Bound-State Effects on Top-Quark Production at Hadron Colliders

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Plan of my talk:

• Introduction: Review of ttbar production at Hadron Colliders

• Bound-State effects on Top-Quark production

• Numerical results: ttbar Invariant-mass distributions

• Summary
Introduction:

• Top-quark discovered at Tevatron (1995)
  \[ \# \sim 8\text{(pb)} \times 3\text{(fb}^{-1}\text{)} \sim 2 \times 10^4 \]

• Mass measurement (CDF and D0 combined)
  \[ m_t = 172.4 \pm 0.7\text{(stat.)} \pm 1.0\text{(syst.)} \]

• At the LHC
  \[ \sigma_{tt}(\text{LHC}) \sim 100 \times \sigma_{tt}(\text{TeV}) \]
  (about half when 14\text{TeV} \rightarrow 10\text{TeV})
  
  8M Top-pairs per 10\text{fb}^{-1} \quad \rightarrow \quad 800,000 \text{ lepton+jets events after cuts}

Detail study will be possible
  \[ \rightarrow \text{check Top property, SM prediction, QCD-Jet physics,} \ldots \]
• Top-quark pair production at hadron colliders

• Parton Subprocesses

\[ q\bar{q} \rightarrow t\bar{t} \quad \text{Color: Octet} \]
\[ |J| = 1 \quad 85\% \quad 10\% \]

\[ gg \rightarrow t\bar{t} \quad \text{Color: Singlet, Octet} \]
\[ |J| = 0, 1, 2, \ldots \quad 15\% \quad 90\% \]

• Hadronic cross-section

\[ \sigma_{tt}(s) = \sum_i \int d\tau \frac{dL_i}{d\tau}(\tau)\hat{\sigma}_i(\hat{s} = \tau s) \]

\[ \frac{dL_i}{d\tau}(\tau, \mu_F) = \int dx_1 dx_2 \delta(\tau - x_1 x_2) f_a(x_1, \mu_F) f_b(x_2, \mu_F) \]
• Perturbative calculations for the **partonic cross-section** :

\[ \hat{\sigma}_i(\hat{s}) = \hat{\sigma}_i^{(0)}(\hat{s}) + \frac{\alpha_s}{\pi} \hat{\sigma}_i^{(1)}(\hat{s}; \mu) + \left( \frac{\alpha_s}{\pi} \right)^2 \hat{\sigma}_i^{(2)}(\hat{s}; \mu) + \cdots \]

• NLO : Dawson,Ellis,Nason(‘88), Beenakker etal.(‘90)
  (Analytic) : Cazkon,Mitov(‘08)

• NLL : Bonciani etal.(‘98),,; NNLL : Moch,Uwer(‘08),

• NNLO(appx.) : Kidonakis,Vogt(‘08)

• Building blocks for full **NNLO** correction :
  Korner etal.(‘06),Dittmaier etal.(‘07),Cazkon etal.(‘07)

**scaling function:**

\[ \sigma_i^{(k)}(\hat{s}) = \frac{\alpha_s^2}{m_t^2} \cdot f_i^{(k)}(\beta) \quad \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \quad \text{velocity of top-quarks in ttbar C.O.M. frame} \]
Recent analysis on the total cross-sections:
(updates the mass, PDF and resummation)

- S.Moch&P.Uwer, arXiv:0804.1476
- M.Cacciari etal., arXiv:0804.2800

Theoretical uncertainties = (Ren & Fac scales) + (PDF)

- ~9% (NLL)
- ~12% (NLO)
- >7% (TeV)
- >4% (LHC)
Cross-Section Measurements at CDF and D0

(Mt=175 GeV is assumed)
• Top-quarks are produced copiously at the LHC (9M per 10fb⁻¹)
  → more detail measurement can be possible than at the Tevatron.

• Theoretical prediction to the cross-sections well-developed.
  → ttbar production as a “Standard Candle”.

• At the LHC, ttbar is produced mainly via gluon-fusion,
  → color-singlet ttbar pairs can be produced.
  (while qqbar dominated and only color-octet at the Tevatron)

Color-Singlet  →  It is known for a long time in e⁺e⁻ collider study that
  ttbar form a broad resonance peak below the threshold, and a
  measurement of the resonance is useful to determine top-quark
  mass, etc.

Our study :  Bound-state can be seen in HCs (especially LHC) ?

Earlier study: Fadin,Khoze,Sjostrand(’90)

Updates + Including ISR/FSR: Hagiwara,Sumino,HY(’08); Kiyo etal.(’08)
• NLO correction near partonic threshold: \((\bar{s} \sim 2m_t, \beta \rightarrow 0)\)

\[
\hat{\sigma}_i^{c,(1)} \sim \hat{\sigma}_i^{c,(0)} \left[ A_i \ln^2(8\beta^2) + B_i^{(c)} \ln(8\beta^2) + C_i^{(c)} \frac{\pi^2}{\beta} + D_i^{(c)} + \mathcal{O}(\beta) \right]
\]

\(i=qq,gg\)

Threshold logs: emission of soft and/or collinear gluon in initial-state and final-state

Coulomb singularity: Coulomb gluon exchange between \(t\) and \(t\)-bar

Hard correction: process dependent

• Factorization of each contributions (at NLO level):
Coulomb singularity ⇒ Binding effects (Green’s function formalism)

= summation of ladder-diagrams + finite width effects

\[ \frac{1}{\alpha_s/\beta} \left( \frac{\alpha_s}{\beta} \right)^2 \]

Fadin, Khoze ('87)

Schrodinger equation:

\[ (E + i\Gamma) - \left\{ -\frac{\nabla^2}{m_t} + V_{QCD}(r) \right\} \right \] \[ G^{(c)}(\vec{x}, E) = \delta^3(\vec{x}) \]

\[ \tilde{\sigma}_{Coul.} = \tilde{\sigma}_0 \left( 1 + \frac{\alpha_s}{\pi} \frac{C^{(c)}\pi^2}{\beta} \right) \rightarrow \tilde{\sigma}_B \equiv \tilde{\sigma}_0 \text{Im}[G^{(c)}(r = 0, E + i\Gamma_t)] \]
Numerically solve it with NLO QCD potential

\[ V^{(c)}_{\text{QCD}}(r) = C^{(c)} \frac{\alpha_s(\mu_B)}{r} \times \left[ 1 + \frac{\alpha_s}{\pi} v_1^{(c)}(r) + \cdots \right] \]

Color-factors :

\[
\begin{aligned}
\text{singlet} & \quad C^{(1)} = -C_F \\
\text{octet} & \quad C^{(8)} = C_A/2 - C_F
\end{aligned}
\]

\[ m_t = 173 \text{ GeV}, \quad \Gamma_t = 1.49 \text{ GeV} \]

\[ \mu_B = 20 \text{ GeV} \sim m_t \alpha_s \]
• Inclusion of ISR & FSR

(NLO level in soft-collinear approximation)

Similar to Higgs/DY process:

\[ \hat{\sigma}_{\text{ISR}} = K_i^{(c)} \int_0^1 dz \hat{\sigma}_{B_i}^{(c)}(z\bar{s}) F_i^{(c)}(z) \]

\[ K_i^{(c)} = 1 + \frac{\alpha_s}{\pi} h_i^{(c)}; \quad h_i^{(c)} = \beta_0 \ln \left( \frac{\mu_R}{2m_t} \right) + \text{const} \]

\[ F_i^{(c)}(z) = \delta(1-z) + \frac{\alpha_s}{\pi} \left[ A_i \left\{ \frac{\ln(1-z)}{1-z} + \ln \left( \frac{\mu_F}{2m_t} \right) \right\} \right. \]

\[ \left. + D_{tt}^{(c)} \left( \frac{1}{1-z} \right) + k_i^{(c)} \delta(1-z) \right] \]

• Hard-vertex factors from NLO quarkonium calc.

Petrelli et al. ('98) + non-decoupling term; HSY ('08)
Results : ttbar invariant-mass distribution

\[ m_{t\bar{t}}^2 = (p_t + p_{T})^2 \]

\[ m_t = 173 \text{ GeV}, \text{CTEQ6M} \]

\[ \frac{d\sigma}{dm_{t\bar{t}}}(s, m_{t\bar{t}}^2) = \sigma_{B,i}^{(c)}(m_{t\bar{t}}^2) \cdot K_i^{(c)} \int_{\tau_0}^{1} \frac{dz}{z} F_i^{(c)}(z) \frac{dL_i}{d\tau}(\tau_0/z) \]

for each initial- and color-state :

Black : Born
Blue : NLO (soft-collinear approx.)
Green : Gr-Fnc. without ISR
Red : Gr-Fnc. with ISR

**gg->tt, color-octet**

**qq->tt, color-octet**
In total at the LHC:

- (In principal) visible peak at $m_{tt} = 2m_t - E_{1S}$ !!!!
- Deform the invariant-mass distribution.
- Enhancement of total cross-section $\sim 1\%$

NLO inv. dist. using MCFM (Campbell, Ellis) 
Figs. from Maltoni, Frederix (‘08)
At the Tevatron:

- Resonance enhancement is small (qq dominance).
- But, threshold region is more significant at the Tevatron.
- Off-shellness may be important.
Summary:

- Top-quark will be produced copiously at the LHC (8M per 10fb⁻¹).
- At the LHC, gluon-fusion process dominates, and the ttbar pair can be color-singlet.
- Due to the attractive force via Coulomb-gluon exchange, ttbar(color-singlet) form a broad resonance below the threshold.
- We have calculated the binding-effect on top-pairs production at HCs, including the ISR/FSR. (NLO, soft-collinear approx.)
- The peak is in principal visible, and deforms the ttbar invariant-mass dist. at the LHC.

- Can we really see the peak at the LHC?
- Is it useful to determine the top-quark mass in a completely different method?
- …
PDF Uncertainties

Tevatron

\[ \sigma_{\text{Tev}}(\text{pb}) = 1.1 \]
\[ \% = 15\% \]
\[ \sigma_{\text{LHC}}(\text{pb}) = 804 \]
\[ \% = 90\% \]

\[ \delta_{\text{PDF}} \sim \%[\sigma_{gg}] \times 40\% \oplus \%[\sigma_{qq}] \times 4\% \sim 7\% \]

LHC

\[ \delta_{\text{PDF}} \sim \%[\sigma_{gg}] \times 4\% \oplus \%[\sigma_{qq}] \times 4\% \sim 4\% \]

<table>
<thead>
<tr>
<th>m=171</th>
<th>gg</th>
<th>qq</th>
<th>tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\text{Tev}}(\text{pb}))</td>
<td>1.1</td>
<td>6.3</td>
<td>7.3</td>
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<tr>
<td>%</td>
<td>15%</td>
<td>85%</td>
<td></td>
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<tr>
<td>(\sigma_{\text{LHC}}(\text{pb}))</td>
<td>804</td>
<td>84</td>
<td>898</td>
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<tr>
<td>%</td>
<td>90%</td>
<td>9%</td>
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</table>

CTEQ6.5

\[ \sigma = 7.61 \pm 0.38(5.1\%) \quad \text{(scales)} \quad +0.49(6.6\%) \quad \text{(PDFs) pb} \]

MRSTW-06

\[ \sigma = 7.93 \pm 0.34(4.3\%) \quad \text{(scales)} \quad +0.24(3.1\%) \quad \text{(PDFs) pb} \]

\[ \text{MRST–CTEQ} = 0.32 \pm 0.45 \text{ pb} \]

LHC

CTEQ6.5

\[ \sigma = 908 \pm 82(9.0\%) \quad \text{(scales)} \quad +30(3.3\%) \quad \text{(PDFs) pb} \]

MRSTW-06

\[ \sigma = 961 \pm 89(9.2\%) \quad \text{(scales)} \quad +11(1.1\%) \quad \text{(PDFs) pb} \]

\[ \text{MRST–CTEQ} = 53 \pm 33 \text{ pb} \]
**NNLO building blocks**

- Real-emission diagrams, squared:
  - $t\bar{t}gg$, $t\bar{t}qg$, $ttqq$ ($g_\sigma^8 = g_\sigma^4 \times g_\sigma^4$)
- 1-loop, $O(\epsilon, \epsilon^2)$:
  - $t\bar{t}$ at 1-loop, squared ($g_\sigma^8 = g_\sigma^{(2+2)} \times g_\sigma^{(2+2)}$)
- 1-loop + real emission diagrams:
  - $ttg + ttq$ at 1-loop, interference with tree-level $ttg + ttq$ ($g_\sigma^8 = g_\sigma^{(3+2)} \times g_\sigma^3$)
- 2-loop
  - $t\bar{t}$ at 2-loops, interference with tree-level $t\bar{t}$ ($g_\sigma^8 = g_\sigma^{(2+4)} \times g_\sigma^2$)

**Real-emission, 1-loop**


**1-loop, $O(\epsilon, \epsilon^2)$**


**NNLO**

$$A_{-2}/\epsilon^2 + A_{-1}/\epsilon + A_{(0)}$$

**2-loop**

$$A_{-2}/\epsilon^2 + A_{-1}/\epsilon + A_{(0)} + A_{(1)} \epsilon + A_{(2)} \epsilon^2$$


qq/gg $\rightarrow$ QQ at 2-loops, analytic, $\sqrt{s} \gg m_Q$

Czakon, arXiv/0803.1400

qq $\rightarrow$ QQ at 2-loops, numerical, $m_Q=0$

Czakon, in progress

gg $\rightarrow$ QQ at 2-loops, numerical, $m_Q=0$
Overview at LHC

- The cross section hierarchy is different at LHC
  - $t$-channel: $\sigma = 240\text{pb}$
  - $tW$ production: $\sigma = 60\text{pb}$
  - $s$-channel: $\sigma = 10\text{pb}$

- Only decays with at least one $e/\mu$ in the final state will be usable at the beginning
Differential cross-sections

- Differential cross-sections: \( \frac{d\sigma}{d^3p_t} \propto |\mathcal{M}|^2 \)
  
  corrections to the matrix-element itself

\[ \mathcal{M} \sim \mathcal{M}_0 \cdot \tilde{G}(p, E) \]

Green's function in momentum space.

- Off-shell effects: \( \sigma_{tt} \rightarrow \sigma_{bWbW} \)
  reduce the cross-section by 2-3%, due to the finite width of top-quarks

\[ \mathcal{O}\left(\frac{\Gamma_t}{m_t}\right) \] correction to the narrow width approximation

\[ \int \frac{d^3p}{(2\pi)^3} \left| \tilde{G}(p, E) \right|^2 f(p, E) < \int \frac{d^3p}{(2\pi)^3} \left| \tilde{G}(p, E) \right|^2 \Gamma_t \equiv \text{Im}G(0, E) \]

--- Unitarity relation

Sumino et al. (’91)
Jezabek, Kuhn, Teubner (’92)
c.f. \( \sigma_{tot} \propto \text{Im}G \)
• Reduction of cross-section near threshold:
  3% (Born), 10% (Green)

• ISR/FSR → Implement to MC generators (e.g. MadEvent):

\[
\hat{\sigma}^{(c)} \sim \hat{\sigma}^{(c)}_0 \left[ 1 - \frac{\alpha_s}{\pi} C^{(c)} \frac{\pi^2}{2\beta} \right] \left[ 1 + \frac{\alpha_s}{\pi} \left\{ A \ln^2 (8\beta^2) + B^{(c)} \ln (8\beta^2) \right\} \right] \left[ 1 + \frac{\alpha_s}{\pi} D^{(c)} \right]
\]

(not included in LO MC)
replace here with Gr.Fnc.

ISR/FSR → Parton Shower

Hard correction
process dependent