An Introduction to Intersecting D-brane Models

-- a very brief introduction --

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1. Introduction

Why particle models in string theory?

1. Inclusion of quantum gravity.

Understanding beyond field theory models through the calculable (in principle)
Yukawa couplings,
gauge couplings,
higher-dimensional interactions.
A long history of the model building based on the heterotic string theory
a special N=1 supersymmetric string theory with $E_8 \times E_8$ gauge symmetry in 10D.

but, no completely realistic model.....

A new framework (recently proposed):

**Intersecting D-brane Models**

based on type II string theories
special N=2 supersymmetric string theories with no gauge symmetry in 10D.

with several D-branes intersecting in compactified extra six-dimensional space
2. Intersecting D-branes

Dp-brane:

- a p-dimensional object in string theory on which the ends of open strings are fixed.

- $U(1)$ gauge symmetry on a single D-brane.
- $U(N)$ gauge symmetry on N D-brane. (multiplicity N)

The gauge field is localized on the $(p+1)$-dimensional world-volume.
Intersecting D6-branes (in type IIA theory)

<table>
<thead>
<tr>
<th>10D:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>D6  :</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

- Open string localized at the intersection point (3D space)
- Chiral fermion localized at the intersecting point (3D space)

Berkooz-Douglas-Leigh, hep-th/9606139
mechanism for chiral fermion (very briefly)

\[
\begin{align*}
D_1 & \to \theta_1 & (X_5, \psi^5, \tilde{\psi}^5) \\
\text{an open string from } D_1 \text{ to } D_2 & \to (X_4, \psi^4, \tilde{\psi}^4)
\end{align*}
\]

\[\begin{align*}
at \sigma = 0 & \\
\begin{cases} 
\psi^4 = e^{-2\pi i \nu} \tilde{\psi}^4 \\ 
\psi^5 = -e^{-2\pi i \nu} \tilde{\psi}^5
\end{cases} & \text{(free edge)} \quad \text{Neumann} \quad \nu = 0, 1/2 \\
\text{(R, NS)} & \\
at \sigma = \pi & \\
\begin{cases} 
\cos \theta_1 \psi^4 + \sin \theta_1 \psi^5 = \cos \theta_1 \tilde{\psi}^4 + \sin \theta_1 \tilde{\psi}^5 \\ 
-\sin \theta_1 \psi^4 + \cos \theta_1 \psi^5 = -(\sin \theta_1 \tilde{\psi}^4 + \cos \theta_1 \tilde{\psi}^5)
\end{cases} & \text{(fixed edge)}
\end{align*}\]
NS-sector states ($\nu = 1/2$)

\[ \psi_{\alpha_1-1/2-n} |0\rangle_{\text{NS}}, \quad n \geq 0 \]  

massive, massless, tachyonic

\[ m^2 = -(\alpha_1 - \frac{1}{2} - n) + \frac{1}{2}(-1 + \alpha_1 + \alpha_2 + \alpha_3) = \frac{1}{2}(-\alpha_1 + \alpha_2 + \alpha_3) + n \]

\[ \tilde{\psi}_{-\alpha_1-1/2-n} |0\rangle_{\text{NS}}, \quad n \geq 0 \]  

massive

\[ m^2 = -(-\alpha_1 - \frac{1}{2} - n) + \frac{1}{2}(-1 + \alpha_1 + \alpha_2 + \alpha_3) = \frac{1}{2}(2\alpha_1 + \alpha_2 + \alpha_3) + n \]

R-sector states ($\nu = 0$)

\[ |S^{4\text{D}}_+\rangle_R \]  

massless chiral fermion

\[ \psi_{\alpha_i-1-n} |S^{4\text{D}}_-\rangle_R, \quad n \geq 0 \]  

massive

\[ m^2 = 1 - \alpha_i + n > 0 \]

\[ \tilde{\psi}_{-\alpha_i-n} |S^{4\text{D}}_-\rangle_R, \quad n \geq 0 \]  

massive

\[ m^2 = \alpha_i + n > 0 \]

\[ \alpha_i \equiv |\theta_i|/\pi, \quad 0 \leq \alpha_i \leq 1/2 \]
representation of chiral fermions

\[ N_a \]

\[ N_b \]

\[ (\Box_a, \bar{\Box}_b) \text{ under } U(N_a) \times U(N_b) \]

intersecting D-branes

the Standard Model may be on intersecting D-branes?
the strength is geometrically determined: \( g_Y \simeq e^{-\frac{A}{2\pi\alpha'}} \)
But, not so simple.....
There are constrains on D-brane configurations.

1) supersymmetry conditions
   not necessary, but for stable configurations

2) RR tadpole cancellation conditions
   for a consistent string theory and 4-dimensional Lorentz invariance
Let’s consider a more concrete set up: intersecting in $T^6 = T^2 \times T^2 \times T^2$.
D6$_a$-brane wrapping on $T^6 = T^2 \times T^2 \times T^2$

wrapping numbers

$[(n_1^a, m_1^a), (n_2^a, m_2^a), (n_3^a, m_3^a)]$

ex.

(1, 0)  (1, 1)  (1, −1)
intersection numbers

number of intersections between $D6_a$ and $D6_b$ branes

$$I_{ab} = \prod_{i=1}^{3} (n^i_a m^i_b - m^i_a n^i_b)$$

$I = 1$ \hspace{2cm} $I = 2$

$(1,0) \times (1,1)$ \hspace{2cm} $(1,0) \times (1,2)$

> If two branes are parallel in some of the three torus, the intersecting number becomes zero.

> Multiple intersecting number means multiple fields with same gauge charges.
supersymmetry conditions

\[ \theta_1^a = \tan^{-1} \left( \frac{R_2^{(1)} m_1^a}{R_1^{(1)} n_1^a} \right) \]

\[ \theta_1^a + \theta_2^a + \theta_3^a = 2\pi n, \quad n \in \mathbb{Z} \]

for all D6\(_a\)-branes

Berkooz-Douglas-Leigh, hep-th/9606139
RR tadpole cancellation conditions in intersecting D-brane systems

\[ + + + = 0 \]

many intersecting D-branes with different multiplicities and winding numbers

constraints on the multiplicities and winding numbers of D-branes
RR tadpole cancellation conditions

\[ \sum_{a} N_{a} n_{a}^{1} n_{a}^{2} n_{a}^{3} = 16, \]

\[ \sum_{a} N_{a} n_{a}^{1} m_{a}^{2} m_{a}^{3} = -16, \]

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Gauss low for RR charges in compact space

\[ T^{6}/(\mathbb{Z}_{2} \times \mathbb{Z}_{2}) \text{orientifold} \]

Cvetic-Shiu-Uranga, hep-th/0107166
A $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold in type IIA theory

$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold projections $\theta$ and $\omega$

Orientifold projection $\Omega R$

$\Omega :$ world-sheet parity

$R : \quad X^{5,7,9} \rightarrow -X^{5,7,9}$
3. Some examples of models

There are many models constructed.

- SUSY or non-SUSY
- type IIA or type IIB
- different compactified spaces

in this review:

### Three generation model by M.Cvetic, G.Shiu and A.M.Uranga

**D6-branes and their winding numbers**

<table>
<thead>
<tr>
<th>D6-brane</th>
<th>winding number</th>
<th>multiplicity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$[(0,1), (0,-1), (2,0)]$</td>
<td>8</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$[(1,0), (1,0), (2,0)]$</td>
<td>2</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$[(1,0), (1,-1), (1,3/2)]$</td>
<td>4</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$[(1,0), (0,1), (0,-1)]$</td>
<td>2</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$[(1,-1), (1,0), (1,1/2)]$</td>
<td>6 + 2</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$[(0,1), (1,0), (0,-1)]$</td>
<td>4</td>
</tr>
</tbody>
</table>

**U(2)**

**U(3) x U(1)**

**tilted torus compactification**
a schematic picture of the configuration
(the relative place of each D-brane has no meaning)
definition of hypercharge

\[ Q_Y \equiv \frac{1}{6} Q_3 - \frac{1}{2} Q_1 + \frac{1}{2} (Q_8 + Q'_8) \]

\( U(3)_c \times U(1) \) of \( C_1 \) brane

Phenomenology of this model is discussed in Cvetić-Langacker-Shiu, PR D66 (2002) 066004.

The Yukawa couplings of this model is discussed in Cvetić-Langacker-Shiu, NP B642 (2002) 139.
Two Main Difficulties

1) Many additional gauge symmetries
   not easy to obtain the Standard Model particle contents
   without any exotics

2) Obtaining non-trivial Yukawa Couplings
   There is no explicit model with non-trivial Yukawa coupling matrices
   with both the hierarchy and flavor mixings in usual intersecting D-brane models
   in which the origin of the generation is the multiple intersections of D-branes.
No no-go theorem for non-trivial Yukawa coupling matrices,

but

there is a common understanding that some new idea are required.
A model with non-trivial Yukawa couplings

D6-branes and their winding numbers

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<tr>
<td>D6₁</td>
<td>[(1, −1), (1, 1), (1, 0)]</td>
<td>4</td>
</tr>
<tr>
<td>D6₂</td>
<td>[(1, 1), (1, 0), (1, −1)]</td>
<td>6 + 2</td>
</tr>
<tr>
<td>D6₃</td>
<td>[(1, 0), (1, −1), (1, 1)]</td>
<td>2 + 2</td>
</tr>
<tr>
<td>D6₄</td>
<td>[(1, 0), (0, 1), (0, −1)]</td>
<td>12</td>
</tr>
<tr>
<td>D6₅</td>
<td>[(0, 1), (1, 0), (0, −1)]</td>
<td>8</td>
</tr>
<tr>
<td>D6₆</td>
<td>[(0, 1), (0, −1), (1, 0)]</td>
<td>12</td>
</tr>
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N. Kitazawa, hep-th/0403278
left-handed sector

\[ [C_\alpha D_\alpha] \sim q_\alpha \]
\[ [N_\alpha D_\alpha] \sim l_\alpha \]
left-handed \( q_{L\alpha} \), \( l_{L\alpha} \)

right-handed \( u_{R\beta} \), \( d_{R\beta} \), \( \nu_{R\beta} \), \( e_{R\beta} \)

Higgs \( H_\alpha, \bar{H}_\alpha \)

**Diagram:**

- **Left-hand side (blue):** D61, \((USp(2))^6\), \(U(2)_L\), \(U(1)\), \(U(3)_c\)
- **Center:** D66, \((USp(2))^6\)
- **Right-hand side (green):** D63, \(U(1)\), \((USp(2))^4\)
- **Bottom:** D62, D64, D65
\[
g \sim \frac{e^{-A/2\pi\alpha'}}{M_s^3}
\]
The Yukawa coupling matrices depend on the geometrical configurations of D6-branes.

dynamical generation of Yukawa couplings

\[
\begin{align*}
\sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g^{u}_{\alpha\beta a}}{M_{s}^{3}} [C_{\alpha}D_{\alpha}] [\bar{C}_{\beta} \bar{D}_{\beta} (-)] [T_{a} T_{a}^{(+)}] & \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} g^{u}_{\alpha\beta a} \frac{\Lambda L \Lambda R \Lambda H}{M_{s}^{3}} q_{\alpha u_{\beta} H_{a}^{(2)}}, \\
\sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g^{d}_{\alpha\beta a}}{M_{s}^{3}} [C_{\alpha}D_{\alpha}] [\bar{C}_{\beta} \bar{D}_{\beta} (+)] [T_{a} T_{a}^{(-)}] & \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} g^{d}_{\alpha\beta a} \frac{\Lambda L \Lambda R \Lambda H}{M_{s}^{3}} q_{\alpha d_{\beta} \bar{H}_{a}^{(1)}}, \\
\sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g^{\nu}_{\alpha\beta a}}{M_{s}^{3}} [N_{\alpha}D_{\alpha}] [\bar{N}_{\beta} \bar{D}_{\beta} (-)] [T_{a} T_{a}^{(+) \text{ }]} & \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} g^{\nu}_{\alpha\beta a} \frac{\Lambda L \Lambda R \Lambda H}{M_{s}^{3}} l_{\alpha \nu_{\beta} H_{a}^{(2)}}, \\
\sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} \frac{g^{e}_{\alpha\beta a}}{M_{s}^{3}} [N_{\alpha}D_{\alpha}] [\bar{N}_{\beta} \bar{D}_{\beta} (+)] [T_{a} T_{a}^{(-)}] & \sim \sum_{\alpha,\beta=1}^{6} \sum_{a=1}^{4} g^{e}_{\alpha\beta a} \frac{\Lambda L \Lambda R \Lambda H}{M_{s}^{3}} l_{\alpha e_{\beta} \bar{H}_{a}^{(1)}}.
\end{align*}
\]
Yukawa coupling matrices can be non-trivial

N.K, T.Kobayashi, N.Marui and N.Okada, hep-th/0406115

dominant $2 \times 2$ sub matrices

$$
\begin{pmatrix}
\bar{c}_L & \bar{t}_L \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1\varepsilon_3 & \varepsilon_1^2\varepsilon_3 \\
0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
c_R \\
t_R \\
\end{pmatrix}
$$

$$
\begin{pmatrix}
\bar{s}_L & \bar{b}_L \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_3 & \varepsilon_1\varepsilon_3 \\
0 & \varepsilon_1 \\
\end{pmatrix}
\begin{pmatrix}
s_R \\
b_R \\
\end{pmatrix}
$$

“$m_c/m_t$” $\sim 10^{-3}$, “$m_s/m_b$” $\sim 10^{-2}$, “$V_{cb}$” $\sim 10^{-2}$

by taking $\varepsilon_1 \sim 0.5$ and $\varepsilon_3 \sim 0.01$

$$
\varepsilon_i \simeq \exp(-A_i/2\pi\alpha'),
$$

$A_i : 1/8$ areas of each three torus $i = 1, 2, 3$
5. Summary and Conclusion

1) some motivations for string phenomenology.
2) a very brief introduction to intersecting D-branes.
3) a very brief introduction to two explicit models.

It would be very interesting to explore more realistic models in this framework.

It is much more interesting to construct models with flux compactifications.

(many difficulties expected may be solved)