Towards the proof of AGT-W relation: Toda 3-point function and $W_{1+\infty}$ algebra

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Gaiotto’s discussion for SU(2) quiver

4d $N=2$ SU(2) quiver gauge theory can be regarded as the two M5-branes theory on 2d Riemann surface with punctures.

S-duality (without permutation of masses) can be interpreted as ‘permutation’ of punctures. [Gaiotto ’09]

- inverse of coupling constants
- interchange of flavor sym.: $[SU(2), xSU(2)]$ gives $[SU(2), xSU(2)]$

For SU(2) gauge theory with four fundamental flavors...

The corresponding theory on 2d surface to 4d theory is...

Gaiotto’s discussion for SU(N) quiver

The generalization to SU(N) quiver has some nontrivial points.

The subgroups of flavor symmetry become, for example,

This means more than one ‘kinds’ of punctures exist for SU(N) quiver. These punctures can be classified by Young tableaux with N boxes.

General: $SU(d_j) \times SU(d_j) \times ... \times SU(N) \times ... \times SU(d_j') \times SU(d_j')$

The 3pt function can be calculated thanks to level-1 degenerate condition.

Inverse Shapovalov matrices is of the form of level expansion.

Divergence in 3pt func.: $\langle Y(0) \rangle^2$ (n = # of Hanany-Witten effect)

Internal momenta: consider weak coupling limit! [Drukker-Passner ‘10]

AGT relation: SU(2) quiver and Liouville

• Partition function of 4d $N=2$ SU(2) quiver gauge theory
• Correlation function of 2d Liouville ($A_1$ Toda) field theory

This means that the generalized AGT relation seems to correspond to Young tableaux of $SU(2)$ quiver and $SU(N)$ quiver.

We can naively assume that 4d $SU(N)$ quiver gauge theory corresponds to 2d $A_{N-1}$ Toda theory.

This means that the generalized AGT relation seems to be restricted as $c=N$.

Towards the proof...

• Concrete form of vertex function (for more than one kinds of punctures)
• Need to restrict d.o.f.s of vertex momenta (N-1 degenerate state)
• Need to calculate conformal blocks (p pants decomposition)

AGT-W relation: SU(N) quiver and $A_{N-1}$ Toda

• $A_{N-1}$ Toda theory and $W_{1+\infty}$ algebra
• Remaining part of AGT-W relation is U(1) factor

• Partition function = $U(1)$ factor $\times$ Correlation function
• Our conjecture: $W_{1+\infty}$ algebra contains $U(1)$ generator in addition to Virasoro and $W_\Lambda$ generators.

Our discussion: chain vectors of CFT with $W_{1+\infty}$ algebra can be written as sum of $W_{1+\infty}$ partition function $\times$ Schur polynomials.

Relation between $W_{1+\infty}$ generators and $W_\Lambda$ generators: easy to obtain if central charge of $W_{1+\infty}$ is restricted as $c=N$.

Chain vector of $W_{1+\infty}$: $\Psi_n(x_1, ... , x_N) = \sum_{y_{1},...,y_{N}} \Psi_n(x_1, ... , x_N)$

Expansion on bases of Schur polynomials:

$\Psi_{\Lambda} = \sum_{\text{Sym}(\mathcal{V})} \mathcal{C}(Y_1, ... , Y_N) \Lambda_{V_1}(x_1) \cdots \Lambda_{V_N}(x_N)$

$\Lambda_{V_i}$ is partition function of $W_{1+\infty}$ algebra

• Future directions: more general cases
• General central charge, quiver gauge group, N>3

[Belavin-Belavin ’11]