Phenomenological Aspects of Global F-theory Compactifications

KEK Workshop on String Phenomenology 2010

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Based on:
1. Motivation

A candidate beyond the Standard Model
→ The **Minimal Supersymmetric Standard Model (MSSM)**

A generic renormalizable superpotential from gauge invariance

\[ W_{\text{Generic}} = W_{\text{Yukawa}} + \mu H_u H_d + W_{\Delta L=1} + W_{\Delta B=1} \]

- **The flavor structure**
- **Dangerous terms**
  - Too rapid proton decay when both terms exist.

\[ W_{\text{Yukawa}} = y_u \bar{u} Q H_u - y_d \bar{d} Q H_d - y_e \bar{e} L H_d \]
\[ W_{\Delta L=1} = \frac{1}{2} \lambda^{ijk} L_i L_j \bar{e}_k + \lambda^{i} L_i Q_j \bar{d}_k + \mu^i L_i H_u \]
\[ W_{\Delta B=1} = \frac{1}{2} \lambda^{i} \bar{u}_i \bar{d}_j \bar{d}_k \]
1. Motivation

The MSSM indicates that the three gauge couplings meet at such a high energy as \(2 \times 10^{16} \text{GeV}\).

→ Grand Unified Theory (GUT)

The matter content for the Georgi–Glashow SU(5) GUT model

\[
10^M \rightarrow (3, 2)_{\frac{1}{6}} + (\bar{3}, 1)_{-\frac{2}{3}} + (1, 1)_{1} \quad 5^H \rightarrow (3, 1)_{-\frac{1}{3}} + (1, 2)_{\frac{1}{2}}
\]

\[
\bar{5}^M \rightarrow (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{-\frac{1}{2}} \quad \bar{5}^H \rightarrow (\bar{3}, 1)_{\frac{1}{3}} + (1, 2)_{-\frac{1}{2}}
\]

The gauge invariant superpotential

\[W_{\text{GUT}} \supset y_u 10^M 10^M 5^H + y_d \bar{5}^M 10^M \bar{5}^H + \lambda \bar{5}^M 10^M \bar{5}^M\]

The flavor structure

Ex. Yukawa matrices are approximately rank 1.

Too rapid proton decays

Hence, this term is unlikely to be present.

What is the origin of those structures?

→ String theory could address the issues.
1. Motivation

Which vacua are suitable for the description of GUT models?

◊ Requirements: (i) Supersymmetric vacua
   (ii) The generation of all the necessary Yukawa couplings
   (iii) Decoupling gravity from GUT gauge theories

(i), (ii) → Heterotic string theory with the $E_8 \times E_8$ gauge symmetry satisfies the two conditions.
   The up-type Yukawa coupling $y_{u,i\ell} 10_{M,i}^{ab} 10_{M,j}^{cd} 5_e^e H_{abcde}$ is difficult to be generated from the other four string theories.

(iii) → Heterotic string theory predicts a special relation between the Newton constant and the GUT gauge coupling.

$$G_N = \frac{\alpha_{\text{GUT}} \alpha'}{4}$$

Witten ‘96

Actually, there is other way, so-called F-theory, which satisfies the above three conditions.
2. F-theory

F theory compactifications are special compactifications of Type IIB string theory.

We compactify Type IIB string theory on a 6-dim background where the string coupling \( \tau = C_0 + i e^{-\phi} \) is NOT constant, but is holomorphic. \( \to \) We get \( N=1 \) supersymmetric four dimensional theory.

Since \( \tau \) has \( SL(2, \mathbb{Z}) \) symmetry in Type IIB string theory, we can think of \( \tau \) as a complex structure modulus of torus.

\[ \text{CY}_4 \quad \text{Not CY}_3 \]

\[ \text{IIB} \quad \text{F} \]

\( \leftarrow 2 \text{ dim} \quad \leftarrow 6 \text{ dim} \)
What is happening where the torus fiber is singular?

At a singular point $z_i$, complex structure behaves as

$$\tau(z) \sim \frac{1}{2\pi i} \ln(z - z_i)$$

So, when circling around $z = z_i$, $\tau$ undergoes monodromy.

$$\tau \rightarrow \tau + 1 \quad (i.e. \ C_0 \rightarrow C_0 + 1)$$

Since $C_0$ magnetically couples to a D7-brane, there is a D7-brane at $z = z_i$! Thus, F-theory incorporates the back-reaction of D7-branes into its geometry.

$z = z_i$ is a (complex) codimension 1 subspace of 6-dim manifold. Hence, the dimension of the branes should be $1+3+(6-2)=1+7$. This is consistent with the dimension of D7-branes.

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Suppose that there is a D7-brane where the $\alpha$-cycle of the torus vanishes, there is a $[p, q]$ 7-brane where the $(p\alpha+q\beta)$-cycle vanishes.

A open string ending on a D7-brane

\[ \text{SL}(2, \mathbb{Z}) \text{ transformation} \]

A $(p, q)$ string ending on a $[p, q]$ 7-brane

The appearance of $[p, q]$ 7-branes introduces “string junctions”.

Schwarz '95

A string junction $\rightarrow$ a gauge field

(in a similar manner as gauge fields couple with the ends of open strings)
The generation of the E-type gauge symmetries with string junctions

**Ex. $E_8$ gauge symmetry**  Gaberdiel, Zwiebach ‘97

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The intersection form

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Gauge symmetries in F-theory

Vanishing two-cycles with the intersection form of the corresponding type of the Dynkin diagram

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The Dynkin diagram of $E_8$
2. F-theory

Focus here to extract gauge theories

Ex. SU(5)
5 D7-branes ↔ 4 vanishing two-cycles

7-branes wrap on the surface
Ex. SU(5) GUT: The up-type Yukawa coupling

SU(6) \supset SU(5): 35 \rightarrow 24 + 1 + 5 + \bar{5}

SO(10) \supset SU(5): 45 \rightarrow 24 + 1 + 10 + \bar{10}
F-theory local models: Focusing only on “small” two-cycles
2. F-theory (Local Models)

Localized fields on 7-branes:

\[ C_3 = A \wedge \omega \quad \omega \in H^{1,1}(CY_4) \]

not \( H^{1,1}(B) \) nor \( H^{1,1}(T^2) \)

\[ \beta^{(3,1)} = \varphi^{(2,0)} \wedge \omega \quad \beta^{(3,1)} \in H^{3,1}(CY_4) \]

Chiral multiplet

\[ A_\mu \quad A_m \quad A_{\bar{m}} \]

Vector multiplet

\[ \varphi_{mn} \quad \bar{\varphi}_{\bar{m}\bar{n}} \]

8 dim gauge fields \quad Adjoint Higgs fields

(transverse deformations of 7-branes)

The Superpotential in F-theory local models

\[ W = \int_S \text{tr}(\varphi^{(2,0)} \wedge (\bar{\partial}A^{(0,1)} + A^{(0,1)} \wedge A^{(0,1)})) \]

F-term condition:

\[ \bar{\partial}A^{(0,1)} + A^{(0,1)} \wedge A^{(0,1)} = 0 \]

\[ \bar{\partial}_A \varphi^{(2,0)} = 0 \]

(D-term condition

\[ g \wedge F^{(1,1)} - \frac{i}{2}[\varphi^{(2,0)}, \bar{\varphi}^{(0,2)}] = 0 \)
Incorporating matters

$S(s_1, s_2)$

$G_\Sigma$

Katz, Vafa ’96
Beasley, Heckman, Vafa 0802
3. Yukawa Couplings in F-theory

Incorporating matters

\[ \det \langle \varphi(s_1, s_2) \rangle = 0 \]

\[ G_\Sigma \supset \langle \varphi \rangle \times G_S = G_{\text{GUT}} \]

Katz, Vafa ‘96
Beasley, Heckman, Vafa 0802
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Incorporating matters
\[ \text{det}(\langle \varphi(s_1, s_2) \rangle) = 0 \]

“off-diagonal components”

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Incorporating matters
\[
\text{det}(\varphi(s_1, s_2)) = 0
\]

"off-diagonal components"

\[
G_\Sigma \supset \langle \varphi \rangle \times G_S = G_{\text{GUT}}
\]

Application to Yukawa couplings:
Ex. SU(3) → U(1) × U(1)

Background Higgs fields:
\[
\langle \varphi_{12} \rangle = \frac{1}{3} \begin{pmatrix}
-2s_1 + s_2 & 0 & 0 \\
0 & s_1 + s_2 & 0 \\
0 & 0 & s_1 - 2s_2
\end{pmatrix}
\]

- Generic → U(1) × U(1)
- s_1=0 → SU(2) × U(1)
- s_2=0 → SU(2) × U(1)
- s_1-s_2=0 → SU(2) × U(1)
- s_1=s_2=0 → SU(3)
3. Yukawa Couplings in F-theory

Yukawa coupling: \[ y = \int_S \text{tr}(\delta \tilde{\varphi} \wedge \delta \tilde{A} \wedge \delta \tilde{A}) \]

\[ y_{ij} = \frac{1 + \sqrt{2}}{g} \int_S \tilde{f}_i(s_2)e^{-\frac{1}{\sqrt{g}}|s_1|^2} \tilde{g}_j(s_1)e^{-\frac{1}{\sqrt{g}}|s_2|^2} e^{-\frac{1}{2\sqrt{g}}|s_1-s_2|^2} ds_1 \wedge d\bar{s}_1 \wedge ds_2 \wedge d\bar{s}_2 \]

\[ = \frac{1 + \sqrt{2}}{g} \int_S (s_1^i s_2^j e^{-\frac{1}{\sqrt{g}}|s_1|^2} e^{-\frac{1}{\sqrt{g}}|s_2|^2} e^{-\frac{1}{2\sqrt{g}}|s_1-s_2|^2} ds_1 \wedge d\bar{s}_1 \wedge ds_2 \wedge d\bar{s}_2 \]

\[ \frac{1}{(2\pi)^2} y_{ij} = \begin{cases} 1, & (i = j = 0) \\ 0, & \text{otherwise} \end{cases} \]

\[ s_1 \rightarrow e^{i\theta} s_1 \]
\[ s_2 \rightarrow e^{i\theta} s_2 \]

Rank 1 from a single Yukawa point!
+ sub-leading corrections from bulk fluxes

Heckman, Vafa 0811
Cecotti, Cheng, Heckman, Vafa 0909
Hence, a single point seems enough to reproduce the flavor structure. However, there is a global constraint between topological invariants.

\[ 3 \# E_6 = 2 \# D_6 + 2g_{10} - 2 \]

→ The number of the up-type Yukawa points are \textbf{EVEN}!

The generic F-theory compactifications predict the up-type Yukawa matrix of the \textit{even rank}.

Solutions:

The physical coupling should be properly normalized and this could induce exponential suppression along the matter curve.

Factorize the locus where matter fields are localized so that the number of the up-type Yukawa point involved with massless fields in the representation of the GUT gauge group becomes one.
Proposed Solutions so far

(i) Global compactifications with $\mathbb{Z}_2$ symmetry

Tatar, Tsuchiya, Watari 0905
H.H. et al 0910, ...

(ii) Rank 5- GUT scenario with an U(1) Flux

Tatar, Watari 0602, ...

(iii) Factorized spectral surface
(using an unbroken U(1) symmetry obtained by taking a special configuration of adjoint Higgs VEV)

Tatar, Tsuchiya, Watari 0905
Marsano, Saulina, Schafer-Nameki 0906, 0912
Grimm, Krause, Weigand 0912,
Chen et al 1005, ...

(iv) Spontaneous R-parity violation

Tatar, Watari 0602, Tatar, Tsuchiya, Watari 0905
Blumenhagen, Grimm, Jurke, Weigand 0908, ...
4. The Dimension-4 Proton Decay Operator in F-theory

☆ Proposed Solutions so far

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The (iii) scenario is not without a theoretical concern. We need to see carefully whether this U(1) is actually unbroken or not.
Note that we break a parent gauge symmetry by adjoint Higgs VEV.

**Ex.** Rank 2 case: $\langle SU(2) \rangle \to$ naively we expect an $U(1)$ symmetry unbroken...

$$a_0(\det(\xi I_{2 \times 2} - \langle \varphi \rangle)) = a_0 \xi^2 + a_2 = 0 \rightarrow \xi = \pm \sqrt{\frac{a_2}{a_0}}$$

$$\langle \varphi(s_1, s_2) \rangle = \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & \circ \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix} \rightarrow \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & \circ \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix}$$

This monodromy kills the $U(1)$ symmetry.

**Factorization:**

$$a_0 \xi^2 + a_2 = (c_0 \xi + c_1)(d_0 \xi + d_1) \rightarrow \langle \varphi(s_1, s_2) \rangle = \begin{pmatrix} \frac{c_1}{c_0} & 0 \\ 0 & -\frac{c_1}{c_0} \end{pmatrix}$$

Then, we have no monodromy and there seems to be an unbroken $U(1)$ symmetry.

This kind of $U(1)$ symmetry seems to be able to be used for prohibiting the dimension-4 proton decay operators when applying to the SU(5) model.

Tatar, Tsuchiya, Watari 0905
Marsano et al 0906, 0912
Grimm et al 0912, Chen et al 1005, ...
Is this scenario actually without a theoretical concern?

**Factorization:** $a_0 \xi^2 + a_2 = (c_0 \xi + c_1)(d_0 \xi + d_1) \rightarrow \xi = \frac{c_1}{c_0}, \xi = \frac{d_1}{d_0} = -\frac{c_1}{c_0}$

$\langle \varphi(s_1, s_2) \rangle = \begin{pmatrix} \frac{c_1}{c_0} & 0 \\ 0 & -\frac{c_1}{c_0} \end{pmatrix}$

There can be a region where $c_0 \approx 0$ and $\langle \varphi \rangle \rightarrow \infty$. The 8D gauge theory approximation becomes subtle.

The monodromy is really reduced when we take into account this region?

We throw away the 8D gauge theory description and reconsider the factorization limit from global F-theory compactifications.
The Dimension-4 Proton Decay Operator in F-theory

The origin of U(1) vector fields.

\[ C_3 = A \wedge \omega \]

\[ \omega \in H^{1,1}(CY_4) \]

not \( H^{1,1}(B) \) nor \( H^{1,1}(T^2) \)

Poincare dual

Globally well-defined two-cycles over \( S \)

Ex. Not globally well-defined two-cycles

Two-cycles could be acted by monodromies.

We can obtain the monodromies by keeping track of the locations of 7-branes since the two-cycles are placed between 7-branes.
4. The Dimension-4 Proton Decay Operator in F-theory

Ex. K3 fib. CY$_4$ with an E$_6$ gauge theory \( \langle \varphi \rangle = \langle \text{SU}(3) \rangle \)

\[
y^2 = x^3 + (a_2 z^3 + f_0 z^4) x + \left( \frac{1}{4} a_3^2 z^4 + a_0 z^5 + g_0 z^6 + a''_0 z^7 \right)
\]

\[
\Delta = 4F^3 + 27G^2 = z^8 \times (\text{degree 6 polynomial})
\]

8 7-branes at \( z=0 \) ↑

(\( E_6 \) gauge symmetry)

8D gauge theory region:
\[
a_0 = a_{0,*} \epsilon_{\eta}, \quad a_2 = a_{2,*} \epsilon_K^2 \epsilon_{\eta}, \quad a_3 = a_{3,*} \epsilon_K^3 \epsilon_{\eta}
\]
\[
|a_{r,*}| \sim \mathcal{O}(1), \quad 0 \neq |\epsilon_K| << 1, \quad 0 \neq |\epsilon_{\eta}| < 1
\]

\[
E_8 \supset E_6 \times \langle \text{SU}(3) \rangle
\]

\[
\langle \text{SU}(3) \rangle \supset S_3 \supset Z_2
\]

The (2+1) factorization

\[
a_0 \xi^3 + a_2 \xi + a_3 = (c_0 \xi^2 + c_1 \xi + c_2)(d_0 \xi + d_1)
\]
4. The Dimension-4 Proton Decay Operator in F-theory

8D gauge theory region:
\[ a_0 = a_0, \epsilon \eta, \quad a_2 = a_2, \epsilon_2 \epsilon \eta, \quad a_3 = a_3, \epsilon_3 \epsilon \eta \]
\[ |a_{r,*}| \sim O(1), \quad 0 \neq |\epsilon_K| \ll 1, \quad 0 \neq |\epsilon_\eta| < 1 \]

Beyond 8D gauge theory region:
\[ a_{0,*} \sim |\epsilon_K|^2 \]
4. The Dimension-4 Proton Decay Operator in F-theory

8D gauge theory region:
\[ a_0 = a_{0,*} \varepsilon_\eta, \quad a_2 = a_{2,*} \varepsilon_K^2 \varepsilon_\eta, \quad a_3 = a_{3,*} \varepsilon_K^3 \varepsilon_\eta \]
\[ |a_{r,*}| \sim \mathcal{O}(1), \quad 0 \leq |\varepsilon_K| << 1, \quad 0 \neq |\varepsilon_\eta| < 1 \]

Beyond 8D gauge theory region
\[ a_{0,*} \sim |\varepsilon_K|^2 \]

Monodromy is beyond \(S_3\)!

\[ a_0 \xi^3 + a_2 \xi + a_3 \]
\[ = (c_0 \xi^2 + c_1 \xi + c_2)(d_0 \xi + d_1) \]

The (2+1) factorization \(\Rightarrow\)

This monodromy is not reduced in the factorization limit \(\Rightarrow\) The U(1) symmetry is broken!
5. Conclusion

F-theory compactifications are good candidates for GUT models. In particular, F-theory can generate the up-type Yukawa coupling.

**Yukawa couplings**

- Approximately rank 1 from a single Yukawa point
- The number of the up-type Yukawa points are generically even.
  - This can be cured from the normalization or the factorization of matter curves.

**The dimension-4 proton decay operator**

- The factorization limit is supposed to generate an U(1) symmetry
- The supposed U(1) symmetry is indeed broken from the monodromy effects beyond the 8D gauge theories.

Which solution is good? Can we compute the effect beyond the 8D gauge theories?