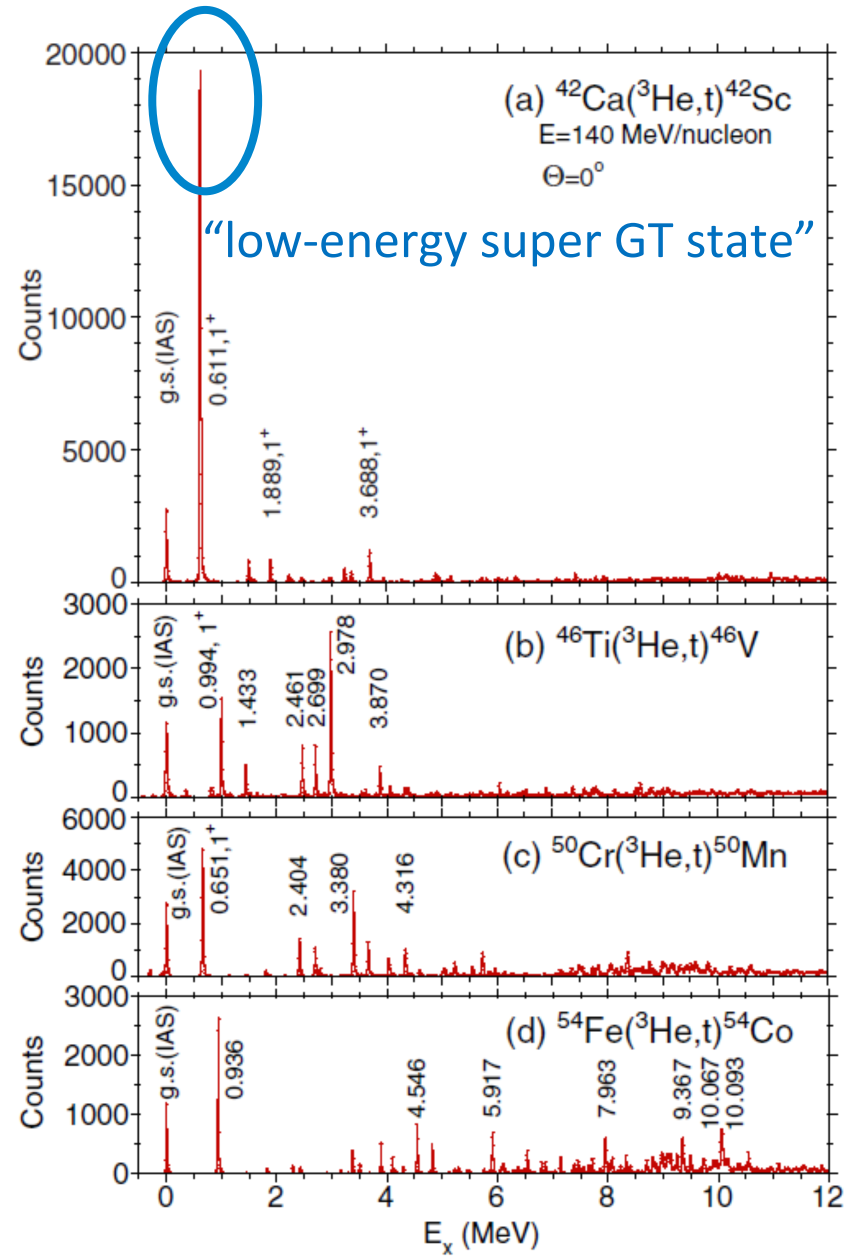


Observed GT strength

- Concentration in the 1^+_1 level for $^{42}Ca \rightarrow {}^{42}Sc$
  - B(GT; $0^+_1 \rightarrow 1^+_1$) = 2.2

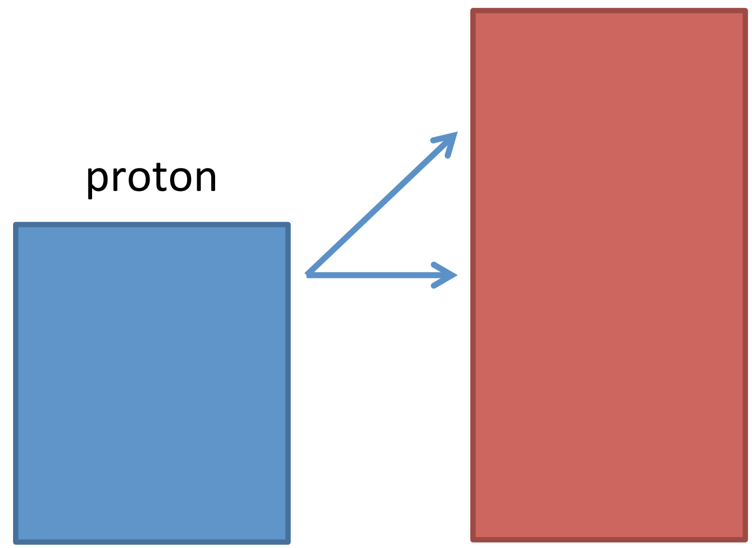


Y. Fujita et al., Phys. Rev. Lett. 112, 112502 (2014).



Ikeda sum rule

- When B(GT) is defined as $B(\mathrm{GT}^{\pm}; i\to f) = |\langle \alpha_f J_f \| \sigma t^{\pm} \| \alpha_i J_i \rangle|^2 / (2J_i+1) = \sum_{\mu M_f} |\langle \alpha_f J_f M_f | \sigma_\mu t^{\pm} | \alpha_i J_i M_i \rangle|^2$,
  $\sum_f B(\mathrm{GT}^-; i\to f) - \sum_f B(\mathrm{GT}^+; i\to f) = 3(N-Z)$ is satisfied.
  (with $t^- |n\rangle = |p\rangle$)
- If GT+ transition is hindered by Pauli blocking, $\sum_f B(\mathrm{GT}^-; i\to f)$ is close to $3(N-Z)$.
  - $\sum_f B(\mathrm{GT}^-; i\to f)$ for ^{42}Ca should be close to 6, if proton excitation to the *pf* shell is small.



Proof of the Ikeda sum rule

The sum of $B(\mathrm{GT})$ values are written as

$$\begin{aligned}
&\sum_f B(\mathrm{GT}^{\pm} i \to f) \\
&= \sum_{\alpha_f J_f \mu M_f} |\langle \alpha_f J_f M_f | \sum_k \sigma_\mu(k) t^{\pm}(k) | \alpha_i J_i M_i \rangle|^2 \\
&= \sum_{\alpha_f J_f \mu M_f} \langle \alpha_i J_i M_i | \sum_{k'} (t^{\pm})^{\dagger}(k') \sigma_\mu^{\dagger}(k') \alpha_f J_f M_f \rangle \langle \alpha_f J_f M_f | \sum_k \sigma_\mu(k) t^{\pm}(k) | \alpha_i J_i M_i \rangle \\
&= \langle \alpha_i J_i M_i | \sum_{\mu k k'} (-1)^\mu \sigma_{-\mu}(k') \sigma_\mu(k) t^{\mp}(k') t^{\pm}(k) | \alpha_i J_i M_i \rangle .
\end{aligned} \tag{1}$$

Here we define

$$O^{\pm} = \sum_{\mu k k'} (-1)^\mu \sigma_{-\mu}(k') \sigma_\mu(k) t^{\mp}(k') t^{\pm}(k) \tag{2}$$

and get

$$\begin{aligned}
O^- - O^+ &= \sum_{\mu k k'} (-1)^\mu \{ \sigma_{-\mu}(k') \sigma_\mu(k) t^+(k') t^-(k) - \sigma_{-\mu}(k') \sigma_\mu(k) t^-(k') t^+(k) \} \\
&= \sum_{\mu k k'} (-1)^\mu \{ \sigma_{-\mu}(k') \sigma_\mu(k) t^+(k') t^-(k) - \sigma_{-\mu}(k) \sigma_\mu(k') t^-(k) t^+(k') \} \\
&= \sum_{\mu k k'} (-1)^\mu \{ \sigma_{-\mu}(k') \sigma_\mu(k) t^+(k') t^-(k) - \sigma_{\mu}(k) \sigma_{-\mu}(k') t^-(k) t^+(k') \} \\
&= \sum_{\mu k} (-1)^\mu \sigma_{-\mu}(k) \sigma_\mu(k) \{ t^+(k) t^-(k) - t^-(k) t^+(k) \} \\
&= \sum_{\mu k} 2 (-1)^\mu \sigma_{-\mu}(k) \sigma_\mu(k) t_z(k) \\
&= \sum_k 2 \{ \sigma_x^2(k) + \sigma_y^2(k) + \sigma_z^2(k) \} t_z(k) \\
&= 6 T_z .
\end{aligned} \tag{3}$$

SU(4)? single-particle transition?

- The situation is similar to what is expected from the SU(4) symmetry (no spin or isospin dependent forces).

Review

Overview of neutron–proton pairing

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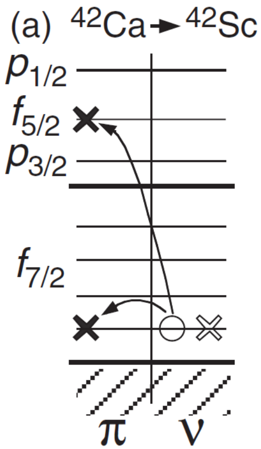
Very recently, Fujita et al. [171] carried out a detailed study of GT excitations for mass number $A = 42, 46, 50$, and 54 using the (^{3}He, t) reaction at the RCNP Facility in Osaka. Of particular interest to us, are the results observed for the charge exchange reaction from the gs of ^{42}Ca to ^{42}Sc. The GT strength for this case appears to be concentrated in the lowest 1^+ state at 0.611 MeV which the authors interpret as a restoration of the SU(4) symmetry in the form of a low-lying collective GT phonon. Shell model calculations with the Kuo–Brown interaction seem to account for the data rather well. With only two valence particles outside closed shells, the direct implication of this observation on the existence of an isoscalar condensate is not clear and shell model indicators do not signal such a condensate. The interpretation of this 1^+ state as an isoscalar phonon, seems at variance with the large $B(M1, 1^+ \rightarrow 0^+)$ which is well described by a $[\nu f_{7/2} - \pi f_{7/2}]^{1^+}$ configuration [158] and other properties to be discussed in Sections 7 and 8. Moreover, low-lying single-particle levels in ^{41}Ca and ^{41}Sc are consistent with jj- rather than LS-coupling. One cannot but notice in Fig. 48 that in the SU(4) limit, the single-l model predicts very small values of the total $B(GT)$.

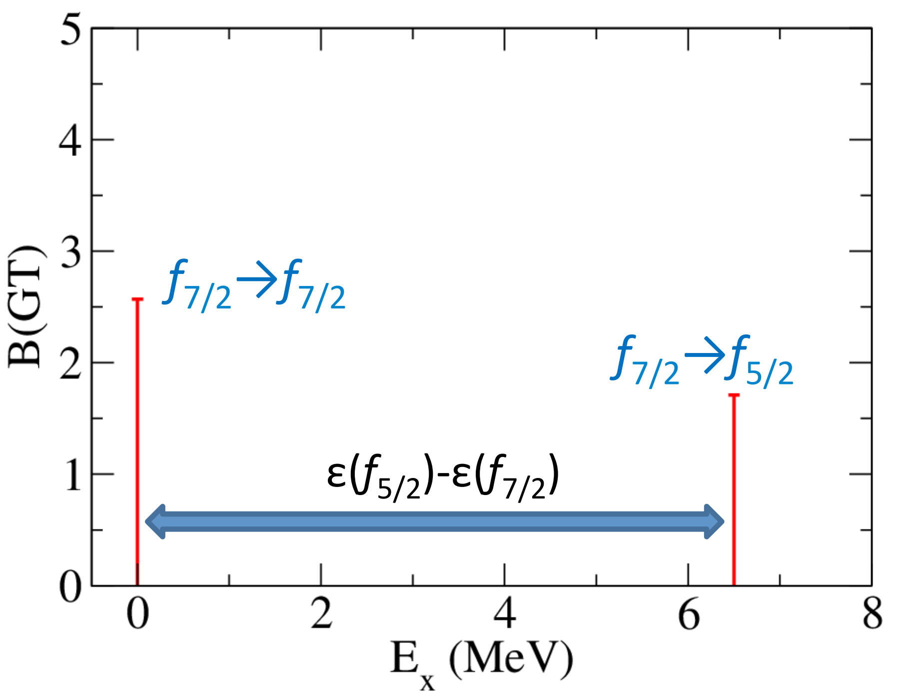
- SU(4) is strongly broken by the existence of spin-orbit splitting.
- Single-particle structure?

From the simple single-particle picture

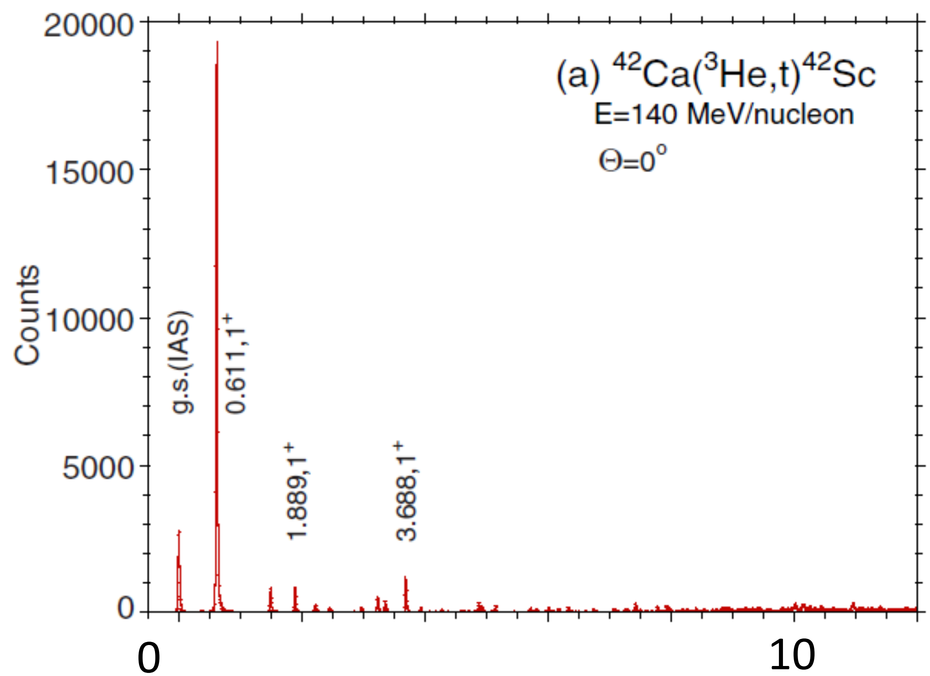


- Concentration in a single state cannot be accounted for by a simple shell-model argument.

B(GT) distribution with pure configurations

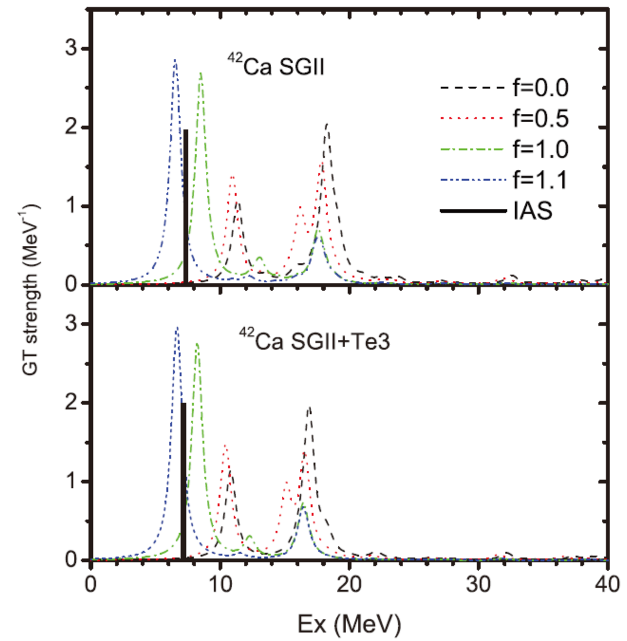


Experimental strength



Essential role of configuration mixing

- RPA calculation
  - Isoscalar pairing is essential.
- Coherence in the GT transition is also obtained by shell-model calc.
- Why is collectivity created?



C. L. Bai et al., Phys. Rev. C 90, 054335 (2014).

Shell-model analysis

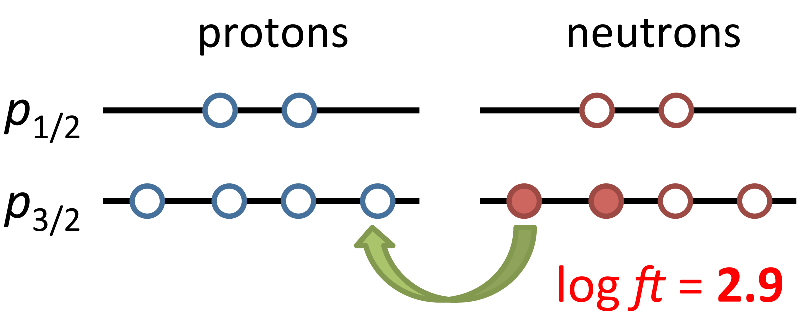
States in ^{42}Sc		Configurations						Transition strengths	
E_x (MeV)	T	$f7 \rightarrow f7$	$f7 \rightarrow f5$	$f5 \rightarrow f7$	$p3 \rightarrow p3$	$p3 \rightarrow p1$	$p1 \rightarrow p3$	ΣM(GT)	B(GT)
0.33	0	1.383	0.548	0.063	0.031	0.024	0.016	2.07	4.28
4.41	0	0.719	−0.742	−0.085	−0.079	−0.073	−0.048	−0.31	0.09
7.41	0	0.193	−0.788	−0.090	0.142	0.060	0.040	−0.44	0.19
8.62	0	−0.151	0.385	0.044	0.109	−0.071	−0.047	0.30	0.09
9.82	1	0.0	1.196	−0.137	0.0	−0.053	0.035	1.04	1.08

Y. Fujita et al., Phys. Rev. C 91, 064316 (2015)

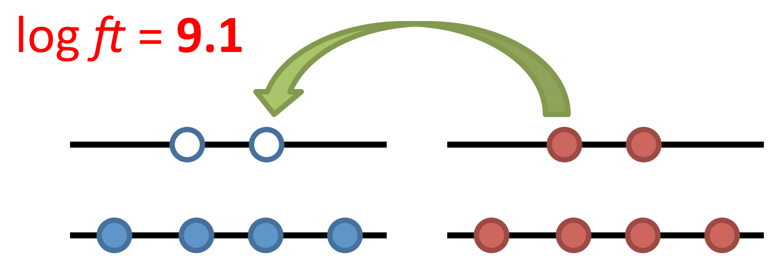
Two-particle vs. two-hole configurations

- A question by someone (sorry, I do not remember who raised)
  - What about two hole systems?

two-particle : $^{6}He \rightarrow {}^{6}Li$

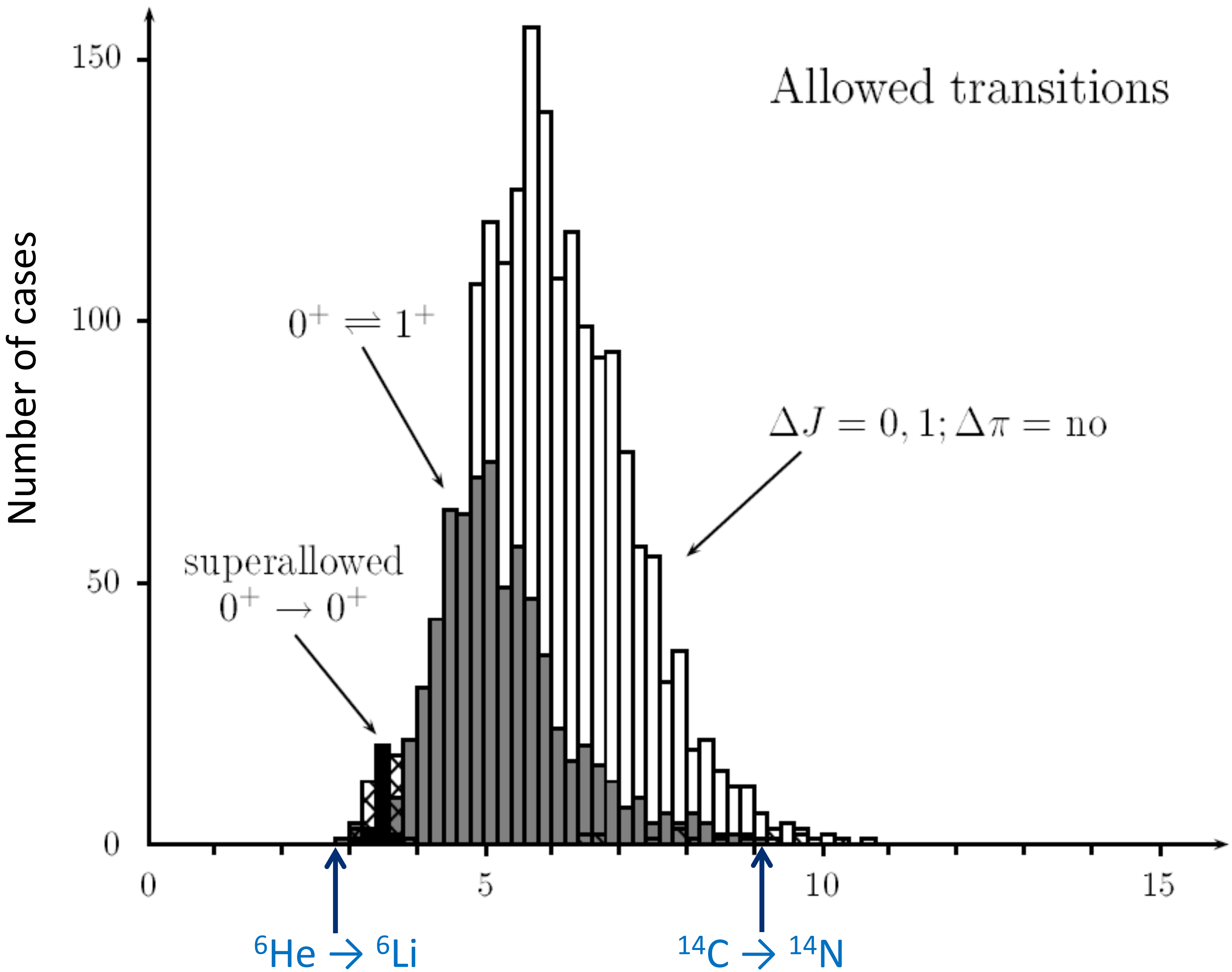


two hole : $^{14}C \rightarrow {}^{14}N$



B(GT)	Two-particle			Two-hole	
	p	*sd*	*pf*	*p*	*sd*
	A=6	A=18	A=42	A=14	A=38
1^+_1	4.7	3.1	2.2	3.5×10^{-6}	0.060
1^+_2		0.13	0.10	2.8	1.5

Distribution of known log *ft*



Questions

- Why does the “low-energy super GT state” appear for two-particle configurations?
- Why is the corresponding B(GT) values for two-hole configurations very small?
- What does those properties tell us about isovector and isoscalar pairing properties, since correlated two nucleons form a “Cooper pair”?

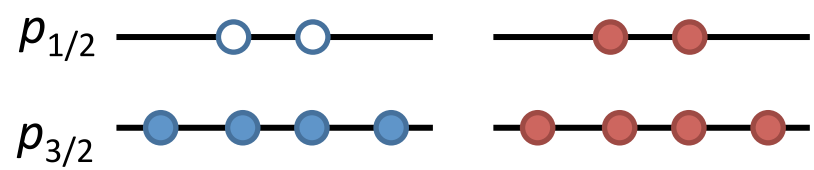
Unified treatment of *p-p* and *h-h* systems

- The following two descriptions are identical
  - Particle Hamiltonian

    $H=\sum_{i} \varepsilon_{i} a_{i}^{\dagger} a_{i} +1/4 \sum_{i,j,k,l} v_{ij;kl} a_{i}^{\dagger} a_{j}^{\dagger} a_{l} a_{k}$
  - Hole Hamiltonian

    $H=\sum_{i} \bar{\varepsilon}_{i} b_{i}^{\dagger} b_{i} +1/4 \sum_{i,j,k,l} \bar{v}_{ij;kl} b_{i}^{\dagger} b_{j}^{\dagger} b_{l} b_{k}$

  when $v_{ij;kl}=\bar{v}_{ij;kl}$ and $\bar{\varepsilon}_{i}=E(i^{-1})$ are satisfied.



Two-body matrix elements are the same, but the single-particle energies are in the reversed order for the hole-hole Hamiltonian.

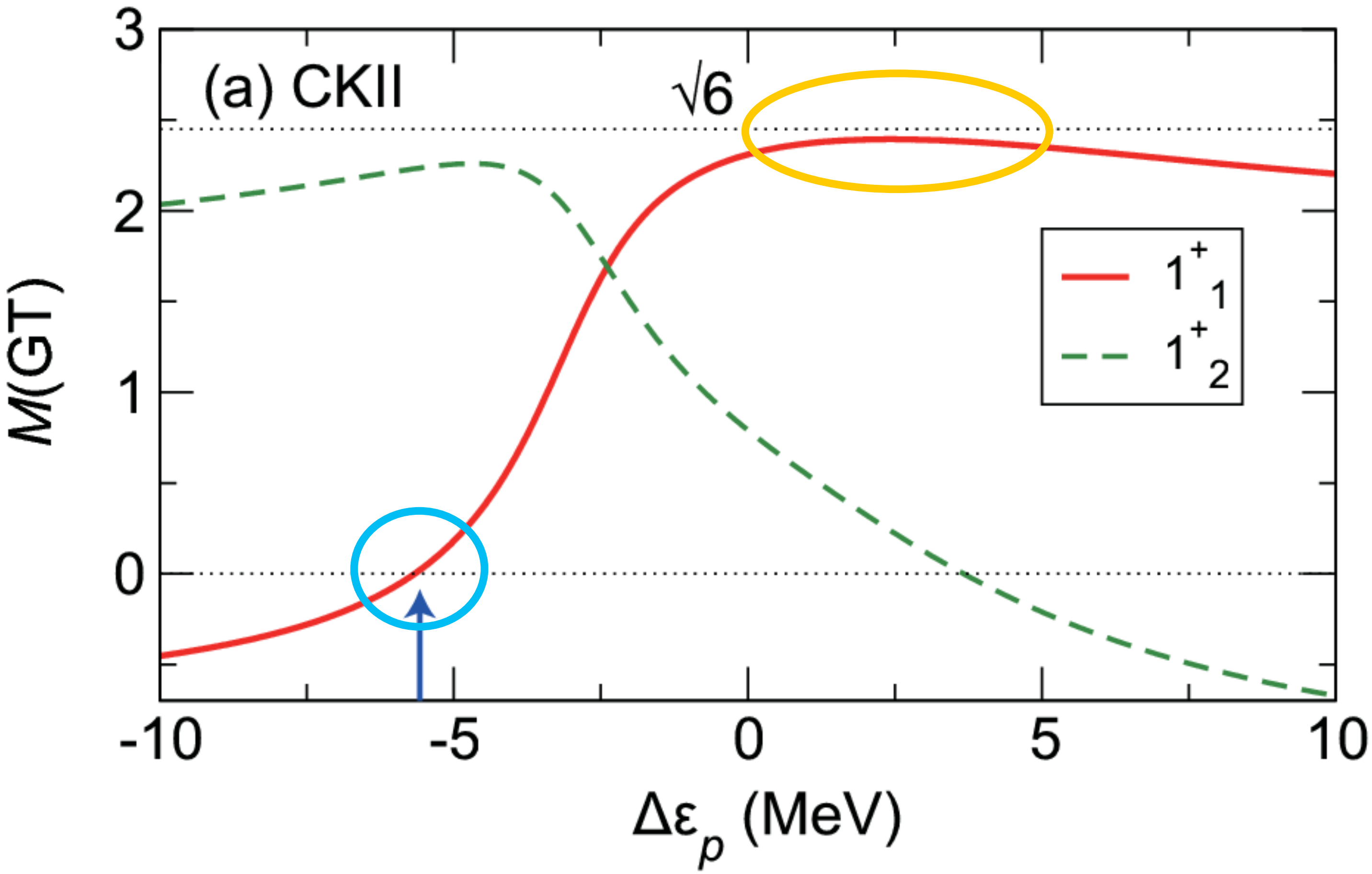
GT strength in 2*p* and 2*h* conf.: *p*-shell case

- As shown in the last slide, both the $^{6}He \rightarrow {}^{6}Li$ (2*p*) and $^{14}C \rightarrow {}^{14}N$ (2*h*) decays can be described as two-nucleon systems with the *same* two-body matrix elements.
- The only difference between them is the single-particle splitting $\Delta\varepsilon_p = \varepsilon(p_{1/2}) - \varepsilon(p_{3/2})$:
  - For the particle case: $\Delta\varepsilon_p = 0.1$ MeV
  - For the hole case: $\Delta\varepsilon_p = -6.3$ MeV

  Taken from the Cohen-Kurath's CKII interaction
- It is interesting to plot the Gamow-Teller matrix element $M(GT) = \langle J_f \| \sigma t^- \| J_i \rangle$ as a function of $\Delta\varepsilon_p$ for given two-body interactions in order to see the dependence on the single-particle energies.

(1) Cohen-Kurath's CKII interaction

- One of the most popular empirical interactions for the *p* shell.
  - 15 two-body matrix elements are deduced by fitting energy levels of *A*=8-16 nuclei.



(2) Pairing interaction

- Isoscalar- and isovector-pairing interactions are defined as

  $$V^{\mathrm{IV}}_{\mathrm{pair}} = G^{\mathrm{IV}} \sum_{\mu} P^{\dagger}_{\mu} P_{\mu}$$

  $$V^{\mathrm{IS}}_{\mathrm{pair}} = G^{\mathrm{IS}} \sum_{\mu} D^{\dagger}_{\mu} D_{\mu}$$

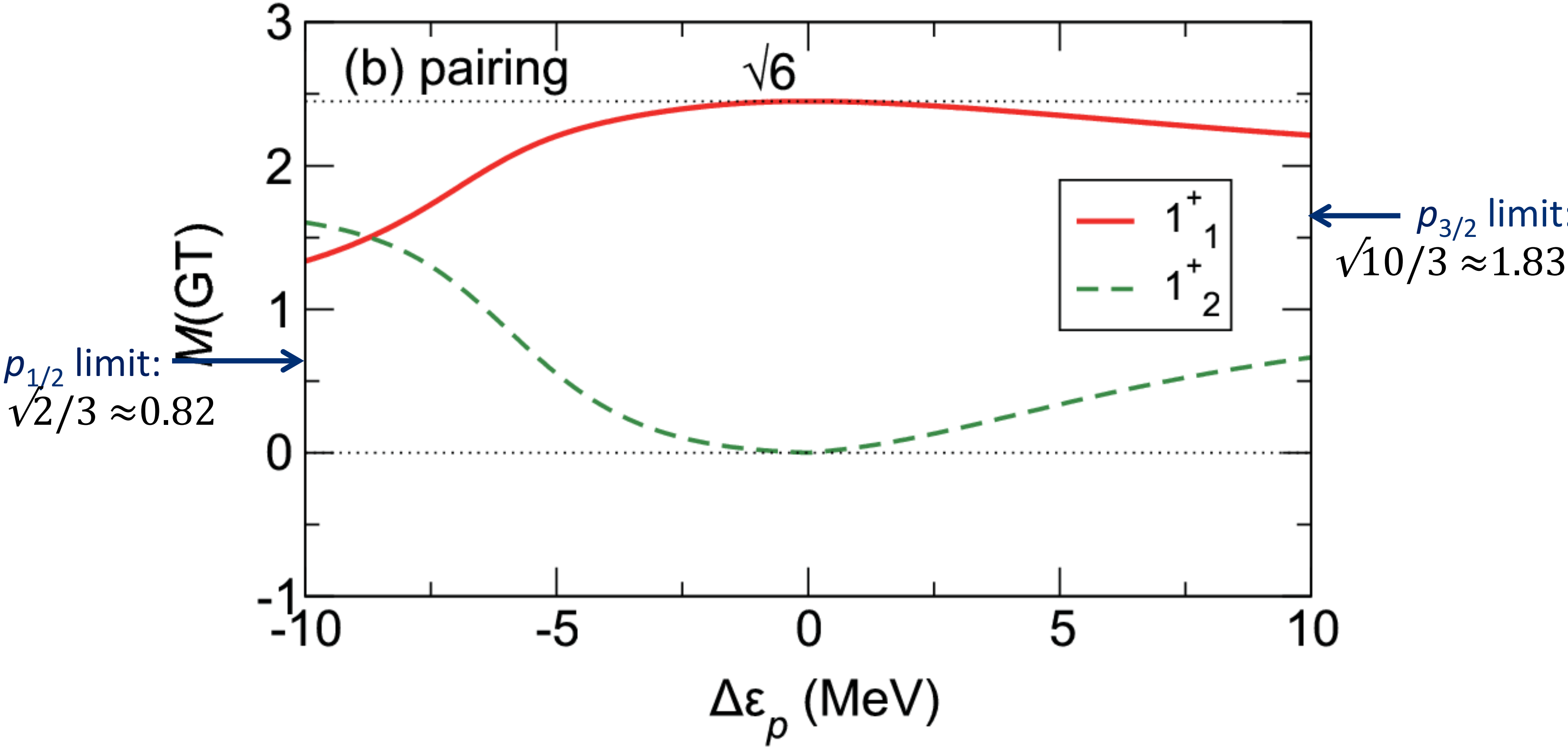
  where $P^{\dagger}_{\mu} = \sqrt{1/2} \sum_{nl} (-1)^{l} \sqrt{2l+1} \left[a^{\dagger}_{nl} \times a^{\dagger}_{nl}\right]^{L=0,S=0,T=1}_{M_L=0,M_S=0,M_T=\mu}$ and

  $D^{\dagger}_{\mu} = \sqrt{1/2} \sum_{nl} (-1)^{l} \sqrt{2l+1} \left[a^{\dagger}_{nl} \times a^{\dagger}_{nl}\right]^{L=0,S=1,T=0}_{M_L=0,M_S=\mu,M_T=0}$.

- Isovector-pair creation operator $P^{\dagger}_{\mu}$ and isoscalar-pair creation operators $D^{\dagger}_{\mu}$ are symmetric in terms of *S* and *T*.

M(GT) with the pairing interaction

- Strengths of G^{IS} and G^{IV} are determined so as to reproduce the mean square (J,T)=(1,0) and (0,1) matrix elements of CKII.



- M(GT): Enhanced for small $\Delta\varepsilon_p$ and no vanishing for $\Delta\varepsilon_p < 0$.
  - Larger than single-particle limits on the both sides

Coherence of the GT matrix element

- When two-nucleon wave functions are decomposed into basis states as $|0_k^+\rangle = \sum_{ab} \alpha_{ab}^{IV}(k)\,|ab\; J=0\; T=1\rangle$ and $|1_k^+\rangle = \sum_{ab} \alpha_{ab}^{IS}(k)\,|ab\; J=1\; T=0\rangle$, the Gamow-Teller matrix element is written as
$M(GT; 1_k^+) = \sum_{abcd} m_{abcd}(1_k^+)$
where $m_{abcd}(1_k^+) = \alpha_{ab}^{IS\,*}(k)\,\alpha_{cd}^{IV}(1)\langle ab\; J=1\; T=0 | \sigma t^{-} | cd\; J=0\; T=1\rangle$.
- Signs of $m_{abcd}(1_k^+)$
  - Constructive or destructive interference—essential for determining large or small B(GT)

Signs of $m_{abcd}(1_k^+)$: CKII

$m_{abcd}(1_k^+) = \alpha_{ab}^{IS*}(k)\,\alpha_{cd}^{IV}(1)\,\langle ab\; J=1\; T=0 | \sigma t^- | cd\; J=0\; T=1 \rangle$.

$\Delta\varepsilon_p = 5$ MeV

```
T=1  J=0  n=1  --> T=0  J=1  n=1

# SPE:    0.0000  (p>)    5.0000  (p<)

                       |      (p> p>)        (p< p<)
                       |     -0.94845       -0.31692
=================================================
(p> p>)  -0.81525  |   -1.83*+0.773   +0.00*+0.258
                       |     -1.41170       +0.00000
-------------------------------------------------
(p> p<)  +0.57862  |   +1.15*-0.549   +1.63*-0.183
                       |     -0.63369       -0.29945
-------------------------------------------------
(p< p<)  +0.02401  |   +0.00*-0.023   +0.82*-0.008
                       |     +0.00000       -0.00621
-------------------------------------------------

sum = -2.351    B(GT) =  5.527
```

$\Delta\varepsilon_p = -5$ MeV

```
T=1  J=0  n=1  --> T=0  J=1  n=1

# SPE:    0.0000  (p>)   -5.0000  (p<)

                       |      (p> p>)        (p< p<)
                       |     +0.45403       +0.89099
=================================================
(p> p>)  -0.00736  |   -1.83*-0.003   +0.00*-0.007
                       |     +0.00610       +0.00000
-------------------------------------------------
(p> p<)  +0.42130  |   +1.15*+0.191   +1.63*+0.375
                       |     +0.22087       +0.61298
-------------------------------------------------
(p< p<)  -0.90689  |   +0.00*-0.412   +0.82*-0.808
                       |     +0.00000       -0.65975
-------------------------------------------------

sum =   0.180    B(GT) =   0.032
```

- Destructive interference for $\Delta\varepsilon_p = -5$ MeV.

Signs of $m_{abcd}(1_k^+)$: pairing

$m_{abcd}(1_k^+) = \alpha_{ab}^{IS*}(k)\,\alpha_{cd}^{IV}(1)\,\langle ab\; J=1\; T=0 | \sigma t^- | cd\; J=0\; T=1 \rangle$.

$\Delta\varepsilon_p = 5$ MeV

T=1 J=0 n=1 --> T=0 J=1 n=1

SPE: 0.0000 (p>) 5.0000 (p<)

	(p> p>) -0.95195	(p< p<) -0.30624
(p> p>) -0.82072	-1.83*+0.781 -1.42644	+0.00*+0.251 +0.00000
(p> p<) +0.56310	+1.15*-0.536 -0.61898	+1.63*-0.172 -0.28160
(p< p<) +0.09658	+0.00*-0.092 +0.00000	+0.82*-0.030 -0.02415

sum = -2.351 B(GT) = 5.528

$\Delta\varepsilon_p = -5$ MeV

T=1 J=0 n=1 --> T=0 J=1 n=1

SPE: 0.0000 (p>) -5.0000 (p<)

	(p> p>) +0.46591	(p< p<) +0.88483
(p> p>) -0.36547	-1.83*-0.170 +0.31088	+0.00*-0.323 +0.00000
(p> p<) +0.76070	+1.15*+0.354 +0.40924	+1.63*+0.673 +1.09915
(p< p<) +0.53644	+0.00*+0.250 +0.00000	+0.82*+0.475 +0.38756

sum = 2.207 B(GT) = 4.870

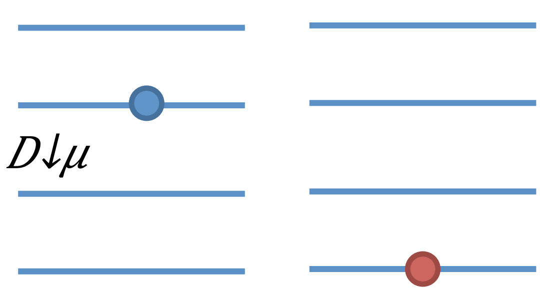
- It looks that the pairing interaction always gives a constructive interference, but realistic interactions do not.

Theorem for the signs of $m_{abcd}(1_k^+)$

When the (J,T)=(0,1) and (1,0) two-body matrix elements are given by the isovector- and isoscalar-pairing interactions with negative G^{IV} and G^{IS}, respectively, all the signs of in two-nucleon systems are the same for *any* valence shell (including multi-j shell) and for *any* single-particle splitting.

$$H^{\text{pair}} = \sum_i \varepsilon_i a_i^\dagger a_i + V^{\text{pair}}$$

$$V^{\text{pair}} = G^{\text{IV}} \sum_\mu P_\mu^\dagger P_\mu + G^{\text{IS}} \sum_\mu D_\mu^\dagger D_\mu$$



$$m_{abcd}(1_k^+) = \alpha_{ab}^{\text{IS}*}(k)\,\alpha_{cd}^{\text{IV}}(1)\,\langle ab\, J=1\, T=0 | \sigma t^- | cd\, J=0\, T=1 \rangle \geq 0$$

Proof of the theorem (1)

- Write down *j-j* coupled two-body matrix elements:

$$\langle ab\, J=0\, T=1 | V^{\text{pair}} | cd\, J=0\, T=1\rangle = G^{\text{IV}} \chi_{ab}^{\text{IV}} \chi_{cd}^{\text{IV}}$$

$$\langle ab\, J=1\, T=0 | V^{\text{pair}} | cd\, J=1\, T=0\rangle = G^{\text{IS}} \chi_{ab}^{\text{IS}} \chi_{cd}^{\text{IS}}$$

with

$$\chi_{ab}^{\text{IV}} = (-1)^{l_a} \sqrt{j_a + 1/2}\, \delta_{ab}$$

$$\chi_{ab}^{\text{IS}} = \sqrt{\frac{2}{1+\delta_{ab}}}\, (-1)^{j_a - 1/2} \sqrt{(2j_a+1)(2j_b+1)} \begin{Bmatrix} 1/2 & j_a & l_a \\ j_b & 1/2 & 1 \end{Bmatrix} \delta_{n_a n_b} \delta_{l_a l_b}$$

- One can easily show that the sign of χ_{ab}^{IV} is $(-1)^{l_a}$ and that that of χ_{ab}^{IS} is $(-1)^{j_b - 1/2}$ by using the exact form of $\begin{Bmatrix} 1/2 & j_a & l_a \\ j_b & 1/2 & 1 \end{Bmatrix}$.

- When the conventions of $|ab\, J=0\, T=1\rangle = (-1)^{l_a} |ab\, J=0\, T=1\rangle$ and $|ab\, J=1\, T=0\rangle = (-1)^{j_b - 1/2} |ab\, J=1\, T=0\rangle$ are taken, all the two-body matrix elements are non positive.

Proof of the theorem (2)

- Matrix element of the Hamiltonian
  $H_{ij}^{\mathrm{pair}} = \delta_{ij}\left(\varepsilon_{a(i)} + \varepsilon_{b(i)}\right) + V_{ij}^{\mathrm{pair}}$
- All the *off-diagonal* matrix elements are non positive when the present phase convention.
- From a version of the Perron-Frobenius theorem, the components of the lowest eigenvector are completely of the same sign.

For a matrix $\begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix}$ with $h_{ij} \leq 0$ ($i \neq j$), t
eigenvector $v_1 = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix}$ satisfies $\alpha_i \geq 0$ for any i.

Proof of the theorem (3)

- Gamow-Teller matrix elements for two-nucleon configurations

Condon-Shortley convention

present convention

Completely the same sign!

Proof of the theorem (4)

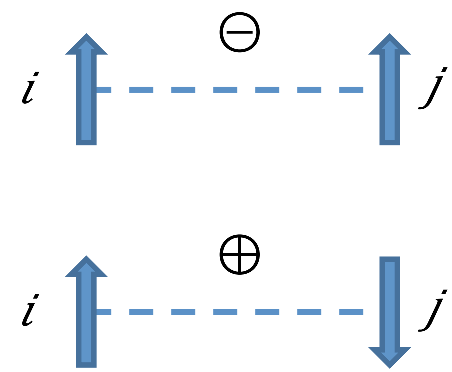
- We go back to $M(GT;1_k^+)=\sum_{abcd} m_{abcd}(1_k^+)$ with $m_{abcd}(1_k^+)=\alpha_{ab}^{\mathrm{IS}\,*}(k)\,\alpha_{cd}^{\mathrm{IV}}(1)\langle ab\,J=1\,T=0|\sigma t^-|cd\,J=0\,T=1\rangle$.
- All of $\alpha_{ab}^{\mathrm{IS}\,*}(1)$, $\alpha_{cd}^{\mathrm{IV}}(1)$, and $\langle ab\,J=1\,T=0|\sigma t^-|cd\,J=0\,T=1\rangle$ have fixed signs when the phase conventions of $|ab\,J=0\,T=1\rangle=(-1)^{l_a}|ab\,J=0\,T=1\rangle$ and $|ab\,J=1\,T=0\rangle=(-1)^{j_b-1/2}|ab\,J=1\,T=0\rangle$ are taken. Therefore, the signs of $m_{abcd}(1_1^+)$ are the same for all possible (a,b,c,d).
- This is the origin of the coherence ("low-energy super GT state") obtained for two-particle configurations.

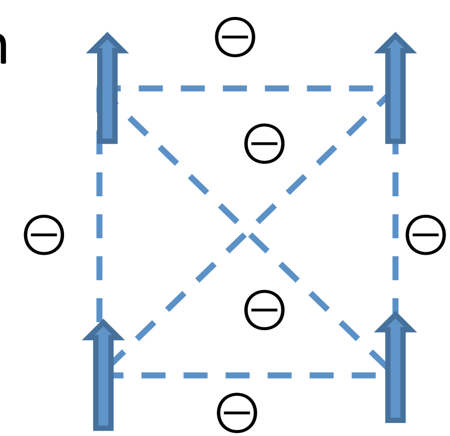
(rough) Proof of the Perron-Frobenius theorem

- Consider a real-symmetric matrix with $H=(h_{ij})$ with $\forall h_{ij} \leq 0$ $(i \neq j)$ and let $v_1 = (\alpha_1, \ldots, \alpha_n)$ be the lowest eigenvector.
- Since v_1 is the lowest eigenvector, there is no vector that satisfies $v H v < v_1 H v_1$.
- If v_1 contains positive and negative components, one can generally assume $\alpha_1 \geq 0, \ldots, \alpha_k \geq 0, \alpha_{k+1} < 0, \ldots, \alpha_n < 0$. Let v_1' be $(\alpha_1, \ldots \alpha_k, -\alpha_{k+1}, -\alpha_n)$.
- Since $v_1 H v_1 = \sum_{ij} \alpha_i h_{ij} \alpha_j$, one gets $v_1 H v_1 - v_1' H v_1' = \sum_{i \leq k, j > k} 2 \alpha_i h_{ij} \alpha_j > 0$. This contradicts that v_1 is the lowest eigenvector.

Graphical image of the P-F theorem

- Represent an off-diagonal matrix element h_{ij} as a bond that connects the site i and j.
- For each site i, a positive (negative) α_i is represented as an upward (downward) arrow.
- The right figure shows favored signs, which are analogous to ferromagnetism and antiferromagnetism.
- The situation of P-F theorem is like parallel spin alignment in ferromagnetism that makes energy stable.





Going back to physics

- The coherence of GT matrix elements is due to the “alignment” of all the two-nucleon configurations, where alignment stands for that coefficients of basis vectors are of the same sign.
- Since this is independent of single-particle energies, such an “aligned” state must be most stable also for two-hole configurations.
- The actual situation is different. This means that some of the off-diagonal matrix elements in the (J,T)=(1,0) and/or (0,1) channels are opposite.

Realistic (J,T)=(0,1) and (1,0) matrix elements

- Questions
  - Isovector or isoscalar?
  - Which matrix elements are different?
  - Any rule about the different signs?
- Examining off-diagonal matrix elements in realistic interactions
  - Empirical and microscopic (*G* matrix)
  - Phase conventions of $|ab\, J=0\ T=1\rangle = (-1)^{l_a} |ab\, J=0\ T=1\rangle$ and $|ab\, J=1\ T=0\rangle = (-1)^{j_b - 1/2} |ab\, J=1\ T=0\rangle$

(J,T)=(0,1) off-diagonal

	CKII	Kuo p
	-4.86	-3.96

(J,T)=(1,0) off-diagonal

	CKII	Kuo p
	-1.56	-1.75
	-3.55	-5.06
	+1.70	+2.31

sd shell

(J,T)=(0,1) off-diagonal

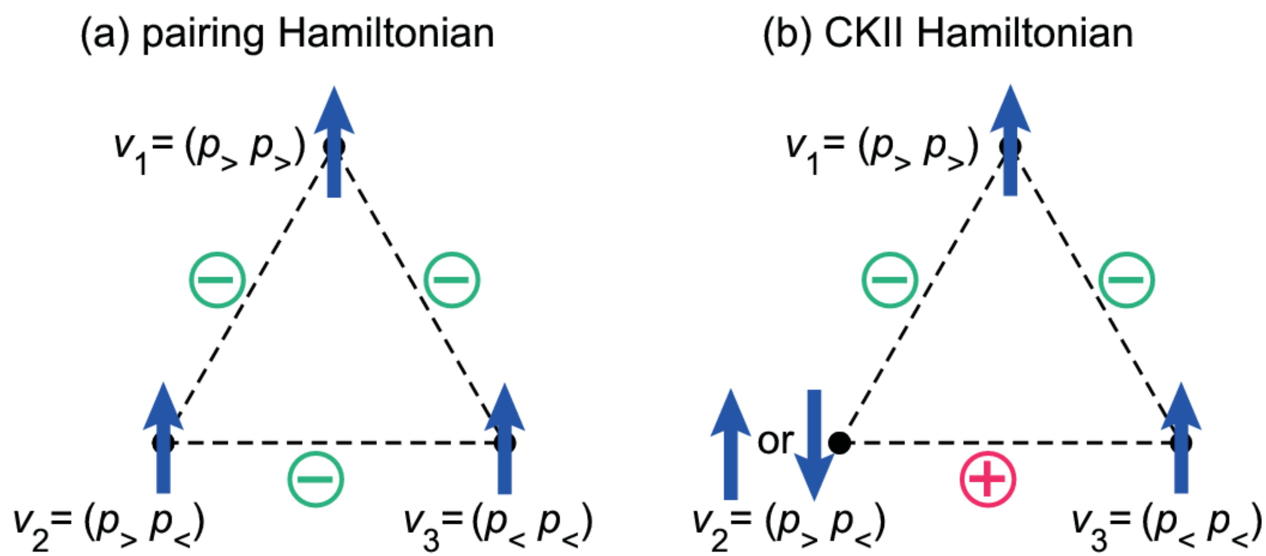
	USD	Kuo sd
	-3.19	-3.79
	-1.32	-0.97
	-1.08	-0.74

(J,T)=(1,0) off-diagonal

	USD	Kuo sd
	-0.72	-1.62
	-2.54	-3.17
	+0.57	+0.04
	+1.10	+0.24
	-1.18	-0.60
	-1.71	-1.91
	-2.10	-1.71
	+0.40	+0.80
	-0.03	+0.21
	-1.25	-0.31

Some isoscalar matrix elements have the opposite sign.

Pairing vs. realistic interactions: *p*-shell case



- Same signs for isovector matrix elements.
- Opposite sign for $p_{3/2}\, p_{1/2}\, V\, p_{1/2}\, p_{1/2}$ in the (J,T)=(1,0) channel. This causes "frustration", which cannot uniquely determine the signs. The actual signs depend on the diagonal matrix elements.
  - Two-particle config.: coherent due to the dominance of v_1 and v_2
  - Two-hole config.: non coherent due to dominance of v_2 and v_3

Origin of difference in sign

- We consider the matrix elements of the delta interaction as a short-range central interaction.
- The j-j coupled matrix elements are written as $abJT\delta(r)cdJT = C_0 \sum_{LS\,(L+S+T=\mathrm{odd})} \eta_{ab}(LSJ)\eta_{cd}(LSJ)$ with $\eta_{ab}(LSJ) = \sqrt{\frac{1-(-1)^{S+T}}{1+\delta_{ab}}}\,\gamma_{LS}^{(J)}(a,b)(-1)^{n_a+n_b}\sqrt{(2l_a+1)(2l_b+1)}\begin{pmatrix} L & l_a & l_b \\ 0 & 0 & 0 \end{pmatrix}$ and $\gamma_{LS}^{(J)}(a,b) = \sqrt{(2j_a+1)(2j_b+1)(2L+1)(2S+1)}\begin{Bmatrix} l_a & 1/2 & j_a \\ l_b & 1/2 & j_b \\ L & S & J \end{Bmatrix}$.
- C_0 depends on (n_a, l_a) etc., but we assume a constant C_0 which is called the surface delta interaction.
- In this case, the $L=0$ contributions are exactly the same as the pairing matrix elements.

L=0 and 2 contributions

- $\langle ab\,JT|V^{\mathrm{SDI}}|cd\,JT\rangle = C_0 \sum_{LS\,(L+S+T=\mathrm{odd})} \eta_{ab}(LSJ)\,\eta_{cd}(LSJ)$
- Restrictions
  - $J=L+S$; S=0 or 1.
  - L is only even when one considers a single-major shell, since $\eta_{ab}(LSJ) \propto \begin{pmatrix} L & l_a & l_b \\ 0 & 0 & 0 \end{pmatrix}$.
- Possible LS
  - Only LS=00 for (J,T)=(0,1)
  - LS=01 and 21 for (J,T)=(1,0)

One must take the LS=21 term into account when fully evaluating the matrix elements of short-range central forces.

Cancellation due to *L*=2

- $abJTV\uparrow\mathrm{SDI}\ cdJT = C\downarrow 0 \sum LS\ (L+S+T=\mathrm{odd})\uparrow\ \eta\downarrow ab\ (LSJ)\eta\downarrow cd$ (LSJ)
- Exact expression of $\eta\downarrow ab\ (L,\ S=1,\ J=1)$ divided by $s=(-1)\uparrow j\downarrow b$

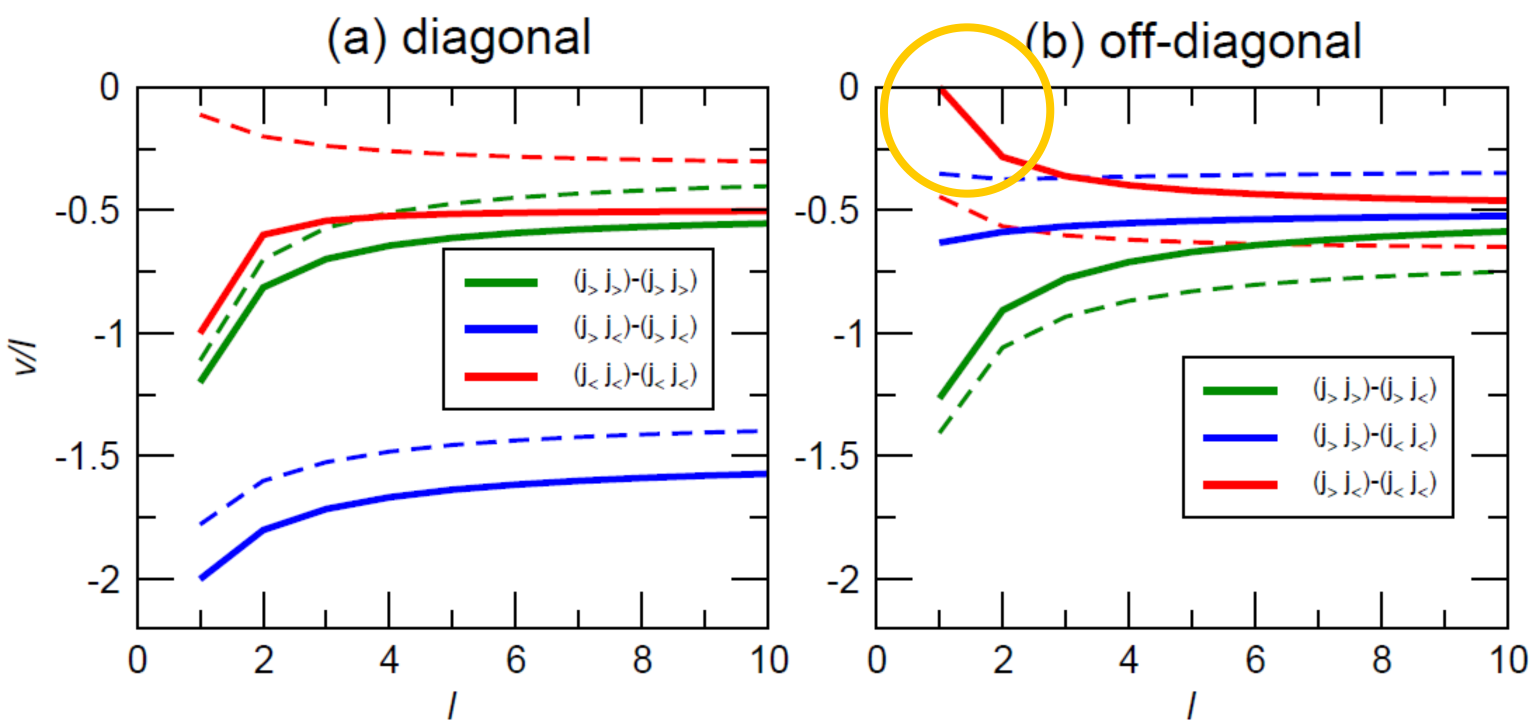
(a,b)	$(j_>, j_>)$	$(j_>, j_<)$	$(j_<, j_<)$	$(j_<, j'_>)$
$L=0$	$\sqrt{\frac{(l+1)(2l+3)}{3(2l+1)}}$	$\sqrt{\frac{8l(l+1)}{3(2l+1)}}$	$\sqrt{\frac{l(2l-1)}{3(2l+1)}}$	
$L=2$	$\sqrt{\frac{2l^2(l+1)}{3(2l+1)(2l+3)}}$	$-\sqrt{\frac{l(l+1)}{3(2l+1)}}$	$\sqrt{\frac{2l(l+1)^2}{3(2l-1)(2l+1)}}$	$-\sqrt{\frac{3(l+1)(l+2)}{2l+3}}$

$l\uparrow' = l-2$

- Clearly, cancellation due to *L*=2 occurs for $j\downarrow> j\downarrow> V\ j\downarrow> j\downarrow<$ and $j\downarrow< j\downarrow< V\ j\downarrow> j\downarrow<$.

Quantitative argument

- $ab\ J{=}1\ T{=}0 V↑$SDI $cd\ J{=}1\ T{=}0\ / l$ with the convention of $(-1)↑ j↓b -1/2\ |ab\ J{=}1\ T{=}0⟩↓$Condon−Shortley

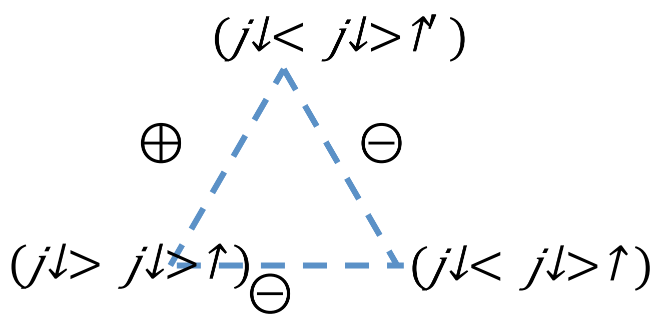


- $j↓< j↓< V$ $j↓> j↓<$ with low-l are strongly cancelled by L=2.
- Finite-range interactions can make $j↓< j↓< V$ $j↓> j↓<$ positive.

Frustration caused by Δ*l*=2 mixing

- Matrix elements concerning $|j_< \; j_>' \rangle$ with $l' = l-2$ (such as $|d_{3/2} \; s_{1/2} \rangle$) does not appear with the *L*=0 terms, and can be an additional source of frustration.
- The sign of $h_{12} \; h_{23} \; h_{31}$ is invariant under phase transformation. When it is positive, the frustration occurs among v_1, v_2 and v_3 $\eta_{ab}(L \; S=1 \; J=1)/(-1)^{j_b - 1/2}$

(a,b)	$(j_>, j_>)$	$(j_>, j_<)$	$(j_<, j_<)$	$(j_<, j_>')$
$L=0$	$\sqrt{\frac{(l+1)(2l+3)}{3(2l+1)}}$	$\sqrt{\frac{8l(l+1)}{3(2l+1)}}$	$\sqrt{\frac{l(2l-1)}{3(2l+1)}}$	
$L=2$	$\sqrt{\frac{2l^2(l+1)}{3(2l+1)(2l+3)}}$	$-\sqrt{\frac{l(l+1)}{3(2l+1)}}$	$\sqrt{\frac{2l(l+1)^2}{3(2l-1)(2l+1)}}$	$-\sqrt{\frac{3(l+1)(l+2)}{2l+3}}$



Tensor-force contributions

- Pointed out by Talmi in terms of the origin of long lifetime of ^{14}C.
- Evaluation with the π+ρ meson exchange tensor force using the $(-1)^{j_b-1/2}\,|ab\,J=1\,T=0\rangle_{\text{Condon-Shortley}}$ convention

p shell

		Matrix element (MeV)
diagonal		+1.40
		+0.59
		-0.78
off-diagonal		+1.16
		-0.22
		+0.43

sd shell

		Matrix element (MeV)
diagonal		+1.05
		+0.51
		-0.23
off-diagonal		+0.76
		-0.25
		+0.21

Pair transfer matrix elements

- It is often discussed that pair transfer probability is a good measure of pairing correlation.
- Isoscalar pair creation and removal probabilities:

$|J_f\, T_f\, ||D^\dagger\, ||J_i\, T_i\, |^2$ and $|J_f\, T_f\, ||D||J_i\, T_i\, |^2$

	Pair creation on vacuum	Pair removal from closure
p	8.1	5.3×10^{-3}
sd	15.7	1.7

Possible effects on pair condensate

- On the basis of strong T=0 attractive force, isoscalar pair condensates are expected to occur especially around N=Z nuclei, similar to well-known isovector pair condensates.
  - $|\Psi\rangle=(\Lambda^{\dagger})^{N}|-\rangle$ with the Cooper pair $\Lambda^{\dagger}=\sum_{ab}\lambda_{ab}[a_{i}^{\dagger}\times a_{j}^{\dagger}]^{J=1\ T=0}$
- There seems to be no strong evidence for isoscalar pair condensates in the actual nuclei.
- As shown in this talk, the signs of the isoscalar pair are not uniquely determined because of frustration. This is a possible reason why isoscalar pair condensates are not well established.

Summary

- We have exactly proven that the GT matrix elements in two-nucleon configurations (2p and 2h) are always in phase with the isovetor- and isoscalar-pairing interactions, which accounts for “low-energy super GT states”.
- The observed hindrance of the GT matrix elements in two-hole configurations is due to “non coherence” of realistic (J,T)=(1,0) matrix elements, which cannot give definite signs in ”Cooper pairs”.
- Difference in sign between isoscalar-pairing and realistic interactions is predominantly caused by L=2 central forces and tensor forces.
- This effect can prevent correlated proton-neutron pair from forming isoescalar-pair condensates, which are not established in experiement.