KEK Theory Center

Neutrino-nucleus interactions in the few GeV region

Theoretical challenges in neutrino scattering studies: weak pion production off the nucleon

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Outline

1. Motivation: Neutrino oscillations, neutrino detectors, nuclear cross sections and systematic errors
2. Llewellyn-Smith: $\Delta(1232)$ & the $\nu_l N \rightarrow l^- N' \pi$ reaction
3. Chiral symmetry and non-resonant contributions
4. Deuteron effects and ANL & BNL data
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6. The $\nu_\mu \ n \rightarrow \mu^- n \pi^+$ channel and the crossed $\Delta$ term: spin 1/2 dof in the $\Delta$ propagator & contact terms
7. Parity-violating contributions to the pion angular differential cross section and T-odd correlations.
8. Conclusions

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Bibliography:


• Hernández E. and Nieves J.: *Neutrino-induced one-pion production revisited: the. $\nu_\mu \ n \rightarrow \mu^- n \pi^+$ channel*, Phys. Rev. D95 (2017) no.5, 053007

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1. Motivation

- Details on the axial structure of hadrons in the free space and inside of nuclei
- Neutrinos are detected through nuclear interactions

Theoretical knowledge of QE, 1π and DIS cross sections is important to carry out a precise neutrino oscillation data analysis...

\[^{12}\text{C} \rightarrow \text{Liquid scintillators}\]
\[^{16}\text{O} \rightarrow \text{Cerenkov detectors}\]
\[^{40}\text{A} \rightarrow \text{TPC's (time projection chambers)}\] ....

\(\Delta(1232)\) RESONANCE PEAK

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Th: NUANCE (D. Casper, 2002)
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Pion production → misidentification of 1 Cherenkov ring events that are assumed to be produced by charged current (CC) QE reactions $\nu_\alpha A \rightarrow l^\alpha A'$

Even distinguishing between $\mu$- and e-like rings

- **Appearance Probability $P(\nu_\mu \rightarrow \nu_e)$**: The CC QE signature $\nu_e A \rightarrow e A'$ used to identify $\nu_e$ can be confused with the NC $1\pi$ production $\nu_\mu A \rightarrow \nu_\mu A'\pi^0$
- **Survival Probability $P(\nu_\mu \rightarrow \nu_\mu)$**: The CC QE signature $\nu_\mu A \rightarrow \mu A'$ used to identify $\nu_\mu$ can be confused with the CC or NC $\nu_{\mu,\tau} A \rightarrow (\nu_{\mu,\tau} \ or \ \mu, \tau) A'\pi$ when only one of the particles emits Cherenkov light. For instance, processes $(\nu_\mu, \mu, \pi)$ might produce an incorrect reconstruction of the neutrino energy $E \rightarrow L/E$ analysis?

Nuclear cross sections are crucial to reduce the systematic errors of oscillation analysis!

There exist dedicated experiments as MINER$\nu$A (FermiLab), which seeks to measure low energy neutrino interactions both in support of neutrino oscillation experiments and also to study the strong dynamics of the nucleon and nucleus that affect these interactions
Neutrino Energy Reconstruction:

\[ \nu_{\mu} + n \rightarrow p \mu^- \quad \text{(bound in the nucleus)} \]

\[ E_{\text{rec}} = \frac{M E_{\mu} - m_{\mu}^2/2}{M - E_{\mu} + |\vec{p}_{\mu}| \cos\theta_{\mu}} \]

QE-like: problem absorbed or not detected pions and...

exp: only 1\(\mu\) (from the lepton vertex). But, for instance if pions are produced:

• pion decays and the extra muon is detected (2 muons in the final state)

• pion is absorbed or not detected (MC corrected if the pion production cross section is well known...)

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Neutrino Energy Reconstruction:

**QE:** $\nu_\mu + n \rightarrow p \mu^- \quad \text{(bound in the nucleus)}$

$$E_{\text{rec}} = \frac{M E_\mu - m_\mu^2 / 2}{M - E_\mu + |\vec{p}_\mu| \cos \theta_\mu}$$

**QE-like:** problem absorbed or not detected pions and $2p2h$ (nucl. effect)

M. Martini, M. Ericson, PRD 87 (2013)

QE Energy Reconstruction will be wrong!!

$2p2h$: $\nu_\mu + NN \rightarrow N'N'' \mu^-$

MC correct for this effect: cross section
Quantitative impact in the determination of the oscillation parameters

Effects of a simple model for QE-like events ....

\[ N_i^{\text{test}}(\alpha) = \alpha \times N_i^{\text{QE}} + (1 - \alpha) \times N_i^{\text{QE-like}} \]

\( \alpha \) parametrizes the fraction of two-nucleon absorption that is neglected in the fit

Reconstructed from naive QE dynamics

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Systematic uncertainties in long-baseline neutrino-oscillation experiments,
Artur M Ankowski and Camillo Mariani, J.Phys. G44 (2017) 054001
\[ \nu + A \rightarrow l + X \]

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PRD93 (2016) 014016 (Watson’s theorem)
PRD95 (2017) 053007 (1/2 dof in Δ propagator)

PRD D76 (2007) 033005
PRD D81 (2010) 085046

PRC 83 (2011) 045501 [\( M_A = 1.049 \text{ GeV} \)]

\[ \theta = 60^\circ \]
\[ E_\nu = 750 \text{ MeV} \]

\[ \Delta + 1p1h1\pi + 2p2h \]

Full model

Only Δ

\[ \text{PRD D76 (2007) 033005} \]

\[ \text{PRD D81 (2010) 085046} \]

\[ \text{PRC 83 (2011) 045501} [M_A = 1.049 \text{ GeV}] \]

\[ \theta = 60^\circ \]
\[ E_\nu = 750 \text{ MeV} \]
Resonance Production

Deficiencies of the Rein Sehgal model! ⇒ Improved models

Electron data ⇒ Resonance vector form factors!
PCAC ⇒ Resonance axial form factors!
Background: chiral symmetry (when possible!)

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Nuclear effects are relevant! (see talk by E. Hernández)

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There exist some discrepancies between theoretical predictions and data!

**Figure 15.** MiniBooNE flux-folded differential $d\sigma/dp_\pi$ cross section for CC1$\pi^0$ production by $\nu_\mu$ in mineral oil. Data are from [27]. Left: predictions from the cascade approach of [184]. The solid curve corresponds to the full model and the dashed one stands for the results obtained neglecting FSI effects. Right: predictions from the GiBUU transport model of [207]. The dashed curves give the results before FSI, the solid curves those with all FSI effects included. Two different form factors $C_5^A(q^2)$, tuned to the ANL and BNL data-sets have been employed and give rise to the systematic uncertainty bands displayed in the figure.

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2. Llewellyn-Smith: $\Delta(1232)$ & the $\nu_l N \rightarrow l^- N' \pi$ reaction

**Theoretical Model** $\nu_l N \rightarrow lN'\pi$, $\nu_l N \rightarrow \nu_l N'\pi$ (C.H. Llewellyn Smith, 1972): weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$. 

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\[ \langle \Delta^+; p_\Delta = p + q | j^{\mu}_{cc+}(0) | n; p \rangle = \bar{u}_\alpha(p_\Delta) \Gamma^{\alpha\mu}(p, q) u(p) \cos \theta_C, \]

\[ \Gamma^{\alpha\mu} = \left[ \frac{C_3^A}{M} (g^{\alpha\mu} q^\gamma - q^\alpha q^\gamma) + \frac{C_4^A}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^A g^{\alpha\mu} + C_6^A}{M^2} q^\mu q^\alpha \right] \]

\[ + \left[ \frac{C_3^V}{M} (g^{\alpha\mu} q^\gamma - q^\alpha q^\gamma) + \frac{C_4^V}{M^2} (g^{\alpha\mu} q \cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2} (g^{\alpha\mu} q \cdot p - q^\alpha p^\mu) \right] \gamma_5, \]

\[ + C_6^V g^{\mu\alpha} \] \gamma_5, \quad C_{3,4,5,6}^A \text{ axial FF's, } C_{3,4,5,6}^V \text{ vector FF's}, \quad \text{furthermore} \]

\[ \mathcal{L}_{\pi N \Delta} = \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^{\dagger} (\partial^\mu \vec{\phi}) \Psi + \text{h.c.}, \quad f^* = 2.14 \]

\[ G^{\mu\nu}(p_\Delta) = \frac{\hat{p}_\Delta + M_\Delta}{p_\Delta^2 - M_\Delta^2 + i M_\Delta \Gamma_\Delta} \left[ -g^{\mu\nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{2}{3} \frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} - \frac{1}{3} \frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right] \]
$eN \to e'\Delta \to e'N'\pi \Rightarrow C_{3,4,5,6}^V$ FF's. CVC $\Rightarrow C_6^V = 0$ and $(M_V = 0.84$ GeV$)$

$$\frac{C_3^V(q^2)}{2.13} = \frac{C_4^V(q^2)}{-1.51} = \frac{1 - \frac{q^2}{0.776M_V^2}}{1 - \frac{q^2}{4M_V^2}} \frac{C_5^V(q^2)}{0.48} = \frac{1}{(1 - q^2/M_V^2)^2} \times \frac{1}{1 - \frac{q^2}{4M_V^2}}$$

$C_{3,4,5,6}^A$ Axial FF’s : $\Delta^{++}$ ($\nu_\mu p \rightarrow \mu^- p \pi^+$) data taken in the ANL and BNL bubble chambers (filled in with deuterium)

Dominant form factor: $C_5^A(q^2)$. $C_3^A(q^2)$ and $C_4^A(q^2)$ contributions are small and we have taken as (Adler’s model 1968)

$$C_4^A(q^2) = -\frac{C_5^A(q^2)}{4}, \ C_3^A(q^2) = 0$$

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PCAC \((\partial_\mu A^\mu \propto m_\pi^2)\) and Goldberger–Treiman

\[
C_5^A(0) \sim \sqrt{\frac{2}{3}} \frac{f_\pi}{m_\pi} f^* = 1.2
\]

\[
C_5^A(q^2) = \frac{1.2}{(1 - q^2/M_{A\Delta}^2)^2} \times \frac{1}{1 - \frac{q^2}{3M_{A\Delta}^2}}, \quad C_6^A(q^2) = C_5^A(q^2) \frac{M^2}{m_\pi^2 - q^2}
\]

\(M_{A\Delta}\) fitted to the \(q^2\) dependence of the \(\nu_\mu p \to \mu^- p \pi^+\) cross section (neutrino energy averaged) with \((M(\pi N) < 1.4\) GeV) measured at ANL and BNL. It varies in the range 0.95 GeV (ANL) – 1.28 GeV (BNL).

E. Paschos, J-Y. Yu and M. Sakuda (PRD69, 014013 (2004)),

\(M_{A\Delta} \sim 1.05\) GeV

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FIG. 12. Differential cross section $d\sigma/dQ^2$ evaluated with the selections $0.5 \leq E_{\nu} < 6.0$ GeV and $M(p\pi^+) < 1.4$ GeV. The curve is the flux-averaged prediction of the Adler model with the dipole form factor and $M_A = 0.95$ GeV.

FIG. 5. The $Q^2$ distribution for (a) the quasielastic and (b) the $\Delta^{++}$ production reactions. The curves are the theoretical predictions obtained from least-squares fits with the fitted $M_A$ values for the $Q^2 < 3.0$ (GeV/c)$^2$. 

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... but only the $\Delta$ pole contribution turns out to be an insufficient model, even at the $\Delta$ peak, and specially close to pion threshold. Close to pion threshold, the pion from the $(\nu_\mu, \mu\pi)$ reaction will not radiate Čerenkov light and thus it would be necessary an improved theoretical model to carry out a proper $L/E$ oscillation analysis.

Such model for the $\nu_l N \rightarrow lN'\pi$, $\nu_l N \rightarrow \nu_l N'\pi$ should include non resonant terms

$\Rightarrow$ Realization of the axial and vector currents, which couple to the $W, Z^0$ bosons, for a system of pions and nucleons.
Non-linear $\sigma$–Model: EFT involving pions and nucleons which implements spontaneous chiral symmetry breaking.

If $\Psi_q = \begin{pmatrix} \Psi_u \\ \bar{\Psi}_d \end{pmatrix}$, the CC and NC, which induce $W(Z^0)N \to N'\pi$

\[
\begin{align*}
    j_{cc\pm}^\mu &= \cos \theta_C \bar{\Psi}_q \gamma^\mu (1 - \gamma_5) \left( -\frac{\tau^1_{\pm 1}}{\sqrt{2}} \right) \Psi_q \\
    j_{nc}^\mu &= \bar{\Psi}_q \gamma^\mu (1 - 2 \sin^2 \theta_W - \gamma_5) \left[ \tau^1_0 \right] \Psi_q \\
    j_{nc}^\mu &= -4 \sin^2 \theta_W s_{em,IS}^\mu - \bar{\Psi}_s \gamma^\mu (1 - \gamma_5) \Psi_s \\
    s_{em}^\mu &= \frac{1}{6} \bar{\Psi}_q \gamma^\mu \Psi_q - \frac{1}{3} \bar{\Psi}_s \gamma^\mu \Psi_s + \frac{1}{\sqrt{2}} \bar{\Psi}_q \gamma^\mu \left[ \tau^1_0 \right] \Psi_q
\end{align*}
\]

$\langle N'\pi| j_{cc+}^\mu(0), j_{cc-}^\mu(0), j_{nc}^\mu(0)|N\rangle$ = ? $\leftarrow$ QCD and its pattern of $S\chi$SB

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Two flavor, $u$ and $d$ with mass $m$, QCD Lagrangian

\[ \mathcal{L}_{QCD} = \bar{\Psi}_q(i\partial - m)\Psi_q + \frac{1}{2g^2} \text{Tr}(F_{\mu\nu} F_{\mu\nu}) \]

with $D^\mu = \partial^\mu - B^\mu$, $F_{\mu\nu} = -[D^\mu, D^\nu]$, $B^\mu = igT^a B^\mu_a$, matrices in the colour space. Chiral symmetry $\Rightarrow$

- $\Psi_q \rightarrow \Psi'_q = e^{-i\vec{v} \cdot \vec{r}/2} \Psi_q$, isospin rotation
- $\Psi_q \rightarrow \Psi'_q = e^{-iA_{\vec{r}} / 2} \Psi_q$, axial–flavor rotation

$\delta \mathcal{L}_{QCD} \propto m$. Currents (Noether)

$\bar{V}^\mu = \bar{\Psi}_q \gamma^\mu \gamma_5 / 2 \Psi_q, \quad \partial_\mu \bar{V}^\mu = 0$

$\bar{A}^\mu = \bar{\Psi}_q \gamma^\mu \gamma_5 / 2 \Psi_q, \quad \partial_\mu \bar{A}^\mu = m \bar{\Psi}_q i\gamma_5 \vec{r} \Psi_q \neq 0$

Charges

$\bar{Q}(t) = \int_{R^3} d^3x \bar{V}^0(\vec{x}, t), \quad \bar{Q}_5(t) = \int_{R^3} d^3x \bar{A}^0(\vec{x}, t)$

$\bar{Q}$ (isospin) and $\bar{Q}_5$ (neglecting $m$) indep. of $t \Rightarrow$ conserved!!

$[Q^i, Q^j] = i\epsilon^{ijk} Q^k, \quad [Q_5^i, Q_5^j] = i\epsilon^{ijk} Q_5^k, \quad [Q^i_5, Q^j_5] = i\epsilon^{ijk} Q_5^k$

$\chi_{SB}: \bar{Q}|0\rangle = 0$ but $\bar{Q}_5|0\rangle \neq 0 \Rightarrow \pi$’s Isotriplet Goldstone bosons from spontaneous chiral symmetry breaking.

Non-linear $\sigma$–model $\Rightarrow$ EFT involving pions and nucleons which implements chiral symmetry and its pattern of spontaneous breaking.

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If $\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$, $U = \frac{f_\pi}{\sqrt{2}} e^{i\vec{\tau} \cdot \vec{\phi}} / f_\pi = \frac{f_\pi}{\sqrt{2}} \xi^2$, with $f_\pi \sim 93$ MeV,

\[ \mathcal{L}_{N\pi} = \bar{\Psi} i \gamma^\mu \left[ \partial_\mu + \mathcal{V}_\mu \right] \Psi - M \bar{\Psi} \Psi + g_A \bar{\Psi} \gamma^\mu \gamma^5 A_\mu \Psi + \frac{1}{2} \text{Tr} \left[ \partial_\mu U^\dagger \partial^\mu U \right] + m_\pi^2 \frac{f_\pi}{2\sqrt{2}} \text{Tr}(U + U^\dagger - \sqrt{2}f_\pi) \]

\[ \mathcal{V}_\mu = \frac{1}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) \quad \mathcal{A}_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) \]

Isospin rotat. $\xi \to T_V \xi T_V^\dagger$, $\Psi \to T_V \Psi$, $T_V = e^{-i \frac{\vec{\tau} \cdot \vec{\sigma}_V}{2}}$

Axial rotat. $\xi \to T_A^\dagger \xi T_A^\dagger = T_\Lambda \xi T_\Lambda^\dagger$, $\Psi \to T_\Lambda \Psi$, $T_{\Lambda,A} = e^{-i \frac{\vec{\tau} \cdot \vec{\sigma}_{\Lambda,A}}{2}}$

Isospin rotat. $\Rightarrow \delta \mathcal{L}_{N\pi} = 0$, Axial rotat. $\Rightarrow \delta \mathcal{L}_{N\pi} \propto m_\pi^2 \neq 0$
Up to order $\mathcal{O}(1/f_\pi^4)$, $\mathcal{L}_{N\pi}$ reads,

$$
\mathcal{L}_{N\pi} = \bar{\Psi} [i\slashed{\partial} - M] \Psi + \frac{1}{2} \partial_\mu \bar{\phi} \partial^\mu \bar{\phi} - \frac{1}{2} m_\pi^2 \bar{\phi}^2 \quad \text{(kinetic)} + \\
\frac{g_A}{f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \frac{\tau}{2} (\partial_\mu \phi) \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma_\mu \tau \left( \bar{\phi} \times \partial^\mu \phi \right) \Psi - \frac{g_A}{6f_\pi^3} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \frac{\tau_2}{2} \partial_\mu \phi - (\bar{\phi} \partial_\mu \phi) \frac{\tau}{2} \phi \right] \Psi \\
- \frac{1}{6f_\pi^2} \left( \bar{\phi}^2 \partial_\mu \phi \partial^\mu \phi - (\bar{\phi} \partial_\mu \phi)(\bar{\phi} \partial^\mu \phi) \right) + \frac{m_\pi^2}{24f_\pi^2} (\bar{\phi}^2)^2 + \mathcal{O}(1/f_\pi^4)
$$

Contact interactions $NN\pi$, $NN\pi\pi$, $NN\pi\pi\pi$ and $\pi\pi\pi\pi$.

**Parameters:** $f_\pi$ and $g_A$. Noether’s currents

$$
\mathcal{L} = \frac{\partial \mathcal{L}_{N\pi}}{\partial (\partial_\mu \varphi_a)} \delta \varphi_a, \quad a = 1, 2, \cdots
$$

up to order $\mathcal{O}(1/f_\pi^3)$ ...
\[ \vec{V}^\mu = \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\psi} \gamma^\mu \vec{\tau} \Psi - \frac{1}{4f_\pi^2} \bar{\psi} \gamma^\mu \left[ \vec{\tau} \phi^2 - \phi (\vec{\tau} \cdot \phi) \right] \Psi \\
- \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3}) , \quad \partial_\mu \vec{V}^\mu = 0 \]

\[ \vec{A}^\mu = f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\psi} \gamma^\mu \gamma_5 \vec{\tau} \Psi + \frac{2}{3f_\pi} \left[ \vec{\phi} (\vec{\phi} \cdot \partial^\mu \phi) - \vec{\phi}^2 \partial^\mu \phi \right] \\
- \frac{g_A}{4f_\pi^2} \bar{\psi} \gamma^\mu \gamma_5 \left[ \vec{\tau} \phi^2 - \phi (\vec{\tau} \cdot \phi) \right] \Psi + \mathcal{O}(\frac{1}{f_\pi^3}) , \quad \underbrace{\partial_\mu \vec{A}^\mu \propto m_\pi^2 ...}_{\text{PCAC}} \]

+ isospin relations \Rightarrow \text{evaluate} \quad \text{CC} \quad \langle N' \pi| j_{cc+}^\mu (0), j_{cc-}^\mu (0)| N \rangle

\[ \langle p\pi^0 | j_{cc+}^\mu (0) | n \rangle = -\frac{1}{\sqrt{2}} \left[ \langle p\pi^+ | j_{cc+}^\mu (0) | p \rangle - \langle n\pi^+ | j_{cc+}^\mu (0) | n \rangle \right] \]

\[ \langle p\pi^- | j_{cc-}^\mu (0) | p \rangle = \langle n\pi^+ | j_{cc+}^\mu (0) | n \rangle \]

\[ \langle n\pi^- | j_{cc-}^\mu (0) | n \rangle = \langle p\pi^+ | j_{cc+}^\mu (0) | p \rangle \]

\[ \langle n\pi^0 | j_{cc-}^\mu (0) | p \rangle = -\langle p\pi^0 | j_{cc+}^\mu (0) | n \rangle = \frac{1}{\sqrt{2}} \left[ \langle p\pi^+ | j_{cc+}^\mu (0) | p \rangle - \langle n\pi^+ | j_{cc+}^\mu (0) | n \rangle \right] \]
\[
\bar{V}^\mu = \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi
\]

\[
- \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3})
\]

\[
\bar{A}^\mu = f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi + 2f_\pi \left[ \vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right]
\]

\[
- \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}(\frac{1}{f_\pi^3})
\]

\[
\left. j^{\mu}_{cc+} \right|_{PF} = \mp i \frac{F_{PF}(q^2)}{f_\pi} \sqrt{2Mg_A \cos \theta_C} \frac{(2k_\pi - q)^\mu}{(k_\pi - q)^2 - m_\pi^2} \bar{u}(\vec{p}') \gamma_5 u(\vec{p})
\]

\[
( - \Rightarrow W^+ p \rightarrow p\pi^+, \quad + \Rightarrow W^+ n \rightarrow n\pi^+ )
\]

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\[ \vec{V}^\mu = \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{gA}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \]

\[ - \frac{\vec{\phi}^2}{3f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3}) \]

\[ \vec{A}^\mu = f_\pi \partial^\mu \vec{\phi} + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + gA \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[ \vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \]

\[ - \frac{gA}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}(\frac{1}{f_\pi^3}) \]

\[ j_{cc+}^\mu \bigg|_{PP} = \mp i F_\rho \left( (q - k_\pi)^2 \right) \frac{\cos \theta_C}{\sqrt{2}f_\pi} \frac{q^\mu}{q^2 - m_\pi^2} \bar{u}(p') \gamma^\mu \gamma_5 \Phi \right] u(p) \]

\[ ( - \Rightarrow W^+ p \rightarrow p\pi^+, \ + \Rightarrow W^+ n \rightarrow n\pi^+ ) \]

\[ F_\rho(t) = \frac{1}{1 - t/m_\rho^2} \]
\[ \vec{V}^\mu = \vec{\phi} \times \partial^\mu \vec{\phi} + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\vec{\phi} \times \vec{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\vec{\tau}}{2} \Psi - \frac{1}{4 f_\pi^2} \bar{\Psi} \gamma^\mu \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi \\
- \frac{\vec{\phi}^2}{3 f_\pi^2} (\vec{\phi} \times \partial^\mu \vec{\phi}) + \mathcal{O}(\frac{1}{f_\pi^3}) \]

\[ \vec{A}^\mu = f_\pi \partial^\mu \vec{\phi} + \frac{1}{2 f_\pi} \bar{\Psi} \gamma^\mu (\vec{\phi} \times \vec{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}}{2} \Psi + \frac{2}{3 f_\pi} \left[ \vec{\phi} (\vec{\phi} \cdot \partial^\mu \vec{\phi}) - \vec{\phi}^2 \partial^\mu \vec{\phi} \right] \\
- \frac{g_A}{4 f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \vec{\tau} \vec{\phi}^2 - \vec{\phi} (\vec{\tau} \cdot \vec{\phi}) \right] \Psi + \mathcal{O}(\frac{1}{f_\pi^3}) \]

\[ j_{cc+}^\mu \bigg|_{CT} = \mp \frac{i \cos \theta_C}{\sqrt{2} f_\pi} \bar{u}(\vec{p}') \gamma^\mu \left( g_A F_{CT}^V (q^2) \gamma_5 - F_{\rho} \left( (q - k_\pi)^2 \right) \right) u(\vec{p}) \]

\( - \rightarrow W^+ p \rightarrow p \pi^+, \quad + \rightarrow W^+ n \rightarrow n \pi^+ \)
\[
\begin{align*}
\bar{V}^\mu & = \bar{\phi} \times \partial^\mu \phi + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\bar{\phi} \times \bar{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\bar{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[ \bar{\tau} \phi^2 - \phi (\bar{\tau} \cdot \phi) \right] \Psi \\
& \quad - \frac{\phi^2}{3f_\pi^2} (\bar{\phi} \times \partial^\mu \phi) + O \left( \frac{1}{f_\pi^3} \right) \\
\bar{A}^\mu & = f_\pi \partial^\mu \phi + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\bar{\phi} \times \bar{\tau}) \Psi + g_A \bar{\Psi} \gamma^\mu \gamma_5 \frac{\bar{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[ \bar{\phi} (\bar{\phi} \cdot \partial^\mu \phi) - \phi^2 \partial^\mu \phi \right] \\
& \quad - \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \bar{\tau} \phi^2 - \phi (\bar{\tau} \cdot \phi) \right] \Psi + O \left( \frac{1}{f_\pi^3} \right)
\end{align*}
\]
... improve the $WNN$ transition vertex

$$
\langle p; \vec{p}' = \vec{p} + \vec{q} | j_{c+c+}^{\alpha} | n; \vec{p}' \rangle = \cos \theta_C \; \bar{u}(\vec{p'}) (V_\alpha^N(q) - A_\alpha^N(q)) u(\vec{p})
$$

$$
V_\alpha^N(q) = 2 \times \left( F_1^V(q^2) \gamma^\alpha + i \mu_V \frac{F_2^V(q^2)}{2M} \sigma^{\alpha \nu} q_\nu \right)
$$

$$
A_\alpha^N(q) = \frac{g_A}{(1 - q^2/M_A^2)^2} \times \left( \gamma^\alpha \gamma_5 + \frac{q^2}{m_\pi^2 - q^2} q^\alpha \gamma_5 \right), \left\{ \begin{array}{l}
g_A = 1.26 \\
M_A = 1.05 \text{ GeV} \
\end{array} \right.
$$

$$
F_1^V(q^2) = \frac{1}{2} (F_1^P(q^2) - F_1^n(q^2)), \quad \mu_V F_2^V(q^2) = \frac{1}{2} (\mu_P F_2^P(q^2) - \mu_n F_2^n(q^2)),
$$

furthermore \textbf{CVC} \Rightarrow \begin{align*}
F_{PF}(q^2) &= F_{CT}^V(q^2) = 2F_1^V(q^2) = F_1^P - F_1^n \\
\end{align*}

Juan Nieves, IFIC (CSIC & UV)
$$\bar{V}^\mu = \bar{\phi} \times \partial^\mu \phi + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 (\bar{\phi} \times \bar{\tau}) \Psi + \bar{\Psi} \gamma^\mu \frac{\bar{\tau}}{2} \Psi - \frac{1}{4f_\pi^2} \bar{\Psi} \gamma^\mu \left[ \bar{\tau} \phi \phi^2 - \bar{\phi} (\bar{\phi} \circ \phi) \right] \Psi$$

$$- \frac{\phi^2}{3f_\pi^2} (\bar{\phi} \times \partial^\mu \phi) + \mathcal{O}\left( \frac{1}{f_\pi^3} \right)$$

$$\bar{A}^\mu = f_\pi \partial^\mu \phi + \frac{1}{2f_\pi} \bar{\Psi} \gamma^\mu (\bar{\phi} \times \bar{\tau}) \Psi + g_A \bar{\Psi} \gamma_5 \frac{\bar{\tau}}{2} \Psi + \frac{2}{3f_\pi} \left[ \phi (\bar{\phi} + \partial^\mu \bar{\phi}) - \phi^2 \partial^\mu \phi \right]$$

$$- \frac{g_A}{4f_\pi^2} \bar{\Psi} \gamma^\mu \gamma_5 \left[ \bar{\tau} \phi \phi^2 - \bar{\phi} (\bar{\phi} \circ \phi) \right] \Psi + \mathcal{O}\left( \frac{1}{f_\pi^3} \right)$$

$\nu N \rightarrow lN^'\pi\pi$, $\nu N \rightarrow \nu N^'\pi\pi$ close to threshold. $N^*(1440)$

degrees of freedom (PRD77 (2008) 053009)
Evaluation of NC $\langle N'\pi| j_{nc}^{\mu}(0)|N \rangle$:

$$j_{nc}^{\mu} = \bar{\Psi}_q \gamma^{\mu}(1 - 2 \sin^2 \theta_W - \gamma_5) \tau^1_0 \Psi_q - 4 \sin^2 \theta_W s_{em,IS}^{\mu} - \bar{\Psi}_s \gamma^{\mu}(1 - \gamma_5) \Psi_s$$

$$s_{em}^{\mu} = \frac{1}{6} \bar{\Psi}_q \gamma^{\mu} \Psi_q - \frac{1}{3} \bar{\Psi}_s \gamma^{\mu} \Psi_s + \frac{1}{\sqrt{2}} \bar{\Psi}_q \gamma^{\mu} \sqrt{2} \Psi_q$$

- ME’s $j_{cc+}^{\mu} \Rightarrow$ ME’s isovector ($\tau^1_0$) $j_{nc}^{\mu}$ contribution
- $\Delta$ does not contribute to the isoscalar $j_{nc}^{\mu}$ part
- $\langle n\pi^+| s_{em,IS}^{\mu} |p \rangle = \langle p\pi^-| s_{em,IS}^{\mu} |n \rangle = \sqrt{2} \langle p\pi^0| s_{em,IS}^{\mu} |p \rangle = -\sqrt{2} \langle n\pi^0| s_{em,IS}^{\mu} |n \rangle$

$$\langle p\pi^0| s_{em,IS}^{\mu} |p \rangle = -\frac{\langle n\pi^0| s_{em}^{\mu}(0) |n \rangle - \langle p\pi^0| s_{em}^{\mu}(0) |p \rangle}{2}$$

Juan Nieves, IFIC (CSIC & UV)
\[
\mathcal{A}_\mu = \bar{\Psi} \gamma^\mu \left( \frac{1 + \tau_z}{2} \right) \Psi + \frac{g_A}{2f_\pi} \bar{\Psi} \gamma^\mu \gamma_5 \left( \tau_-^1 \phi^\dagger + \tau_+^1 \phi \right) \Psi + i \left( \phi^\dagger \partial^\mu \phi - \phi \partial^\mu \phi^\dagger \right) + \cdots
\]

CT, PF do not contribute \(\Rightarrow\) PN and PNC \(\Rightarrow\) ME's of \(s_{em,I\S}^\mu\)

- ME's \(j_{\text{nc,str}}^\mu = \bar{\Psi}_s \gamma^\mu (1 - \gamma_5) \Psi_s : \text{nucleon strange content}\)

\[
\langle \rho^0 | j_{\text{nc,str}}^\mu (0) | p \rangle = -i \frac{g_A}{2f_\pi} \bar{u}(\vec{p}^\prime) \left\{ k_\pi^\nu \gamma_5 \frac{p^\mu + q^\mu + M}{(p + q)^2 - M^2 + i\epsilon} \left[ V_{N,s}^\mu (q) - A_{N,s}^\mu (q) \right] \right. \\
+ \left. \left[ V_{N,s}^\mu (q) - A_{N,s}^\mu (q) \right] \frac{p^\mu - q^\mu + M}{(p^\prime - q)^2 - M^2 + i\epsilon} k_\pi^\nu \gamma_5 \right\} u(\vec{p}) \iff \text{PN + PNC}
\]

\[
V_{N,s}^\mu (q) \approx F_1^s (q^2) \gamma^\mu + i \mu_s \frac{F_2^s (q^2)}{2M} \sigma^{\mu\nu} q_\nu, \quad A_{N,s}^\mu (q) = \sum_{G_P} G_1^s (q^2) \gamma^\mu \gamma_5 + G_2^s q^\mu \gamma_5 \not\text{don't contr.}
\]

Juan Nieves, IFIC (CSIC & UV)
Results:

\[ \Rightarrow \text{CC : } \nu_l(k) + N(p) \rightarrow l^-(k') + N(p') + \pi(k_{\pi}) \]

\[
\frac{d^5\sigma_{\nu l}}{d\Omega(k')dE'd\Omega(k_{\pi})} = \frac{|k'|}{|k|} \frac{\alpha^2}{4\pi^2} \int_0^{+\infty} \frac{d|k_{\pi}| |k'|^2}{E_{\pi}} L^{(\nu)}_{\mu\sigma} (W^{\mu\sigma}_{\text{CC}\pi})^{(\nu)}
\]

Juan Nieves, IFIC (CSIC & UV)
\begin{align*}
(W_{CC\pi}^{\mu\sigma})^{(\nu)} & = \frac{1}{4M} \sum_{\text{spins}} \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2E_N'} \delta^4(p' + k_\pi - q - p) \langle N'\pi|j_{cc+}^{\mu}(0)|N\rangle \langle N'\pi|j_{cc+}^{\sigma}(0)|N\rangle^* \\
L^{(\nu)}_{\mu\sigma} & = (L^{(\nu)}_s)_{\mu\sigma} + i(L^{(\nu)}_a)_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta \\
\implies CC: \quad \bar{\nu}_l(k) + N(p) & \rightarrow l^+(k') + N(p') + \pi(k_\pi) \\
L^{(\bar{\nu})}_{\mu\sigma} & = L^{(\nu)}_{\sigma\mu}, \quad j_{cc+}^{\sigma} \leftrightarrow j_{cc-}^{\sigma} \\
\implies NC: \quad \nu(k) + N(p) & \rightarrow \nu(k') + N(p') + \pi(k_\pi) \\
j_{cc+}^{\sigma} & \leftrightarrow \frac{1}{2} j_{nc}^{\sigma}, \quad (W_{NC\pi}^{\mu\sigma})^{(\nu)} = (W_{NC\pi}^{\mu\sigma})^{(\bar{\nu})} \\
\text{Note} \quad \underbrace{(E', \theta')}_{\text{outgoing lepton}} & \leftrightarrow q^2, \quad \underbrace{W^2 = (p + q)^2}_{\text{\piN inv. mass}} \\
\text{Juan Nieves, IFIC (CSIC & UV)}
\end{align*}
\[ \int_{M+m_\pi}^{1.4 \text{ GeV}} dW \frac{d\sigma_{\nu_\mu\mu^-}}{dq^2 dW}, \quad \nu_\mu p \rightarrow \mu^- p \pi^+ \]

PCAC predicts \( \sim 1.2 \)

Fit to ANL:

\[ C_5^A(0) = 0.867 \pm 0.075, \quad M_{\Delta} = 0.985 \pm 0.082 \text{ GeV} \]
Juan Nieves, IFIC (CSIC & UV)
**$\sigma_{NC}/\sigma_{CC}$ ANL cross sections at $E = 0.6 - 1.2$ GeV**

<table>
<thead>
<tr>
<th></th>
<th>ANL</th>
<th>Our results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_+ = \sigma(\nu p \rightarrow \nu n \pi^+)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$</td>
<td>0.12 ± 0.04</td>
<td>0.12 - 0.10</td>
</tr>
<tr>
<td>$R_0 = \sigma(\nu p \rightarrow \nu p \pi^0)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$</td>
<td>0.09 ± 0.05</td>
<td>0.18 - 0.14</td>
</tr>
<tr>
<td>$R_- = \sigma(\nu n \rightarrow \nu p \pi^-)/\sigma(\nu p \rightarrow \mu^- p \pi^+)$</td>
<td>0.11 ± 0.022</td>
<td>0.12 - 0.09</td>
</tr>
</tbody>
</table>

**NC: Cross sections ($10^{-38}$ cm$^2$) for $\langle E \rangle = 2.2$ GeV (no cut in $W$)**

<table>
<thead>
<tr>
<th></th>
<th>CERN</th>
<th>Our results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\nu p \rightarrow \nu p \pi^0)$</td>
<td>0.130 ± 0.020</td>
<td>0.105±0.006</td>
</tr>
<tr>
<td>$\sigma(\nu p \rightarrow \nu n \pi^+)$</td>
<td>0.080 ± 0.020</td>
<td>0.091±0.003</td>
</tr>
<tr>
<td>$\sigma(\nu n \rightarrow \nu n \pi^0)$</td>
<td>0.080 ± 0.020</td>
<td>0.104±0.006</td>
</tr>
<tr>
<td>$\sigma(\nu n \rightarrow \nu p \pi^-)$</td>
<td>0.110 ± 0.030</td>
<td>0.082±0.003</td>
</tr>
</tbody>
</table>

Juan Nieves, IFIC (CSIC & UV)
Below the $\tau$ prod. threshold, Distinguish $\nu_\tau$ from $\bar{\nu}_\tau$?

Juan Nieves, IFIC (CSIC & UV)
4. Deuteron effects and ANL & BNL data

K.M. Graczyk et al. [Phys. Rev. D 80, 093001 (2009)]

- ANL and BNL data were measured in deuterium
  - Deuteron effects were estimated by L. Alvarez-Ruso et al. [Phys. Rev. C 59, 3386 (1999)] to reduce the cross section by 5-10%.
- Large uncertainties in the neutrino flux normalization, 10% for BNL data and 20% for ANL data.

K.M. Graczyk et al. made a combined fit to both ANL&BNL data, assuming that only the $\Delta$ mechanism contributed, including deuteron effects, and treating flux uncertainties as systematic errors. They found

$$C_5^A(0) = 1.19 \pm 0.08, \quad M_{A\Delta} = 0.94 \pm 0.03 \text{ GeV}$$

for a pure dipole parameterization for $C_5^A(q^2)$. Good agreement with the off-diagonal GTR is found! **No background terms included!**

**Background terms included**

PRD 81 085046 (2010): We included background terms in a combined fit to ANL & BNL data that took into account deuteron effects and flux normalization uncertainties.

We used a simpler dipole parameterization for $C_5^A(q^2)$

$$C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_{A\Delta}^2)^2}$$

Using Adler’s constraints we obtained

$$C_5^A(0) = 1.00 \pm 0.11, \quad M_{A\Delta} = 0.93 \pm 0.07 \text{ GeV}$$

$C_5^A(0)$ compatible with its GTR value ($\sim 1.2$) at the 2$\sigma$ level.
In some of the fits we relaxed Adler’s constraints allowing

\[ C_{3,4}^A(q^2) = C_{3,4}^A(0) \frac{C_{5}^A(q^2)}{C_{4}^A(0)} \]

exploring the possibility of extracting some direct information on \( C_{3,4}^A(0) \)

<table>
<thead>
<tr>
<th>( \chi^2 / \text{dof} )</th>
<th>( C_{5}^A(0) )</th>
<th>( M_{A\Delta/GeV} )</th>
<th>( C_{3}^A(0) )</th>
<th>( C_{4}^A(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I(^*) (only ( \Delta P ))</td>
<td>1.08 ± 0.10</td>
<td>0.92 ± 0.06</td>
<td>Ad</td>
<td>Ad</td>
</tr>
<tr>
<td>II(^*)</td>
<td>0.95 ± 0.11</td>
<td>0.92 ± 0.08</td>
<td>Ad</td>
<td>Ad</td>
</tr>
<tr>
<td>III (only ( \Delta P ))</td>
<td>1.13 ± 0.10</td>
<td>0.93 ± 0.06</td>
<td>Ad</td>
<td>Ad</td>
</tr>
<tr>
<td>IV</td>
<td>1.00 ± 0.11</td>
<td>0.93 ± 0.07</td>
<td>Ad</td>
<td>Ad</td>
</tr>
<tr>
<td>V</td>
<td>1.08 ± 0.14</td>
<td>0.91 ± 0.10</td>
<td>(-1.0 \pm 1.4)</td>
<td>Ad</td>
</tr>
<tr>
<td>VI</td>
<td>1.08 ± 0.14</td>
<td>0.86 ± 0.15</td>
<td>Ad</td>
<td>(-1.0 \pm 1.3)</td>
</tr>
<tr>
<td>VII</td>
<td>1.07 ± 0.15</td>
<td>1.0 ± 0.3</td>
<td>1 ± 4</td>
<td>(-2 \pm 4)</td>
</tr>
</tbody>
</table>

* No deuteron effects included.
Comparison with ANL & BNL data

\[ \nu_\mu d \rightarrow \mu^- p \pi^+ n \]

68% confidence level bands are shown. The total experimental errors shown contain flux uncertainties that are considered as systematic errors and have been added in quadratures to the statistical ones.

Later we included the D13(1520) resonance [E. Hernández., J. Nieves and M.J. Vicente-Vacas, PRD 87 (2013) 113009]
5. Unitarity corrections and Watson’s theorem

Watson’s final-state-interaction theorem (unitarity and time-reversal invariance): The phase of an amplitude leading to a final state with two strongly interacting particles in a given partial wave is the same as the scattering phase of that pair, $\delta$. [PRD 88 (1952) 1163]
Optical theorem in partial waves

\[ SS^\dagger = 1 \iff i \left( T - T^\dagger \right) = T^\dagger T \]

\[ a + b \to 1 + 2 \]

\[ i \left[ \langle \lambda_1 \lambda_2 | T J | \lambda_a \lambda_b \rangle - \langle \lambda_a \lambda_b | T J | \lambda_1 \lambda_2 \rangle^* \right] \sim \sum_{\lambda_1' \lambda_2'} \langle \lambda_1 \lambda_2 | T J^\dagger | \lambda_1' \lambda_2' \rangle \langle \lambda_1' \lambda_2' | T J | \lambda_a \lambda_b \rangle \]
Optical theorem in partial waves

\[ SS^\dagger = 1 \iff i \left( T - T^\dagger \right) = T^\dagger T \]

\[ a + b \to 1 + 2 \]

\[ i \left[ \langle \lambda_1 \lambda_2 | T_J | \lambda_a \lambda_b \rangle - \langle \lambda_a \lambda_b | T_J | \lambda_1 \lambda_2 \rangle^* \right] \sim \sum_{\lambda_1' \lambda_2'} \langle \lambda_1 \lambda_2 | T_J^\dagger | \lambda_1' \lambda_2' \rangle \langle \lambda_1' \lambda_2' | T_J | \lambda_a \lambda_b \rangle \]

Using CM helicity states \(|p; JM\lambda_1 \lambda_2\rangle\) and \textbf{invariance under time reversal},

\[ \langle \lambda_1 \lambda_2 | T_J | \lambda_a \lambda_b \rangle \substack{a+b \to 1+2} = \langle \lambda_a \lambda_b | T_J | \lambda_1 \lambda_2 \rangle \substack{1+2 \to a+b} \]

\[ \mathbb{R} \ni \text{Im} \langle \lambda_1 \lambda_2 | T_J | \lambda_a \lambda_b \rangle \sim \sum_{\lambda_1' \lambda_2'} \langle \lambda_1 \lambda_2 | T_J^\dagger | \lambda_1' \lambda_2' \rangle \langle \lambda_1' \lambda_2' | T_J | \lambda_a \lambda_b \rangle \in \mathbb{R} \]

Juan Nieves, IFIC (CSIC & UV)
Considering intermediate strong interacting $\pi N$ states, Watson's theorem for the weak $WN \rightarrow N\pi$ process implies,

$$\sum_{J} \langle \lambda'_{N} | T_{J}^{\dagger}(s) | \lambda''_{N} \rangle \langle \lambda''_{N} | T_{J}(s) | \lambda_{N} \lambda_{W} \rangle \in \mathbb{R}$$

In terms $\pi N \ | p; L S J M \rangle$ states

$$\sum_{L} \sqrt{\frac{2L + 1}{2J + 1}} \left( L \frac{1}{2} J |0 \lambda'_{N} \lambda''_{N} \right) \langle L \frac{1}{2} J | T_{J} | L \frac{1}{2} J \rangle^{*} \langle L \frac{1}{2} J | T_{J} | \lambda_{N} \lambda_{W} \rangle \in \mathbb{R}$$

For $J = 3/2, T = 3/2$ and neglecting the $L = 2$ multipole,

$$\left\langle P_{33} | T_{J=3/2, T=3/2}^{WN\rightarrow N\pi} | J = 3/2, M = \lambda_{N} - \lambda_{W}, \lambda_{N} \lambda_{W} \right\rangle \times e^{-i\delta_{P_{33}}(s)} \in \mathbb{R}$$

There is a total of 6 $[(\lambda_{N} = \pm \frac{1}{2}) \times (\lambda_{W} = 0, \pm 1)]$ amplitudes which should have the same phase $(\delta_{P_{33}}(s), s = (p_{N} + p_{\pi})^{2})$.
Using CM three momentum helicity states \( |p; \theta \phi \lambda_1 \lambda_2 \rangle \)

\[
|P_{33} M \rangle = \int d\Omega \sum_{\lambda} \sqrt{\frac{3}{4\pi}} \mathcal{D}^{3*}_{M\lambda}(\phi, \theta, -\phi) \left( \frac{1}{2} \frac{3}{2} |0\lambda\lambda\rangle |p; \theta \phi \lambda \rangle
\]

\[
|p; \theta = 0 \phi = 0\lambda_N \lambda_W \rangle = \sum_{J} \sqrt{\frac{2J + 1}{4\pi}} |p; JM = \lambda_N - \lambda_W, \lambda_N \lambda_W \rangle
\]

\[
\int d\Omega \sum_{\lambda} \mathcal{D}^{3}_{\lambda_N - \lambda_W \lambda}(\phi, \theta, -\phi) \left( \frac{1}{2} \frac{3}{2} |0\lambda\lambda\rangle \langle p'; \theta \phi \lambda | T_{J=\frac{3}{2}, T=\frac{3}{2}}^{W_N \rightarrow N_{\pi}} |p; 00\lambda_N \lambda_W \rangle e^{-i\delta_{P_{33}}} \in \mathbb{R}
\]

related to \( \bar{u}(p', \lambda)(O_\mu \epsilon^{\mu}_{\lambda_W})u(p, \lambda_N) \)

There is a total of 6 \( [(\lambda_N = \pm \frac{1}{2}) \times (\lambda_W = 0, \pm 1)] \) amplitudes which should have the same phase \( (\delta_{P_{33}}(s), s = (p_N + p_\pi)^2) \).

Juan Nieves, IFIC (CSIC & UV)
We force the correct phase for two different linear combinations of these amplitudes that correspond to the two multipoles where the $\Delta$ mechanism (vector and axial contributions) is dominant. For instance, in the case of the vector $\Delta$ contribution, this is the $M_{1+}$ multipole. We denote the corresponding axial multipole as $A_\Delta$.

We follow a generalization of M.G. Olsson’s procedure [NPB 78 (1974) 55] introducing two small phases $\phi_{V,A}(s, q^2)$ which correct the vector and axial $\Delta$ contributions such that

$$\text{Im} \left[ \left( T^V_A(s, q^2)e^{i\phi_{V,A}(s, q^2)} + T^A_B(s, q^2) \right)^{M_{1+}; A_\Delta} e^{-i\delta_{P33}(s)} \right] = 0$$
We include chiral background terms in a combined fit to ANL & BNL data that takes into account deuteron effects, flux normalization uncertainties and unitarity corrections (Watson’s theorem).
$C_s^A(0)$ compatible with its GTR value ($\sim 1.2$) at the 1σ level.

moderately small vector and axial Olsson phases!
Juan Nieves, IFIC (CSIC & UV)

$C_3^A(0)$ compatible with its GTR value ($\sim 1.2$) at the 1σ level.

$\nu_\mu \ p \rightarrow \mu^- \ p \pi^+$

$W_{\pi N} < 1.4 \text{ GeV}$

$\sigma \left[ 10^{-38} \text{ cm}^2 \right]$ vs $Q^2 \left[ \text{GeV}^2 \right]$

$\nu_\mu \ n \rightarrow \mu^- \ p \pi^0$

$W_{\pi N} < 1.4 \text{ GeV}$

$\sigma \left[ 10^{-38} \text{ cm}^2 \right]$ vs $E_\nu \left[ \text{GeV} \right]$

problems?
ANL and BNL reanalyzed data: C. Wilkinson, P. Rodrigues, S. Cartwright, L. Thompson, and K. McFarland, PRD 90 (2014) 112017 (similar results)

This underprediction of experimental data is a common problem to other models.
6. The $\nu_\mu n \rightarrow \mu^- n\pi^+$ channel ...

- sensitive to the propagation of spin $1/2$ dof in the $\Delta$ propagator
- large contribution to the $\nu_\mu n \rightarrow \mu^- n\pi^+$ channel

<table>
<thead>
<tr>
<th>reaction</th>
<th>spin-isospin factor direct term</th>
<th>spin-isospin factor crossed term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\mu p \rightarrow \mu^- p\pi^+$</td>
<td>$\sqrt{3}$</td>
<td>$1/\sqrt{3}$</td>
</tr>
<tr>
<td>$\nu_\mu n \rightarrow \mu^- p\pi^0$</td>
<td>$2/\sqrt{3}$</td>
<td>$-2/\sqrt{3}$</td>
</tr>
<tr>
<td>$\nu_\mu n \rightarrow \mu^- n\pi^+$</td>
<td>$1/\sqrt{3}$</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

Juan Nieves, IFIC (CSIC & UV)
In the zero width limit, the $\Delta$ propagator is given by

$$G_{\mu\nu}(p\Delta) = \frac{P_{\mu\nu}(p\Delta)}{p_{\Delta}^2 - M_{\Delta}^2 + i\epsilon}$$

with

$$P_{\mu\nu}(p\Delta) = -(\hat{\gamma} + M_{\Delta}) \left[ g_{\mu\nu} - \frac{1}{3} \gamma_{\mu\gamma} \gamma_{\nu} - \frac{2}{3} \frac{p_{\Delta}^\gamma p_{\Delta}^\nu}{M_{\Delta}} + \frac{1}{3} \frac{p_{\Delta}^\mu \gamma_{\nu} - p_{\Delta}^\nu \gamma_{\mu}}{M_{\Delta}} \right]$$

In an EFT the strength of the contact terms have to be fitted to experiment. According to this, we propose a minimal modification of our model, in which the contact terms that derive from the spin 1/2 part of the $\Delta$ propagator are multiplied by an extra parameter (low energy constant), that will be fitted to data.

see also discussion of consistent couplings to select spin-3/2 dof [V. Pascalutsa, Phys. Lett. B 503 (2001) 85]

the spin-1/2 component does not propagate giving rise to contact interactions

In an EFT the strength of the contact terms have to be fitted to experiment. According to this, we propose a minimal modification of our model, in which the contact terms that derive from the spin 1/2 part of the $\Delta$ propagator are multiplied by an extra parameter (low energy constant), that will be fitted to data.

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\[
\frac{P_{\mu\nu}(p_{\Delta})}{p_{\Delta}^2 - M_{\Delta}^2 + i\epsilon} \rightarrow \frac{P_{\mu\nu}(p_{\Delta}) + c \left( P_{\mu\nu}(p_{\Delta}) - \frac{p_{\Delta}^2}{M_{\Delta}^2} P_{\mu\nu}^{3/2}(p_{\Delta}) \right)}{p_{\Delta}^2 - M_{\Delta}^2 + i\epsilon} = \frac{P_{\mu\nu}(p_{\Delta})}{p_{\Delta}^2 - M_{\Delta}^2 + i\epsilon} + c \delta P_{\mu\nu}(p_{\Delta})
\]

\[
\rightarrow \frac{P_{\mu\nu}(p_{\Delta})}{p_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\Delta}} + c \delta P_{\mu\nu}(p_{\Delta})
\]

\[
= \frac{p_{\Delta}^2}{M_{\Delta}^2} \frac{P_{\mu\nu}^{3/2}(p_{\Delta})}{p_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\Delta}} + \frac{(1 + c)(p_{\Delta}^2 - M_{\Delta}^2) + icM_{\Delta}\Gamma_{\Delta}}{p_{\Delta}^2 - M_{\Delta}^2 + iM_{\Delta}\Gamma_{\Delta}} \delta P_{\mu\nu}(p_{\Delta})
\]

- the LEC $c$ is a free parameter that will be fitted to data
- $c = 0$ original model
- $c = -1$ only propagation of spin-3/2 dof (consistent $\pi N\Delta$ coupling, see V. Pascalutsa) in the $\Delta$ propagator, up to finite $\Delta$ width corrections.
We fit only to reanalyzed data

perfect agreement with the PCAC prediction $\sim 1.2$

better description!

$\nu_\mu n \to \mu^- n \pi^+$ data included in the fit!

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Olsson phases are significantly smaller than in the previous model. This means the present model without the phases is closer to satisfying Watson theorem!
The $\nu_\mu p \rightarrow \mu^- p \pi^+$ at higher energies for $W_{\pi N} < 1.4$ GeV

Besides, the terms that come with the $C_3^A$ and $C_4^A$ nucleon-to-Delta axial form factors become more relevant at higher energies, since larger $q^2$ values are allowed. Deviations from Adler's constraints ($C_3^A(q^2) = 0$, $C_4^A(q^2) = -C_5^A(q^2)/4$), that we implement so far, might play a role in describing the data at higher energies.
Effect of the new terms in pion photoproduction

The model for pion photoproduction is constructed from the vector part of our weak pion production model, including the implementation of Watson theorem.

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Effect of the new terms in pion electroproduction

In collaboration with J.E. Sobczyk

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in collaboration with J.E. Sobczyk

discrepancies for momentum transfers above 1 GeV, but cross sections are much smaller!
Parity violation

7. Parity violating....

- Explicit dependence on $\phi_{\pi}^*$
- $A^*, B^*, C^*, D^*$ and $E^*$ are functions of $E, q^2, W_{\pi N}$ and $\theta_{\pi}^*$
- For NC, $C^*$ and $E^*$ are the same for $\nu$ and $\bar{\nu}$

$$
\frac{d^5 \sigma_{\nu l}}{d\Omega(\hat{k}')dE'd\Omega^*(\hat{k}_{\pi})} = \frac{|\vec{k}'|}{|\vec{k}|^2} \frac{G^2}{4\pi^2} \left\{ \begin{array}{l}
\left\{ A^* + B^* \cos \phi_{\pi}^* + C^* \cos 2\phi_{\pi}^* \right\} \\
\left\{ D^* \sin \phi_{\pi}^* + E^* \sin 2\phi_{\pi}^* \right\}
\end{array} \right\}
$$

\begin{itemize}
  \item Similar to $eN \rightarrow e'N\pi$
  \item parity violating
  \item these are also T-odd terms
\end{itemize}

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\[ A^*, B^*, C^*, D^*, E^* \text{ vs } \cos \theta_{\pi}^* \]

\( E = 1.5 \text{ GeV}, \ W = M_\Delta \)

and \( q^2 = -0.5 \text{ GeV}^2 \)

Reactions \( \nu_\mu n \rightarrow \mu^- p \pi^0 \)

and \( \bar{\nu}_\mu p \rightarrow \mu^+ n \pi^0 \)

- Only direct \( \Delta \)
  \( C_A^A(0) = 1.2, \ M_\Delta = 1.05 \text{ GeV} \)

- Full model \( C_A^A(0) = 1.2, \ M_\Delta = 1.05 \text{ GeV} \)

- Full model \( C_A^A(0) = 0.867, \ M_\Delta = 0.985 \text{ GeV} \)

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... new **NC neutrino–antineutrino asymmetries**

\[
\frac{1}{2} \left( \frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi} \right) \bigg|_\nu = (B_s + B_a) \cos \phi_\pi + (D_s + D_a) \sin \phi_\pi
\]

\[
\frac{1}{2} \left( \frac{d\sigma(\phi_\pi)}{d\phi_\pi} - \frac{d\sigma(\phi_\pi + \pi)}{d\phi_\pi} \right) \bigg|_{\bar{\nu}} = (B_s - B_a) \cos \phi_\pi + (D_s - D_a) \sin \phi_\pi
\]

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results from the old-model (no Watson theorem, no modified propagator etc..). Update is needed!
\[ L_{\mu\sigma}^{(\nu)} = (L_s^{(\nu)})_{\mu\sigma} + i(L_a^{(\nu)})_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k'_\mu - g_{\mu\sigma} k \cdot k' + i \epsilon_{\mu\sigma\alpha\beta} k'^\alpha k_\beta \]

By construction (similar for both CC and NC),

\[ W^{\mu\sigma} = W_s^{\mu\sigma} + iW_a^{\mu\sigma}, \quad W_{s,a}^{\mu\nu} = (W_{s,a}^{\mu\nu})^{PC} + (W_{s,a}^{\mu\nu})^{PV} \]

\[
(W_s^{\mu\nu})^{PC} = W_1 g^{\mu\nu} + W_2 p^\mu p^\nu + W_3 q^\mu q^\nu + W_4 k_\pi^\mu k_\pi^\nu + \cdots \\
(W_a^{\mu\nu})^{PC} = W_{14} \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta + W_{15} \epsilon^{\mu\nu\alpha\beta} p_\alpha k_\pi^\beta + W_{16} \epsilon^{\mu\nu\alpha\beta} q_\alpha k_\pi^\beta + \cdots \\
(W_s^{\mu\nu})^{PV} = W_8 (q_\mu \epsilon_{\nu,\alpha\beta\gamma} k_\pi^\alpha p^\beta q^\gamma + q_\nu \epsilon_{\mu,\alpha\beta\gamma} k_\pi^\alpha p^\beta q^\gamma) + \cdots \\
(W_a^{\mu\nu})^{PV} = W_{11} (q^\mu p^\nu - q^\nu p^\mu) + W_{12} (q^\mu k_\pi^\nu - q^\nu k_\pi^\mu) + \cdots 
\]

**Under Parity**

\[ L_{\mu\nu}^{(\nu)} \rightarrow (L^{\nu\mu})^{(\nu)}, \quad (W_{\mu\nu})^{PC} \rightarrow (W^{\nu\mu})^{PC}, \quad (W_{\mu\nu})^{PV} \rightarrow - (W^{\nu\mu})^{PV} \]

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- $d^5\sigma/d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)$ is not inv. under parity, since the pseudovector $\hat{k} \times \hat{k}'$ is used to define the $Y$ axis.

- $d^3\sigma/d\Omega(\hat{k}')dE'$ scalar, except for the factor $|\vec{k}'|/|\vec{k}| \Rightarrow$ parity violation disappears when performing the $\int d\Omega^* (\hat{k}_\pi)$

$\nu_\mu n \rightarrow \mu^- p\pi^0 \ (E = 1.5 \text{ GeV, } W = M_\Delta, \ q^2 = -0.5 \text{ GeV}^2)$

- Non-resonant terms are needed to produce non-vanishing parity violating structure functions

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8. Conclusions: Model for CC and NC weak pion production off the nucleon,

- In addition to the $\Delta$ resonance, we include **non-resonant contributions** $\leftrightarrow$ QCD $S\chi$SB.
- Non resonant contributions are important $\Rightarrow$ re-adjust of $C_S^{A}(q^2)$. GTR prediction $C_S^{A}(0) \sim 1.2$.
  - Fit to ANL $\Rightarrow$ $C_S^{A}(0) = 0.867 \pm 0.075$
  - Fit to ANL & BNL + normalization uncertainties + deuteron effects $\Rightarrow$ $C_S^{A}(0) = 1.00 \pm 0.11$
  - Fit to ANL & BNL + normalization uncertainties + deuteron effects + **unitarity corrections (Watson’s theorem)** $\Rightarrow$ $C_S^{A}(0) = 1.12 \pm 0.11$, but poor description of $\nu_\mu p \rightarrow n \pi^+$ reaction
  - Addition of extra contact interaction terms that mostly cancel the propagation of spin-1/2 dof in the $\Delta$ propagator (related to the use of a consistent $\pi N\Delta$ coupling, see V. Pascalutsa) $\Rightarrow C_S^{A}(0) = 1.18 \pm 0.07$ and much better description of data, including the $\nu_\mu p \rightarrow n \pi^+$ reaction and pion photo- and electro-production. Olsson phases become also much smaller. Nevertheless, FSI effects on single pion production off the deuteron might induce corrections on the nucleon spectator approximation, and they might be of special relevance precisely in the $\nu_\mu p \rightarrow n \pi^+$ channel (T. Sato et al.)

- There exist parity violation (T-odd correlations) effects due to the interferences between the non resonant and $\Delta$ contributions.
- $\nu - \bar{\nu}$ asymmetries might be used to distinguish $\nu_\tau$ from $\bar{\nu}_\tau$ below the $\tau -$ lepton production threshold.

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