Determination of exotic structure of light hadrons

Takayasu SEKIHARA (RCNP, Osaka Univ.)

in collaboration with

Hiroyuki KAWAMURA (Juntendo Univ)

and Shunzo KUMANO (KEK)

[1] H. Kawamura, S. Kumano, and T. S. , *Phys. Rev.* D88 (2013) 034010.
[2] T. S. and S. Kumano, *Phys. Rev.* C89 (2014) 025202.
[3] T. S. and S. Kumano, arXiv:1409.2213 [hep-ph] (revision coming soon).

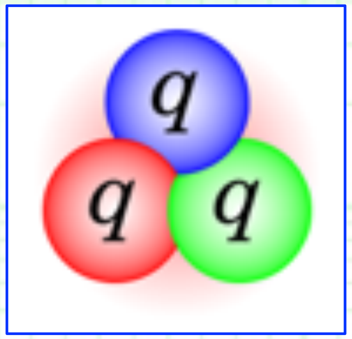
Contents

1. Introduction

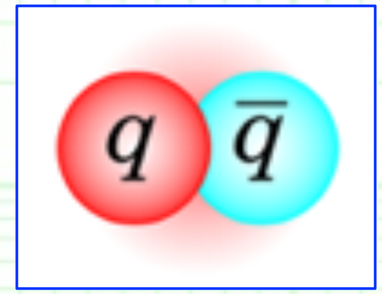
1. Introduction

++ Hadrons ++

- **Hadrons --- Interact with each other by strong interaction.**



Baryons
(p, n, Λ, ...)

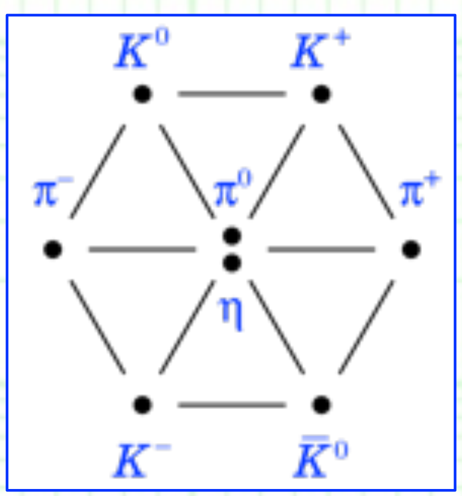


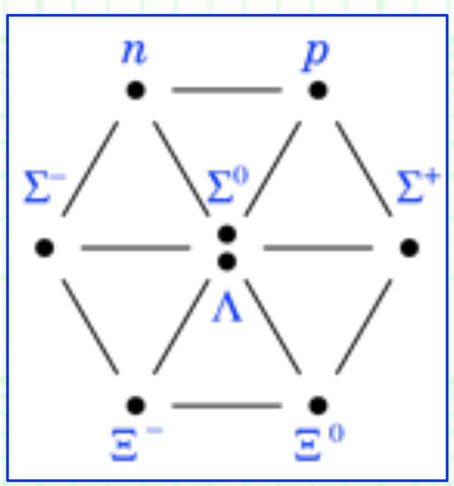
Mesons
(π, K, ρ, ...)

- **Why we know that baryons (mesons) are composed of qqq ($q\bar{q}$) ?**
  - □ We can construct color singlet states minimally from qqq and $q\bar{q}$.

--- QCD, fundamental theory of strong interaction, restricts observables to be color singlet.

  - □ Excellent successes of constituent quark models.





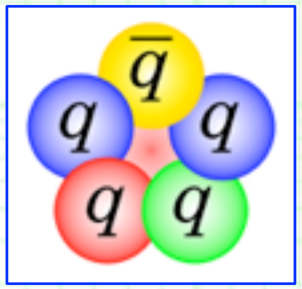
--- Classifications with qqq and $q\bar{q}$,
mass spectra,
magnetic moments,
transition amplitudes, ...

  - □ Parton distribution inside nucleons. □ ...

1. Introduction

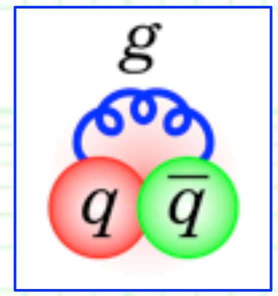
++ Exotic hadrons and their structure ++

- **Exotic hadrons --- not same quark component as ordinary hadrons**
  = not qqq nor $q\bar{q}$.

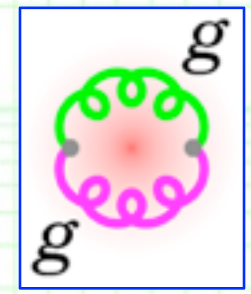


Penta-quarks

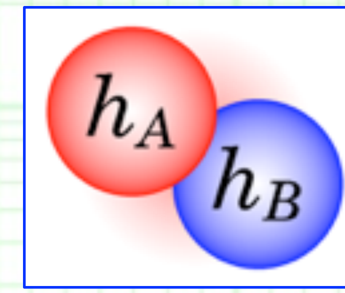
Tetra-quarks



Hybrids



Glueballs



Hadronic molecules

...

--- Actually some hadrons cannot be described by the quark model.

- **Do exotic hadrons really exist ?**
- **If they do exist, how are their properties ?**
  --- Re-confirmation of quark models.
  --- Constituent quarks in multi-quarks ? **"Constituent" gluons ?**
- **If they do not exist, what mechanism forbids their existence ?**

<-- We know very few about hadrons (and dynamics of QCD).

1. Introduction

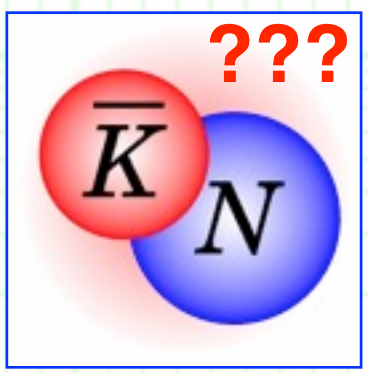
++ Exotic hadrons and their structure ++

- **Exotic hadrons** --- not same quark component as ordinary hadrons = not qqq nor $q\bar{q}$.
  - Candidates: $\Lambda(1405)$, the lightest scalar mesons, $X\ Y\ Z$, ...

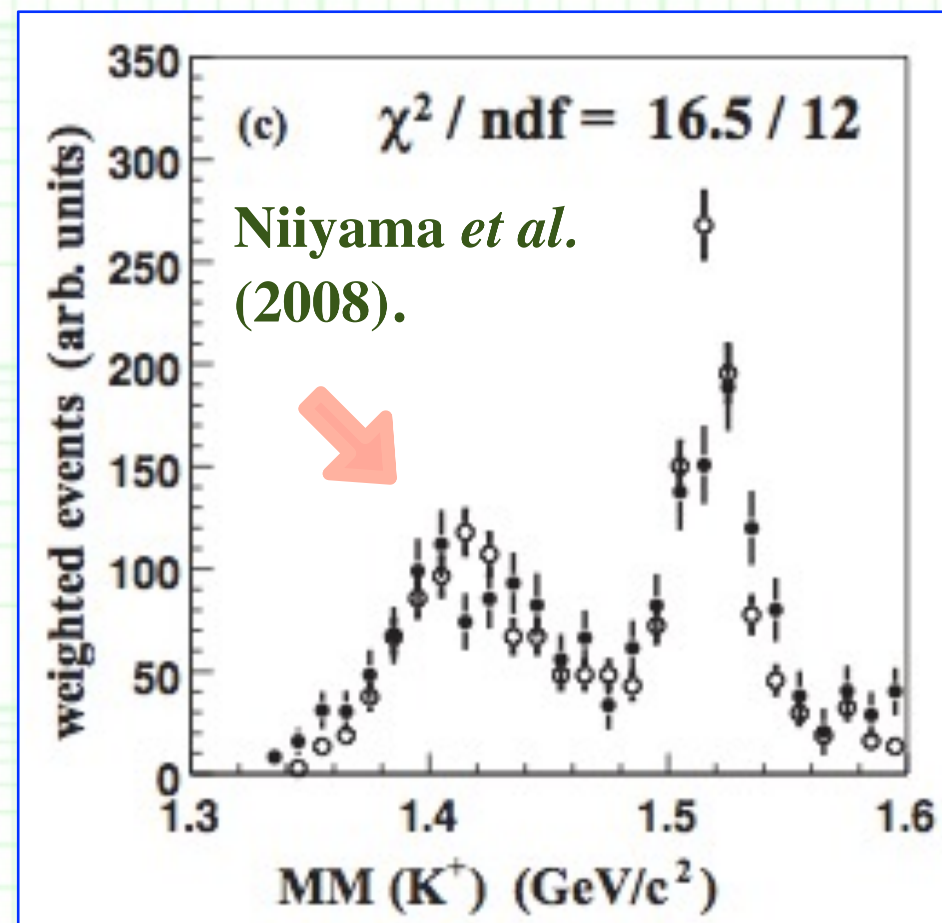
- $\Lambda(1405)$ --- Minimal quark configuration: uds.
  - $\Lambda(1405)$ is the lightest baryon with $J^P = 1/2^-$, although it contains a strange quark.
  - Strongly attractive $\bar{K}N(I = 0)$ interaction.

--> $\Lambda(1405)$ is a $\bar{K}N$ quasi-bound state ???

Dalitz and Tuan (1960), ...



--- A hadronic molecule ?



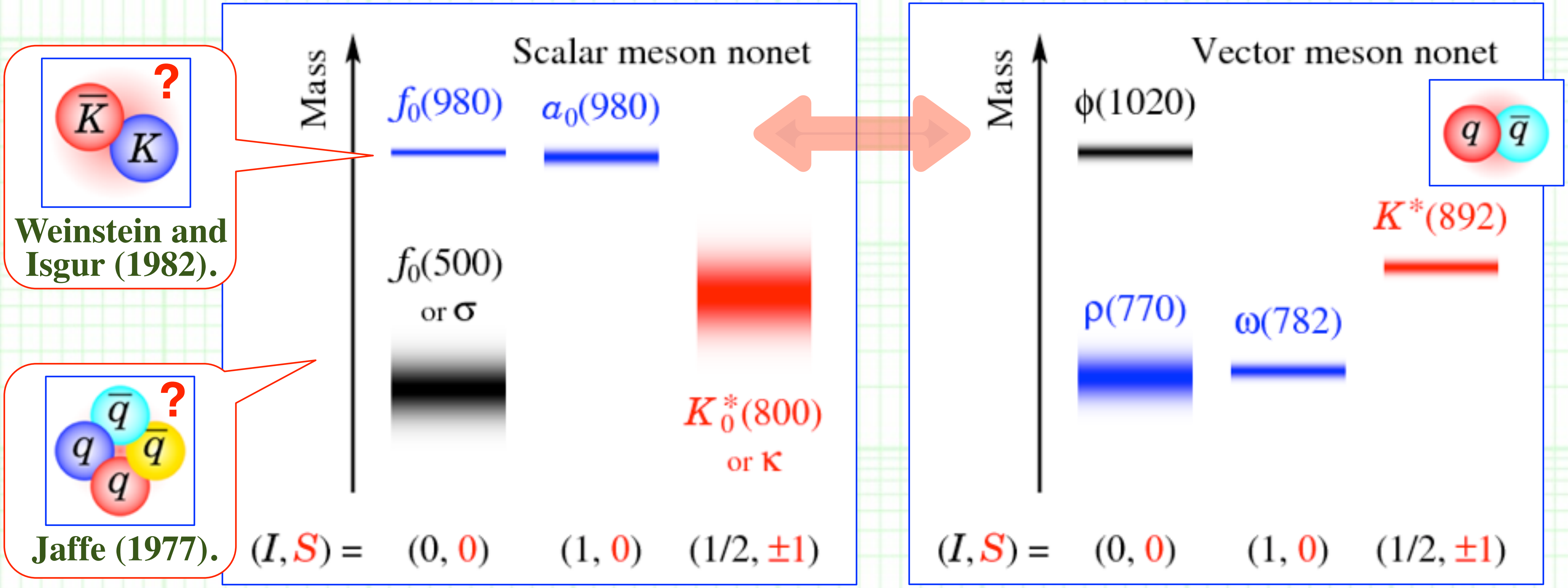
$\gamma\ p \to K^+\ \pi^{\pm}\ \Sigma^{\mp}$

1. Introduction

++ Exotic hadrons and their structure ++

- **Exotic hadrons --- not same quark component as ordinary hadrons = not qqq nor $q\bar{q}$.**
  - **Candidates: $\Lambda(1405)$, the lightest scalar mesons, X Y Z, ...**

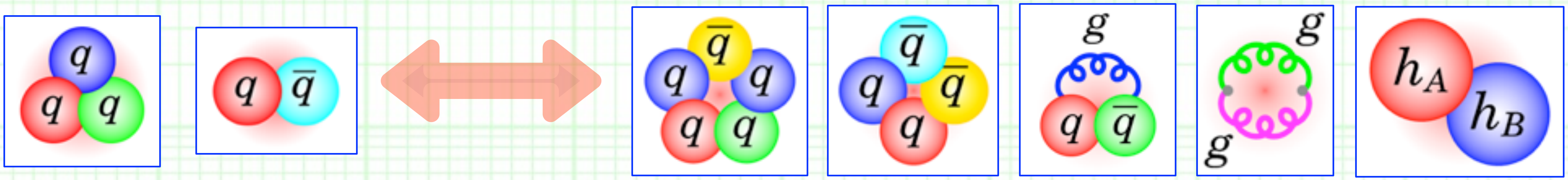
- **The lightest scalar meson nonet.**
  - **Inverted spectrum from the $q\bar{q}$ configuration.**



1. Introduction

++ Identify exotic hadrons ++

- **How can we identify exotic hadrons, especially in Exps.?**



- **Naïve: compare with predictions from constituent quark models.**
  - **Mass, width, couplings, etc. of exotic hadrons do not match the predictions from constituent quark models.**

  --> The constituent quark models can support
  the exotic nature of exotic hadrons = not qqq nor $q\bar{q}$.

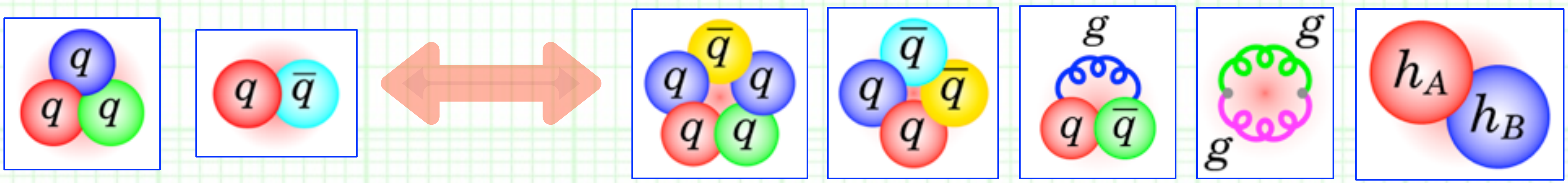
- **However, constituent quark models (or, in general, any effective models) cannot provide undoubted evidences of the exotic nature, because constituent quarks are not "universal" for hadrons.**
  - **Constituent quarks are not asymptotic states of QCD !**

--> We need some approaches which do not rely on effective models of QCD to identify the exotic hadrons.

1. Introduction

++ Identify exotic hadrons ++

- **How can we identify exotic hadrons, especially in Exps.?**



--- What are crucial differences between ordinary and exotic ?

□ **Spatial structure (= spatial size) of hadronic molecules.**

--- Loosely bound hadronic molecules will have large spatial size.

T. S. , T. Hyodo and D. Jido (2008), (2011); T. S. and T. Hyodo (2013).

□ **# of constituents is different.**

--- However, # of constituents is usually not conserved

due to the creation/annihilation of $q\bar{q}$ (*e.g.* $\bar{K}N$ <--> *uds* transition).

--> "Count" it by using the counting rule in high energy scattering.

H. Kawamura, S. Kumano and T. S. , *Phys. Rev.* D88 (2013) 034010.

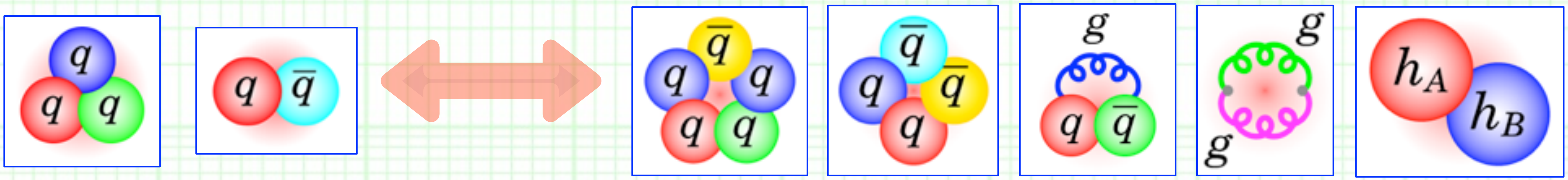
□ **Compositeness is introduced to identify hadronic molecules.**

Hyodo, *Int. J. Mod. Phys.* A28 (2013) 1330045; T. S. , T. Hyodo and D. Jido, arXiv:1411.2308.

1. Introduction

++ Identify exotic hadrons ++

- How can we identify exotic hadrons, especially in Exps.?



--- What are crucial differences between ordinary and exotic ?

□ Spatial structure (= spatial size) of hadronic molecules.

--- Loosely bound hadronic molecules will have large spatial size.

T. S. , T. Hyodo and D. Jido (2008), (2011); T. S. and T. Hyodo (2013).

□ # of constituents is different.

--- However, # of constituents is usually not conserved
due to the creation/annihilation of $q\bar{q}$ (*e.g.* $\bar{K}N$ <--> *uds* transition).

--> "Count" it by using the counting rule in high energy scattering.

H. Kawamura, S. Kumano and T. S. , *Phys. Rev.* D88 (2013) 034010.

□ Compositeness is introduced to identify hadronic molecules.

Hyodo, *Int. J. Mod. Phys.* A28 (2013) 1330045; T. S. , T. Hyodo and D. Jido, arXiv:1411.2308.

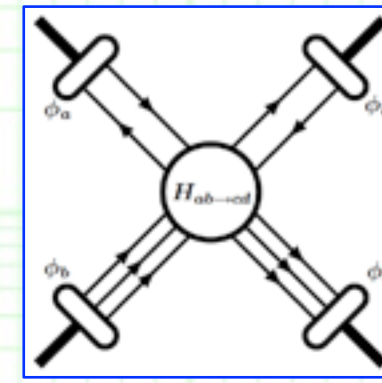
2. Hadron productions and the counting rule in hard exclusive process

2. Hard exclusive process

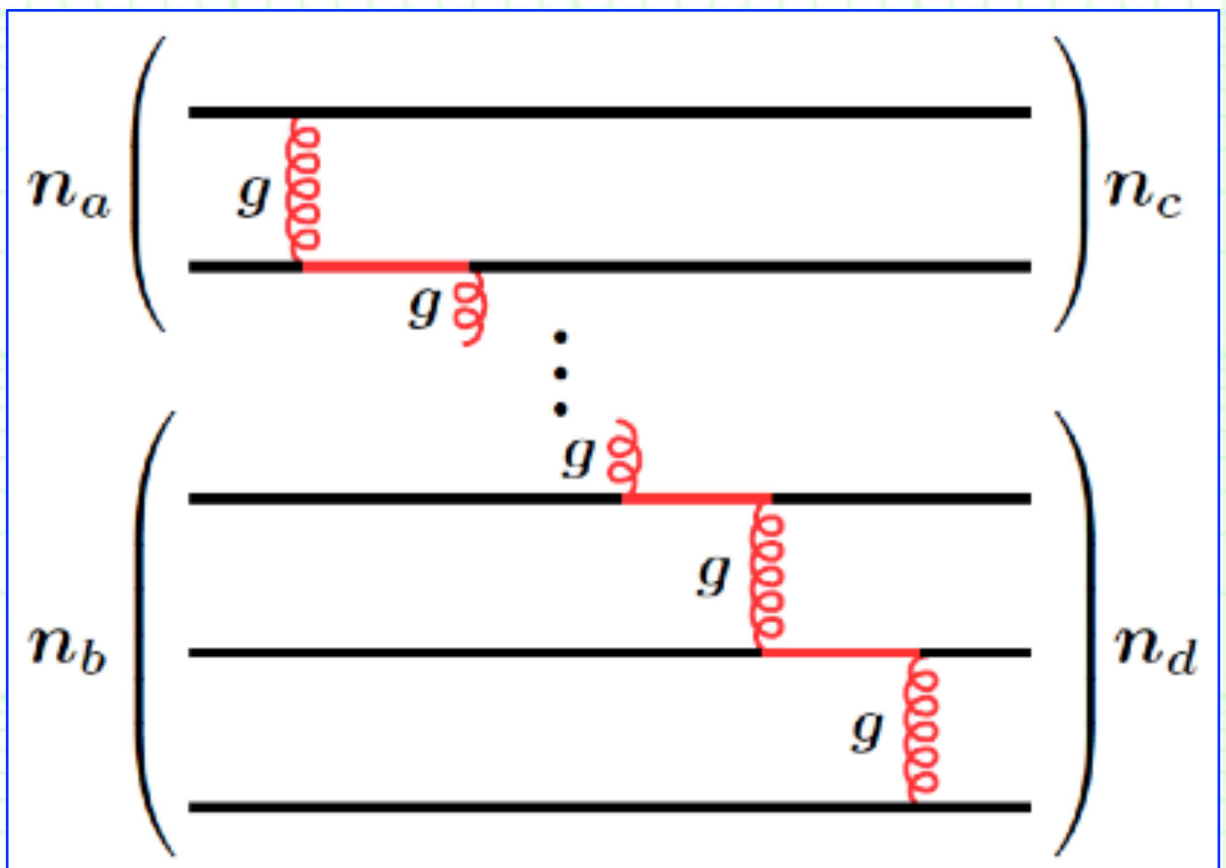
++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



Brodsky and Farar ('73, '75); Matveev *et al.* ('73).



- Consider *a b* --> *c d* reaction in a large-angle exclusive process.

--- # of constituents: $n_a + n_b + n_c + n_d$.

- □ Connect quarks by gluons.
- □ Each gluon propagator ~ 1 / *s*.
- □ Each quark propagator ~ 1 / $s^{1/2}$.

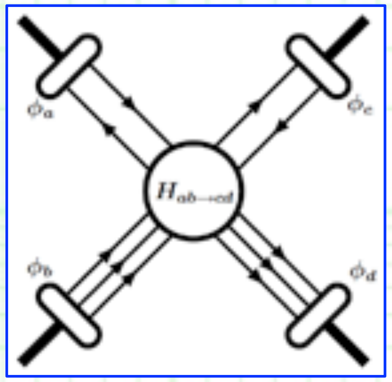
--> Count the power of 1 / *s* to obtain the scaling law.

2. Hard exclusive process

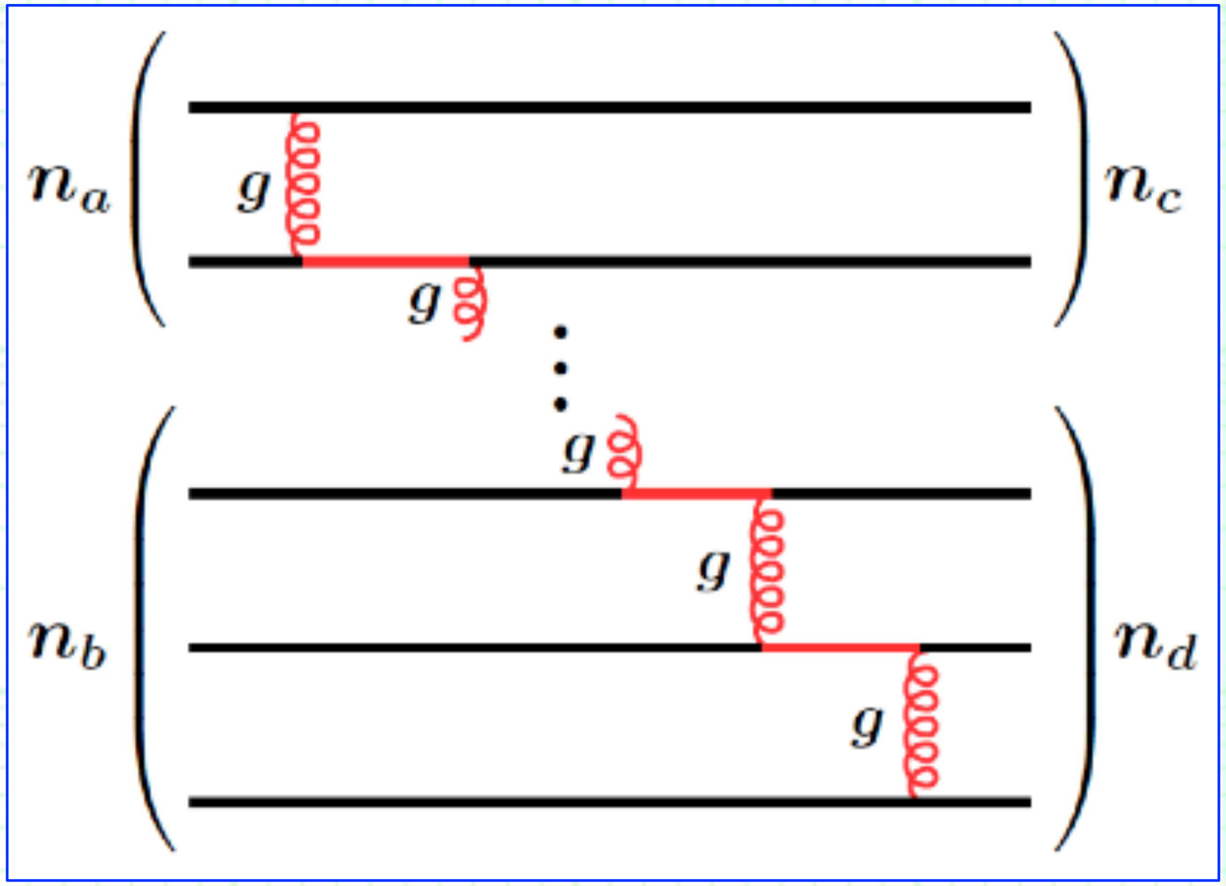
++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges in exclusive reactions at high energy and high momentum transfer region:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



Brodsky and Farar ('73, '75); Matveev *et al.* ('73).



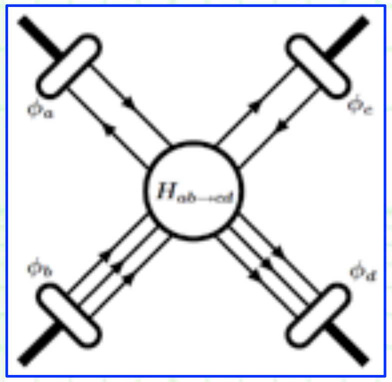
- Consider *a b* --> *c d* reaction in a large-angle exclusive process.
  1. High momentum reaction so as to apply pQCD.
  2. Large scattering angle so as to share the momenta.

--- Applicable to any hadrons as long as we can observe them.

2. Hard exclusive process

++ Counting rule for constituent quarks ++

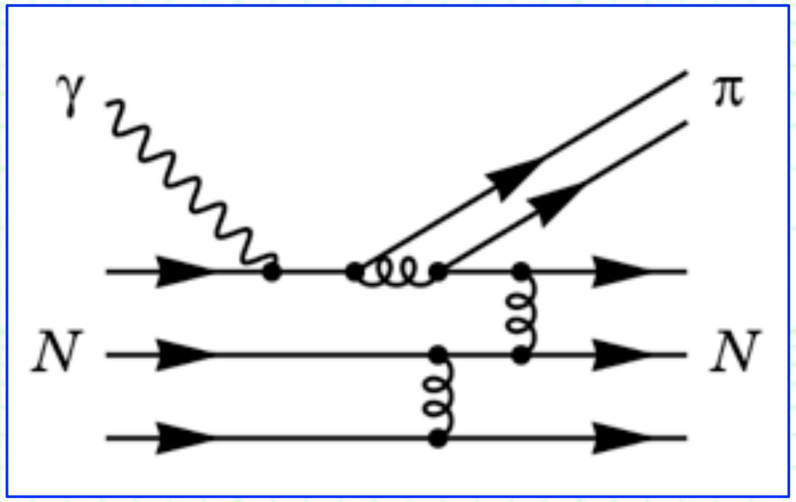
- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:



$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$

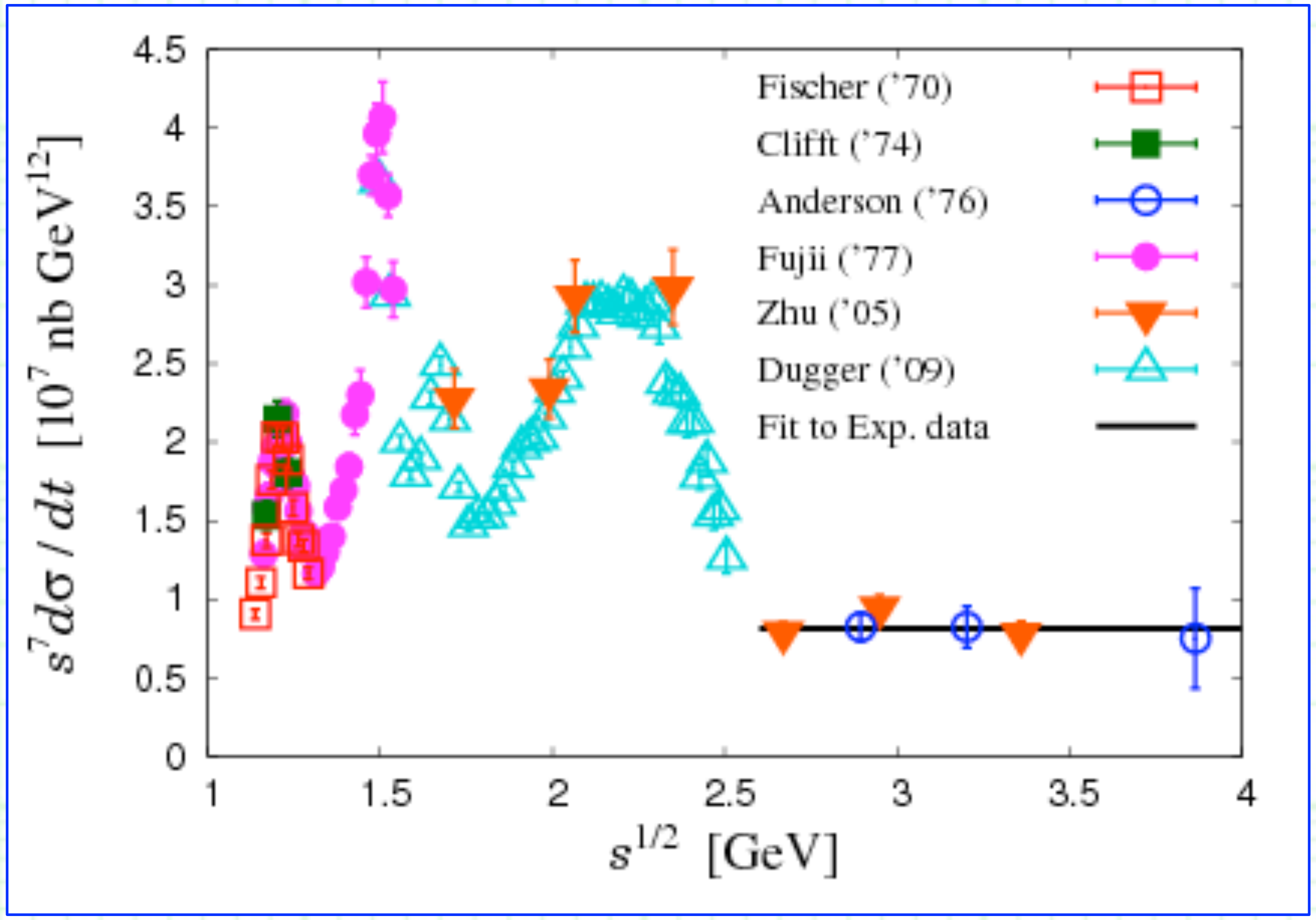
Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- Ex. 1: $\gamma\, p \to \pi^+ n$ at $\theta_{\rm cm} = 90^{\rm o}$.



n = 1+3+2+3 = 9.

--- At High energy and high momentum transfer region, propagators scales as $\sim 1/t \sim 1/u \sim 1/s$.



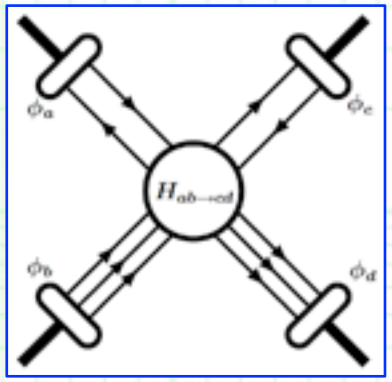
L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003;
H. Kawamura, S. Kumano, and T. S. (2013).

2. Hard exclusive process

++ Counting rule for constituent quarks ++

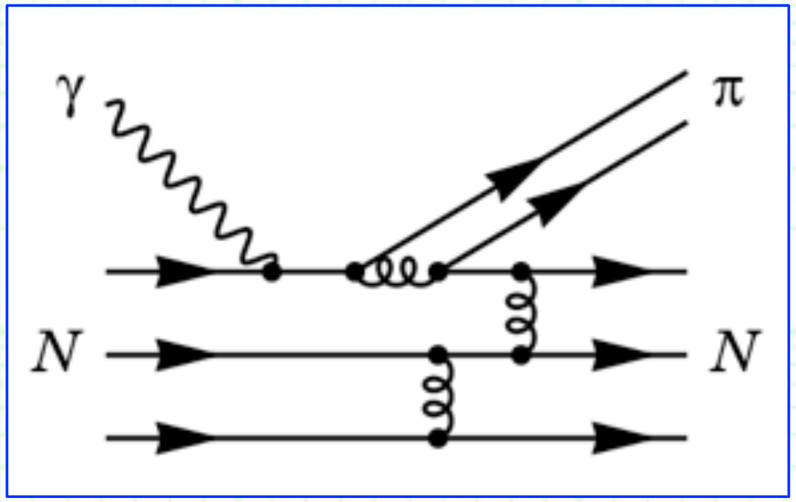
- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



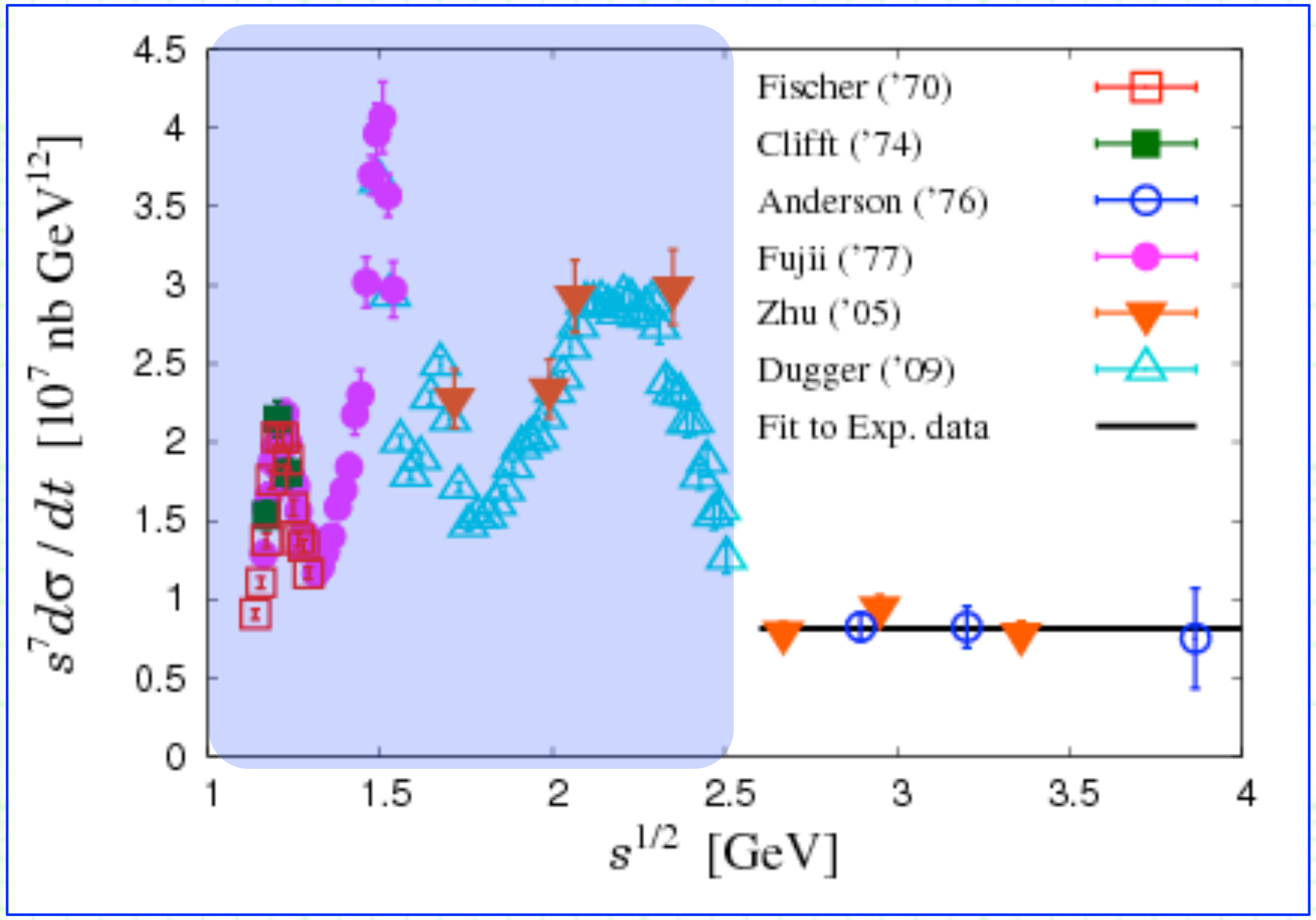
Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- **Ex. 1:** $\gamma\, p \dashrightarrow \pi^+ n$ at $\theta_{\rm cm} = 90^{\rm o}$.



$n = 1+3+2+3$
$= 9.$

--- At High energy and high momentum transfer region, **propagators scales as** $\sim 1/t \sim 1/u \sim 1/s$.



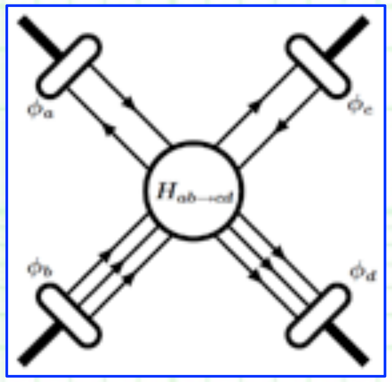
L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003;
H. Kawamura, S. Kumano, and T. S. (2013).

2. Hard exclusive process

++ Counting rule for constituent quarks ++

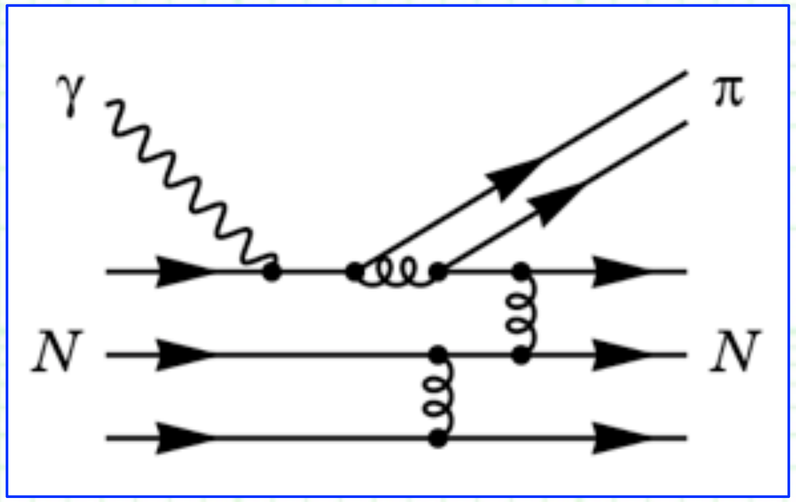
- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



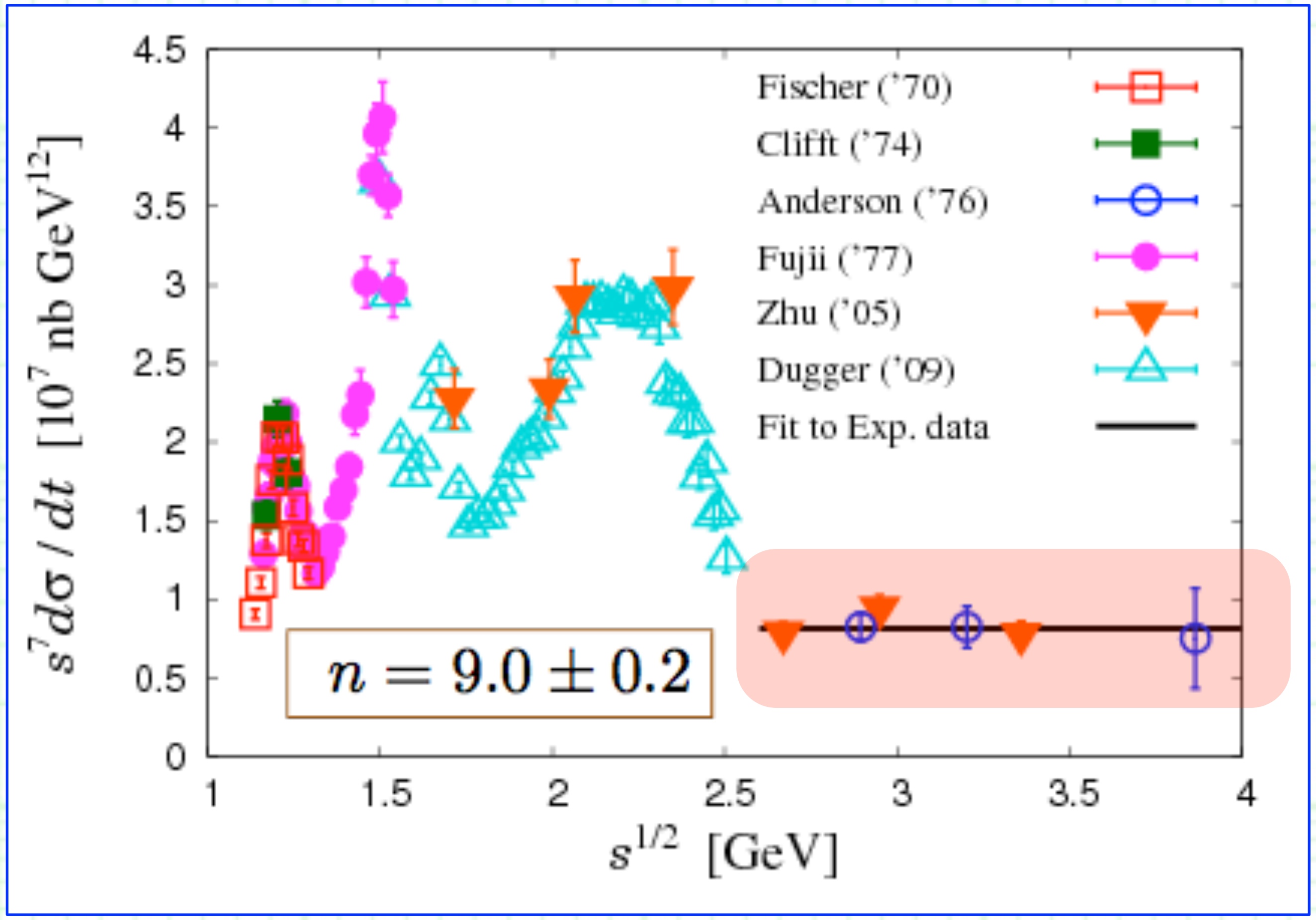
Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- **Ex. 1:** $\gamma\, p \to \pi^+ n$ at $\theta_{\rm cm} = 90^{\rm o}$.



n = 1+3+2+3 = 9.

--- At High energy and high momentum transfer region, **propagators scales as** $\sim 1/t \sim 1/u \sim 1/s$.

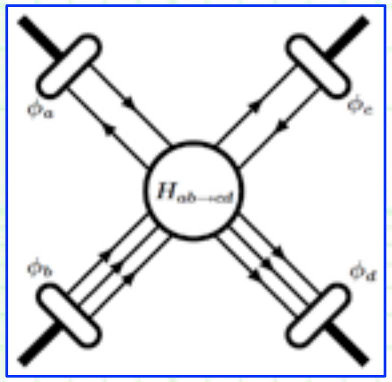


L.Y. Zhu *et al.*, *Phys. Rev. Lett.* 91 (2003) 022003;
H. Kawamura, S. Kumano, and T. S. (2013).

2. Hard exclusive process

++ Counting rule for constituent quarks ++

- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:



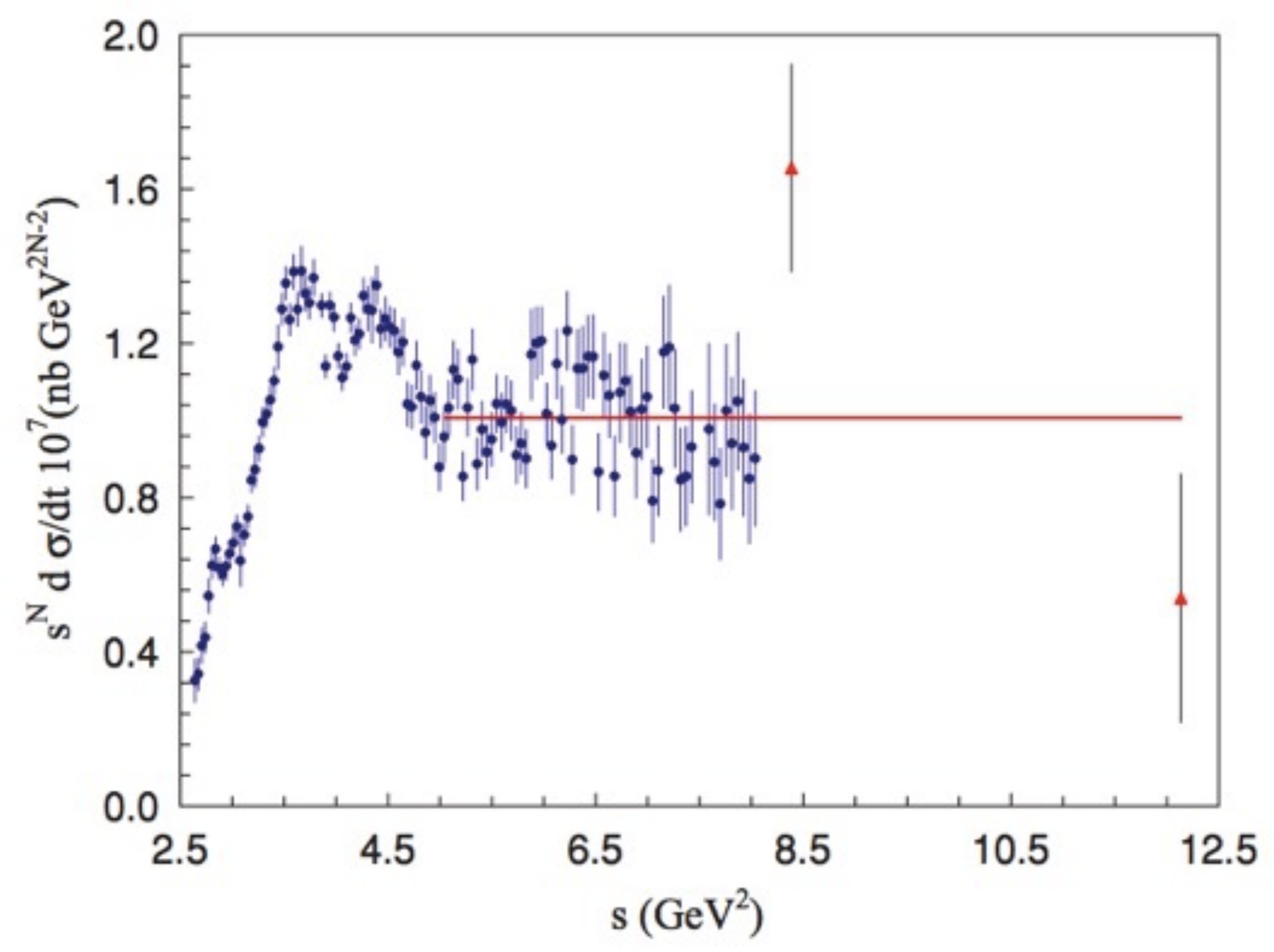
$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$

Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- Ex. 2: $\gamma\, p \dashrightarrow K^+ \Lambda$ at $\theta_{cm} = 90^o$.

--- n = 1+3+2+3 = 9.

  - CLAS and SLAC data.



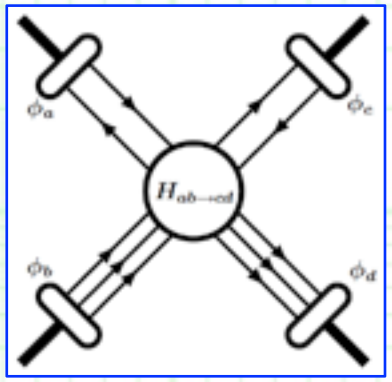
Schumacher and Sargsian, *Phys. Rev.* C83 (2011) 025207.

2. Hard exclusive process

++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$

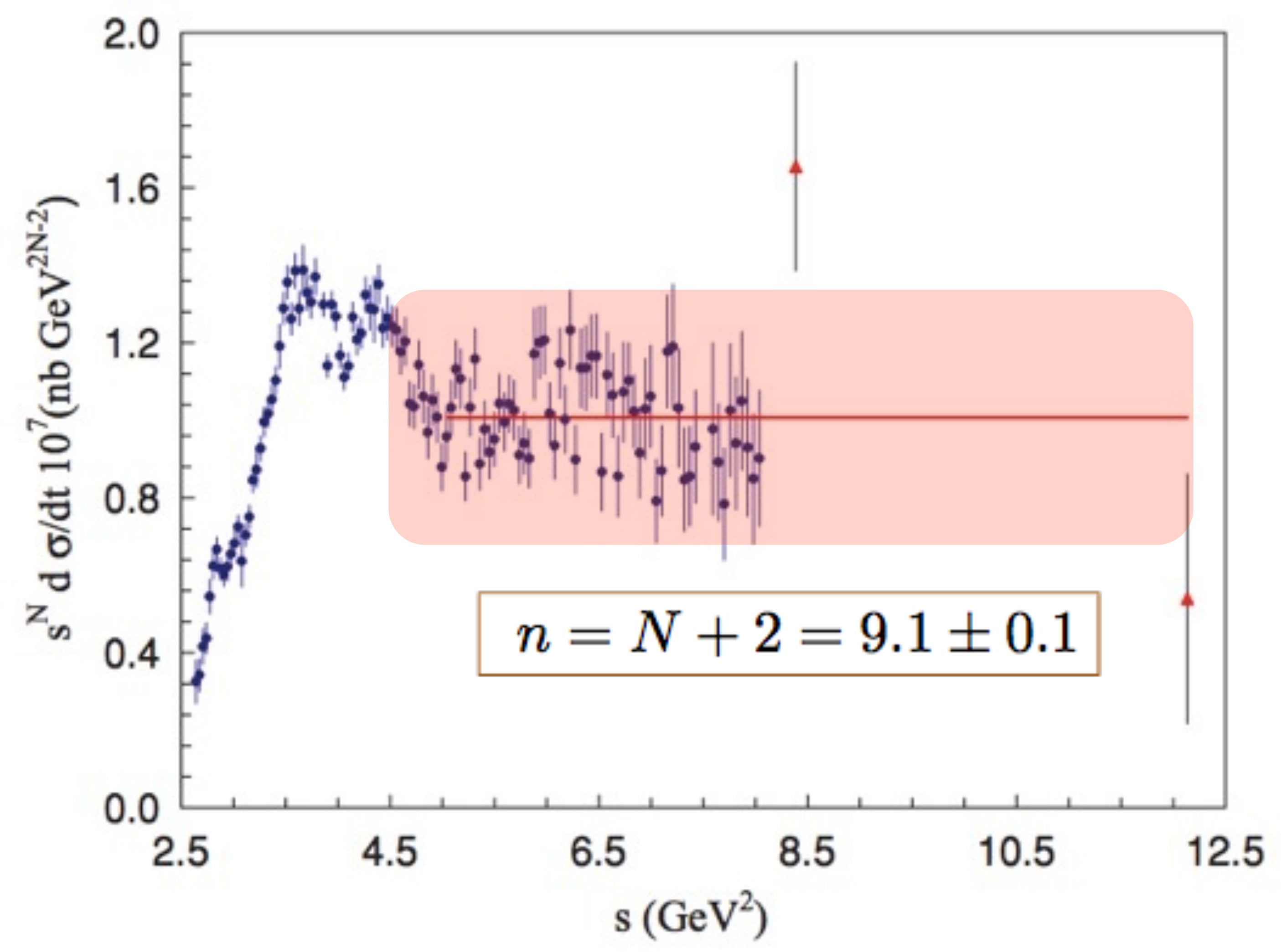


Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- **Ex. 2:** $\gamma\, p \dashrightarrow K^+ \Lambda$ at $\theta_{\rm cm} = 90^{\rm o}$.

--- $n = 1+3+2+3 = 9.$

  - CLAS and SLAC data.



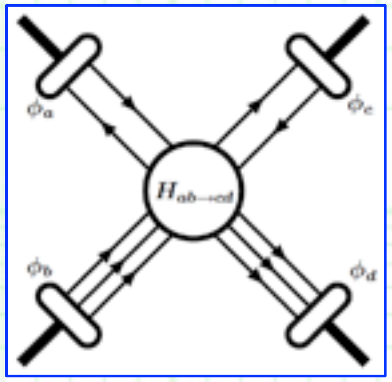
Schumacher and Sargsian, *Phys. Rev.* C83 (2011) 025207.

2. Hard exclusive process

++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges **in exclusive reactions at high energy and high momentum transfer region**:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



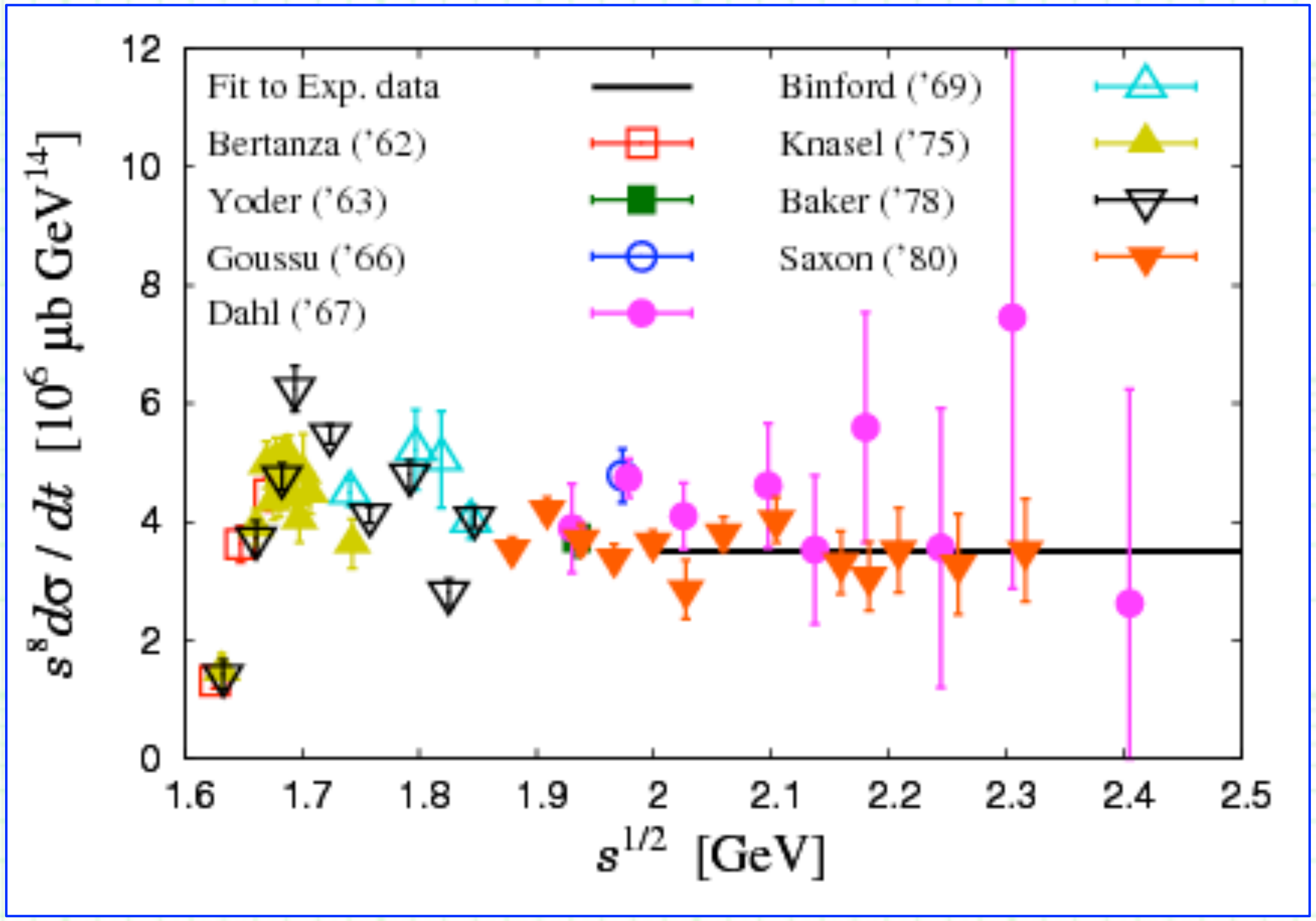
Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- **Ex. 3:** $\pi^- p \to K^0 \Lambda$ at $\theta_{\rm cm} = 90^o$.

--- $n = 2+3+2+3 = 10.$

  - **Exp. data in wide energy range** have been taken in 1960's ~ 1980's: $\sqrt{s}$ = [1.6 GeV, 2.4 GeV].

  Bertanza ('62); Yoder ('63); Goussu ('66); Dahl ('69); Binford ('69); Knasel ('75); Baker ('78); Saxon ('80).



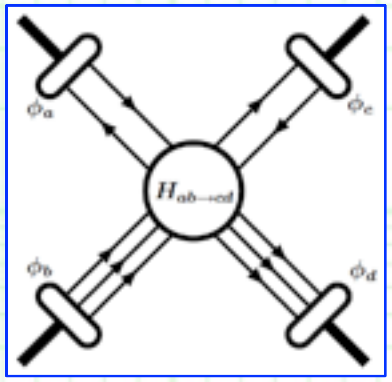
H. Kawamura, S. Kumano, and T. S. (2013).

2. Hard exclusive process

++ Counting rule for constituent quarks ++

- **The constituent counting rule** emerges in exclusive reactions at high energy and high momentum transfer region:

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



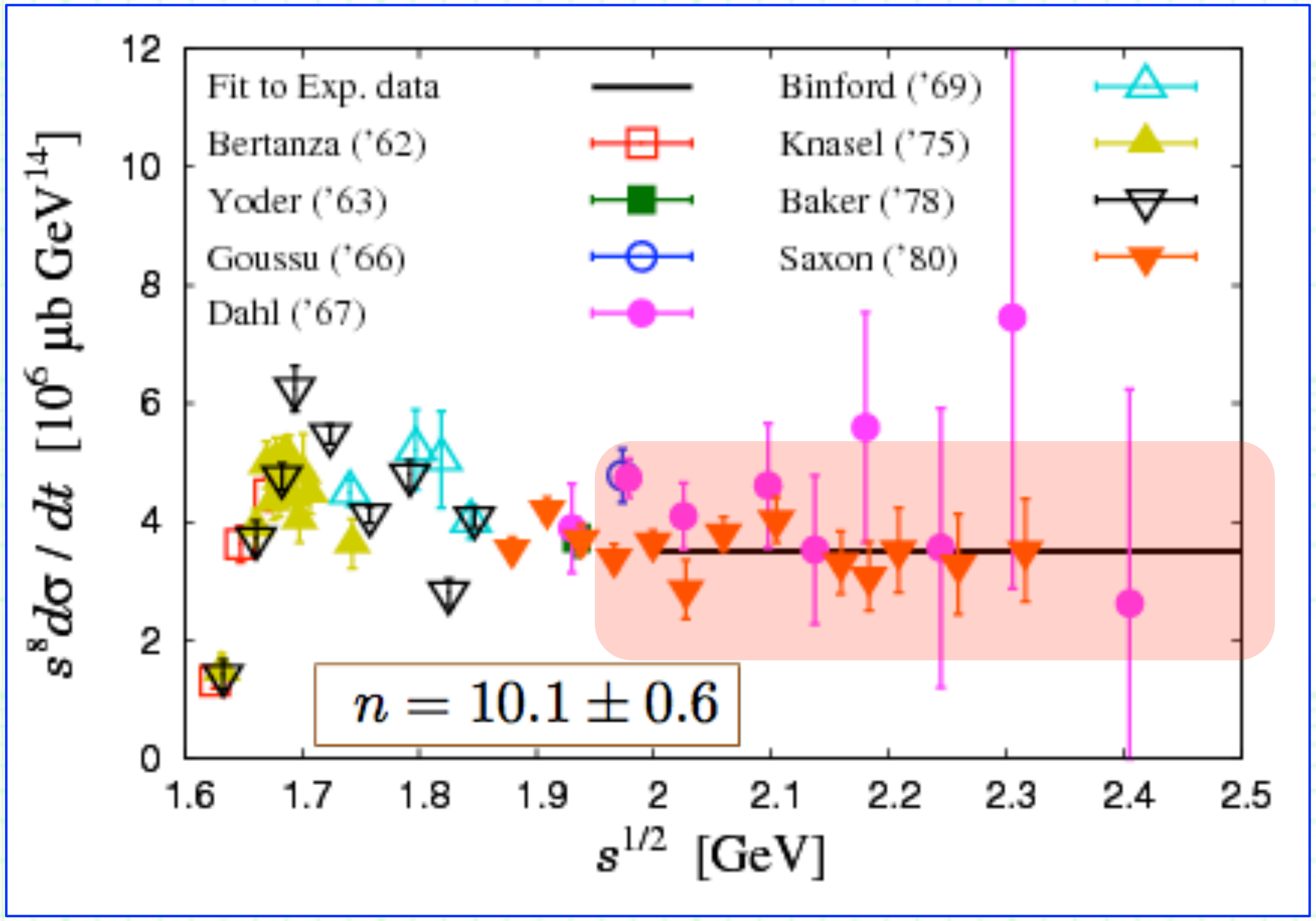
Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- Ex. 3: $\pi^{-} p \to K^0 \Lambda$ at $\theta_{\rm cm} = 90^o$.

--- $n = 2+3+2+3 = 10.$

- Exp. data in wide energy range have been taken in 1960's ~ 1980's: $\sqrt{s}$ = [1.6 GeV, 2.4 GeV].

  Bertanza ('62); Yoder ('63); Goussu ('66); Dahl ('69); Binford ('69); Knasel ('75); Baker ('78); Saxon ('80).



H. Kawamura, S. Kumano, and T. S. (2013).

2. Hard exclusive process

++ Counting rule for constituent quarks ++

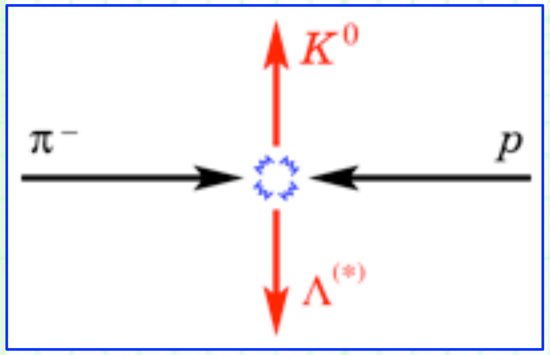
- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:

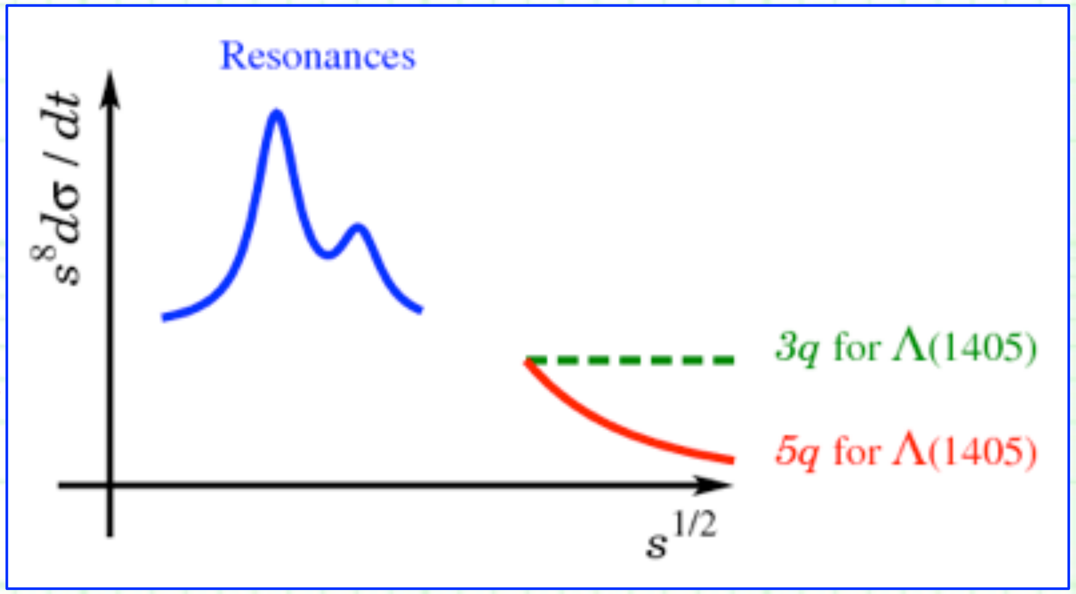
$$\boxed{\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d}$$

Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- Then how cross section of $\pi^- p \to K^0 \Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ behaves at high energy and high momentum transfer region?

$n \overset{?}{=} 2+3+2+3 = 10.$





2. Hard exclusive process

++ Counting rule for constituent quarks ++

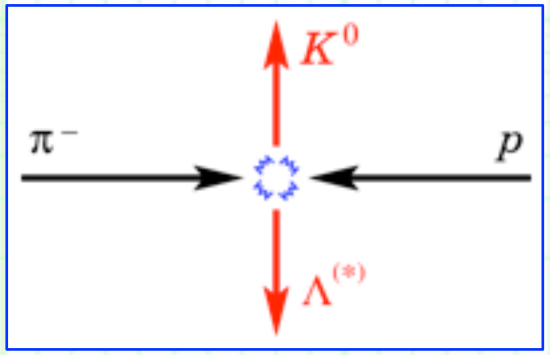
- The constituent counting rule emerges in exclusive reactions at high energy and high momentum transfer region:

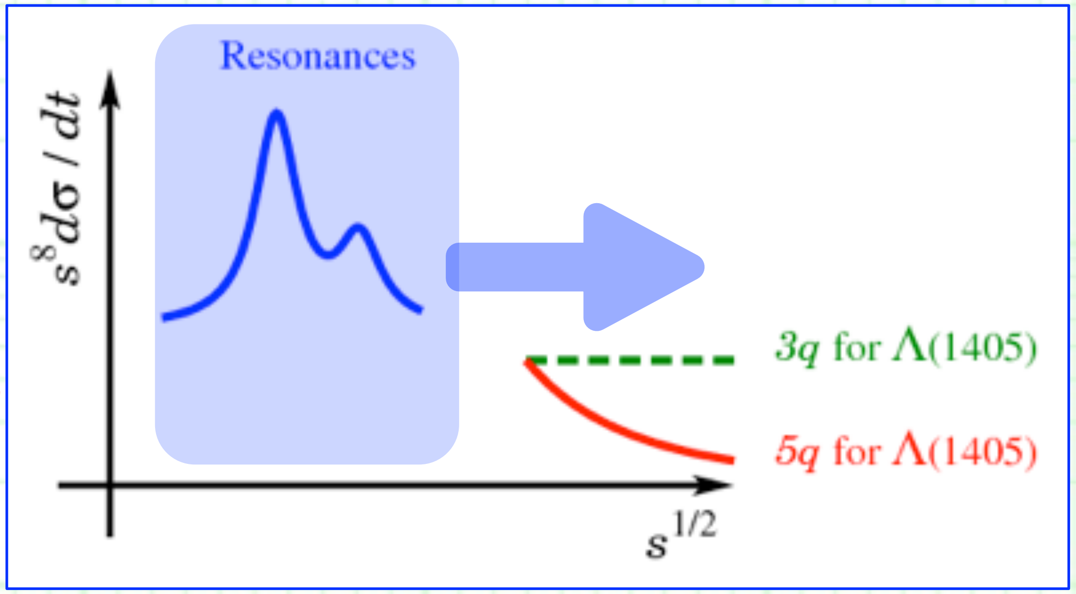
$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$

Brodsky and Farar ('73, '75); Matveev *et al.* ('73).

- Then how cross section of $\pi^{-} p \dashrightarrow K^0 \Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ behaves at high energy and high momentum transfer region?

$n \overset{?}{=} 2+3+2+3 = 10.$





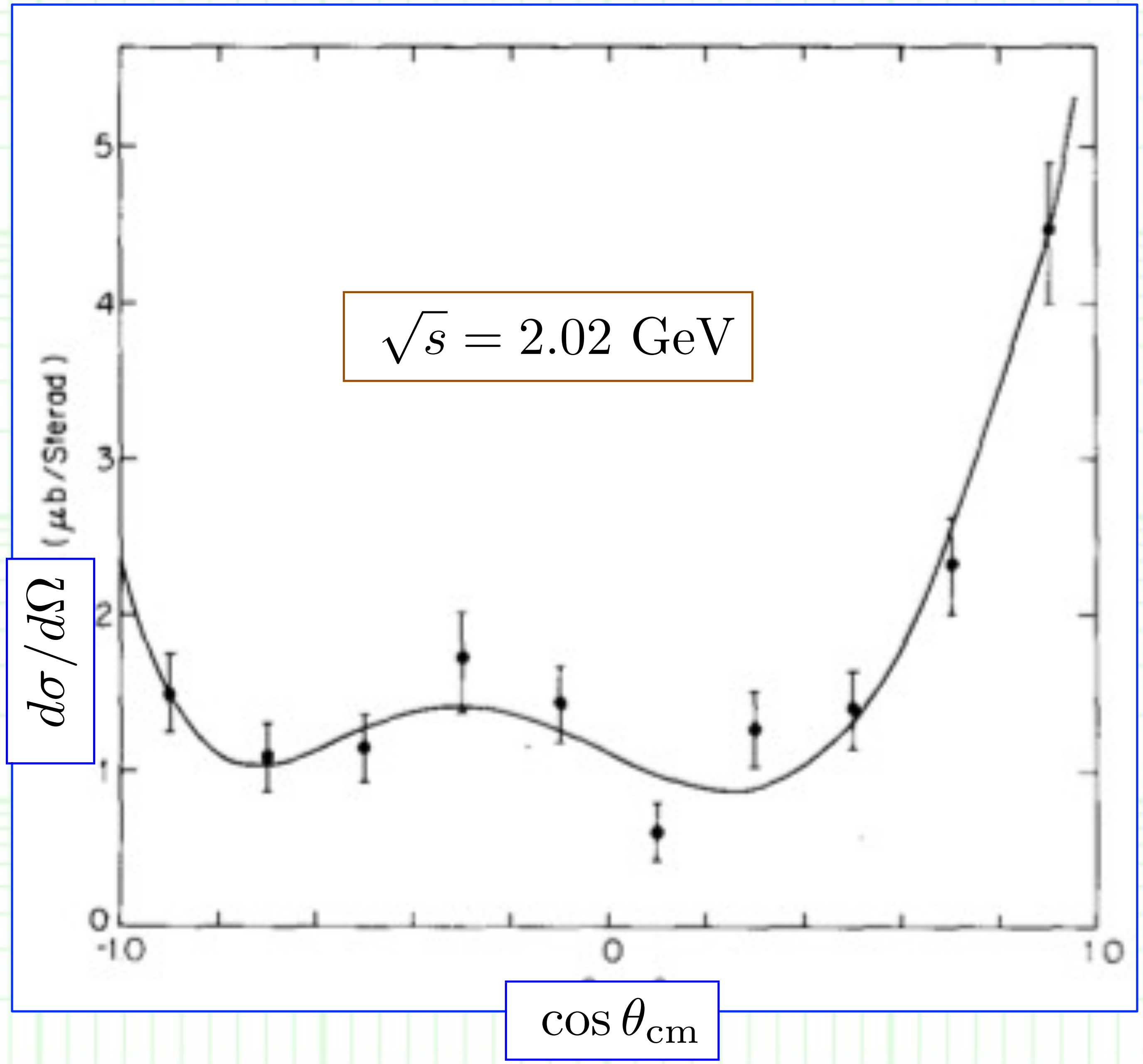
--> We "estimate" cross section of $\pi^{-} p \dashrightarrow K^0 \Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ as a function of s from the resonance region to the pQCD one.

2. Hard exclusive process

++ Λ(1405) production: Experimental data ++

- Now we consider
  $\pi^- p \to K^0\,\Lambda(1405)$ reaction.

--- Very few Exp. data have been taken, and (as far as I know) only one data is available for $d\sigma / dt$ at $\theta_{cm} = 90^o$:



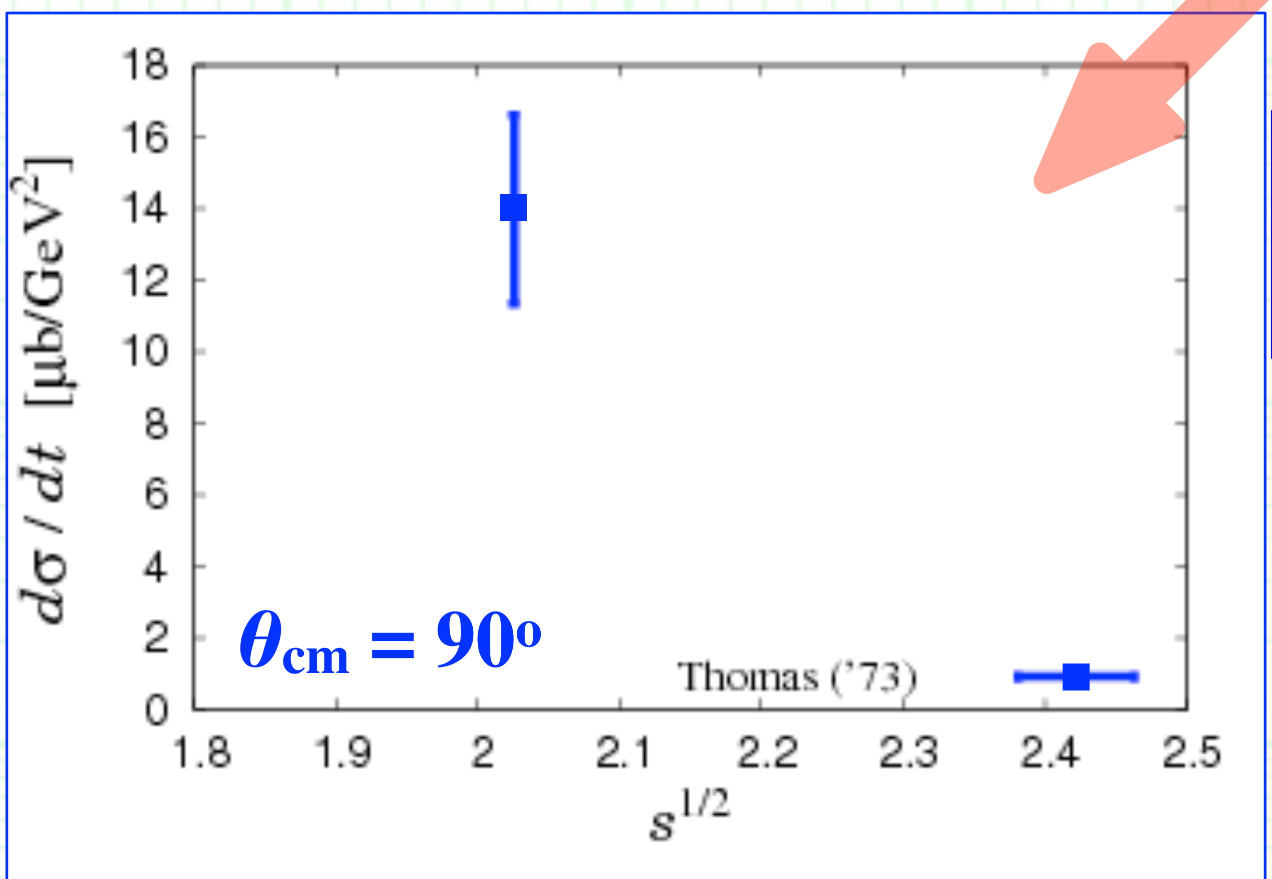
Thomas *et al.*, *Nucl. Phys.* B56 (1973) 15.

2. Hard exclusive process

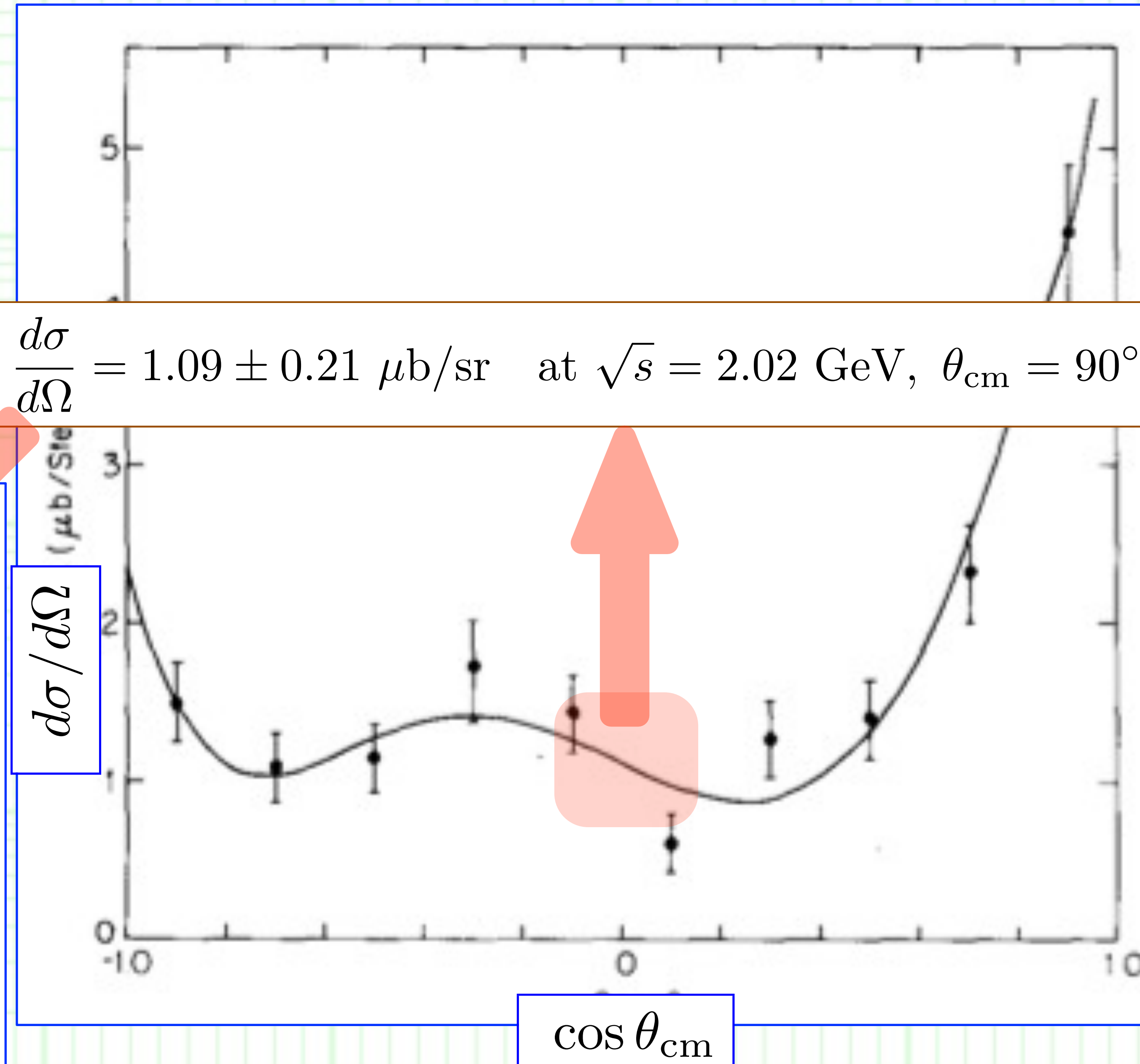
++ Λ(1405) production: Experimental data ++

- Now we consider
  $\pi^{-} p \to K^0\, \Lambda(1405)$ reaction.

--- Very few Exp. data have been taken, and (as far as I know) only one data is available for $d\sigma / dt$ at $\theta_{cm} = 90^o$:



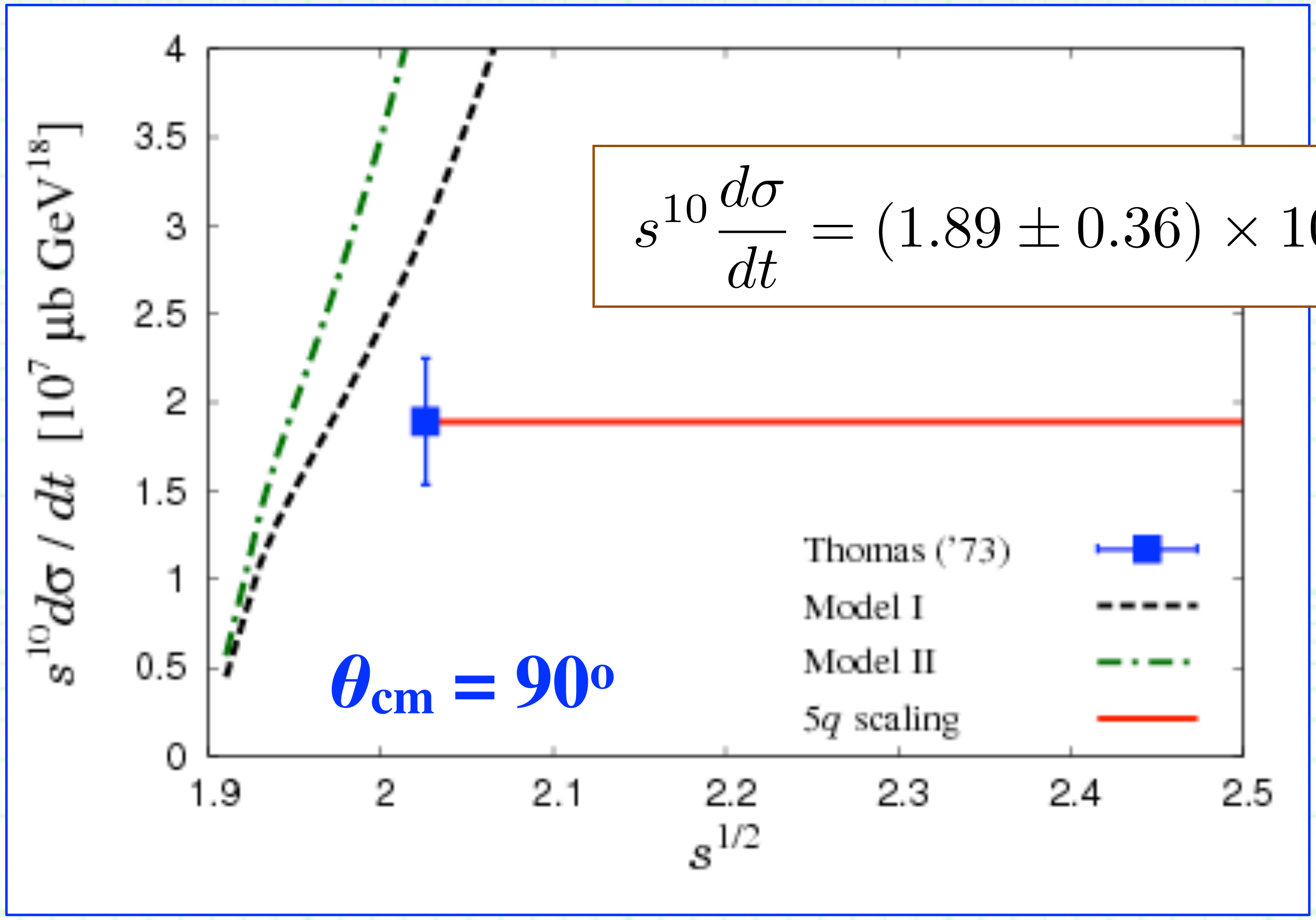
$$\frac{d\sigma}{d\Omega} = 1.09 \pm 0.21\ \mu\mathrm{b/sr} \quad \text{at } \sqrt{s} = 2.02\ \mathrm{GeV},\ \theta_{\mathrm{cm}} = 90^\circ$$



Thomas *et al.*, *Nucl. Phys.* B56 (1973) 15.

2. Hard exclusive process

++ Λ(1405) production: Estimation ++



$$s^{10}\frac{d\sigma}{dt} = (1.89 \pm 0.36) \times 10^7 \mu\text{b GeV}^{18} \quad \text{at } \sqrt{s} = 2.02 \text{ GeV}$$

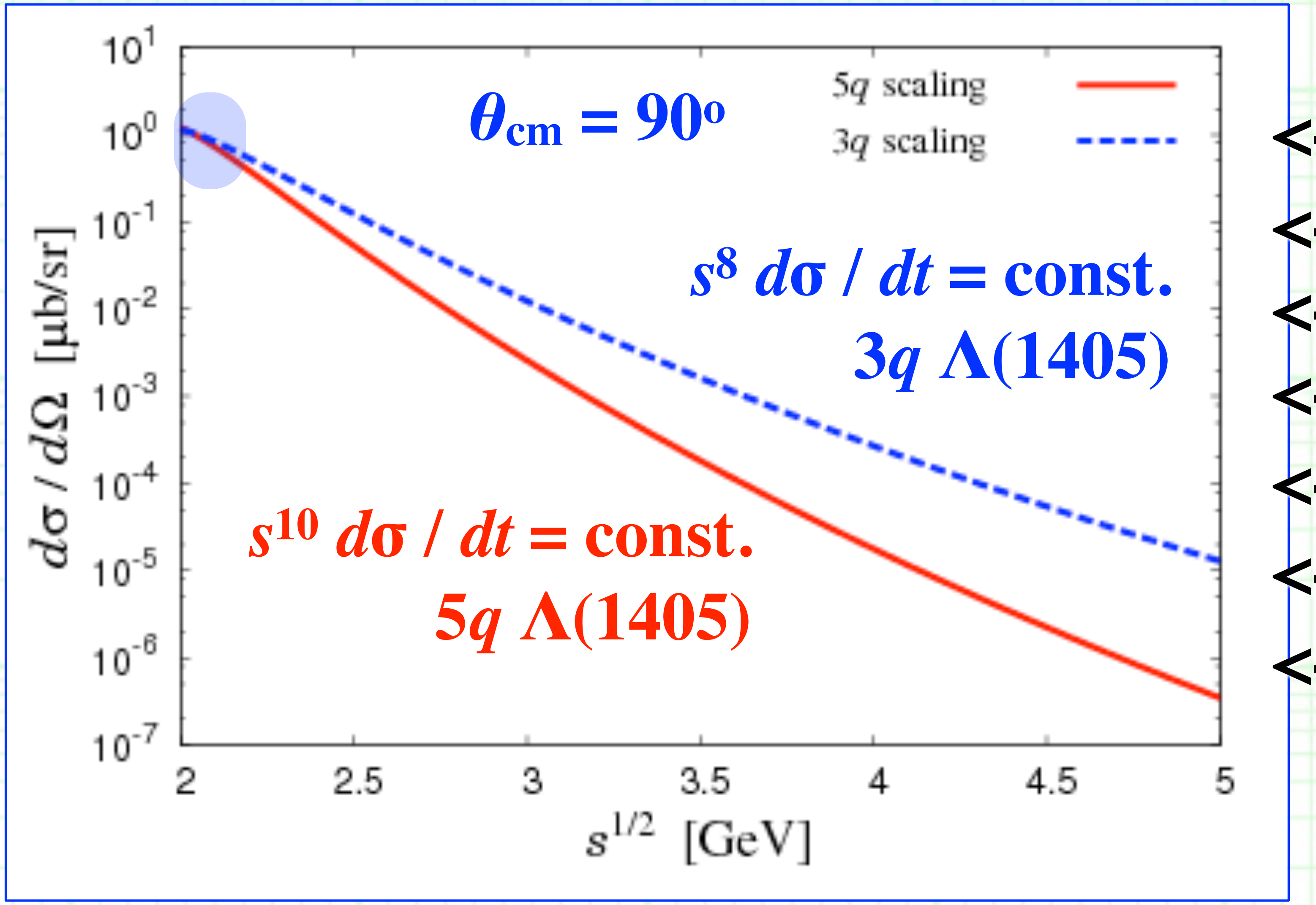
- **If Λ(1405) is a 5*q* state (including a $\bar{K}N$ molecule), the cross section scales as $s^{10}\, d\sigma / dt = \text{const.}$ (the red straight line).**

--- Theoretical calculation (Model I & II) of $\pi^- p \to K^0 \Lambda(1405)$ reaction from the chiral unitary model. Hyodo *et al.*, *Phys. Rev.* C68 (2003) 065203.

2. Hard exclusive process

++ Λ(1405) production: Estimation ++

- Estimate cross section at higher energies by using Exp. data at $\sqrt{s}$ = 2.02 GeV with $s^{10}\, d\sigma / dt$ = const. or $s^{8}\, d\sigma / dt$ = const.



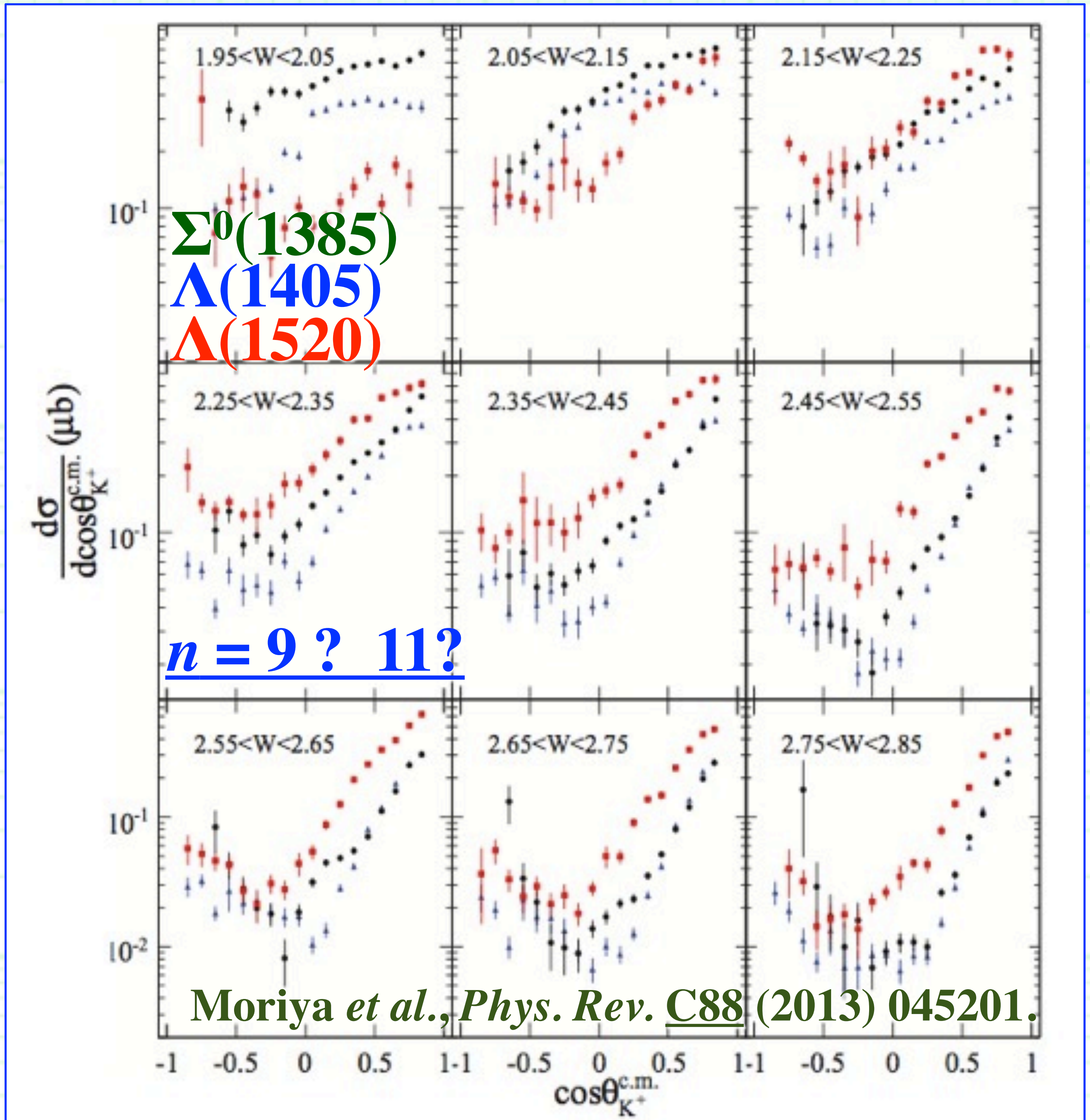
<-- 1 μb
<-- 100 nb
<-- 10 nb
<-- 1 nb
<-- 100 pb
<-- 10 pb
<-- 1 pb

- Ratio of the cross section for 3q and 5q Λ(1405) is about 10:1 (~ 10 nb : 1 nb) at $\sqrt{s}$ = 3 GeV and more at higher energies.

2. Hard exclusive process

++ How about Λ(1405) photoproduction? ++

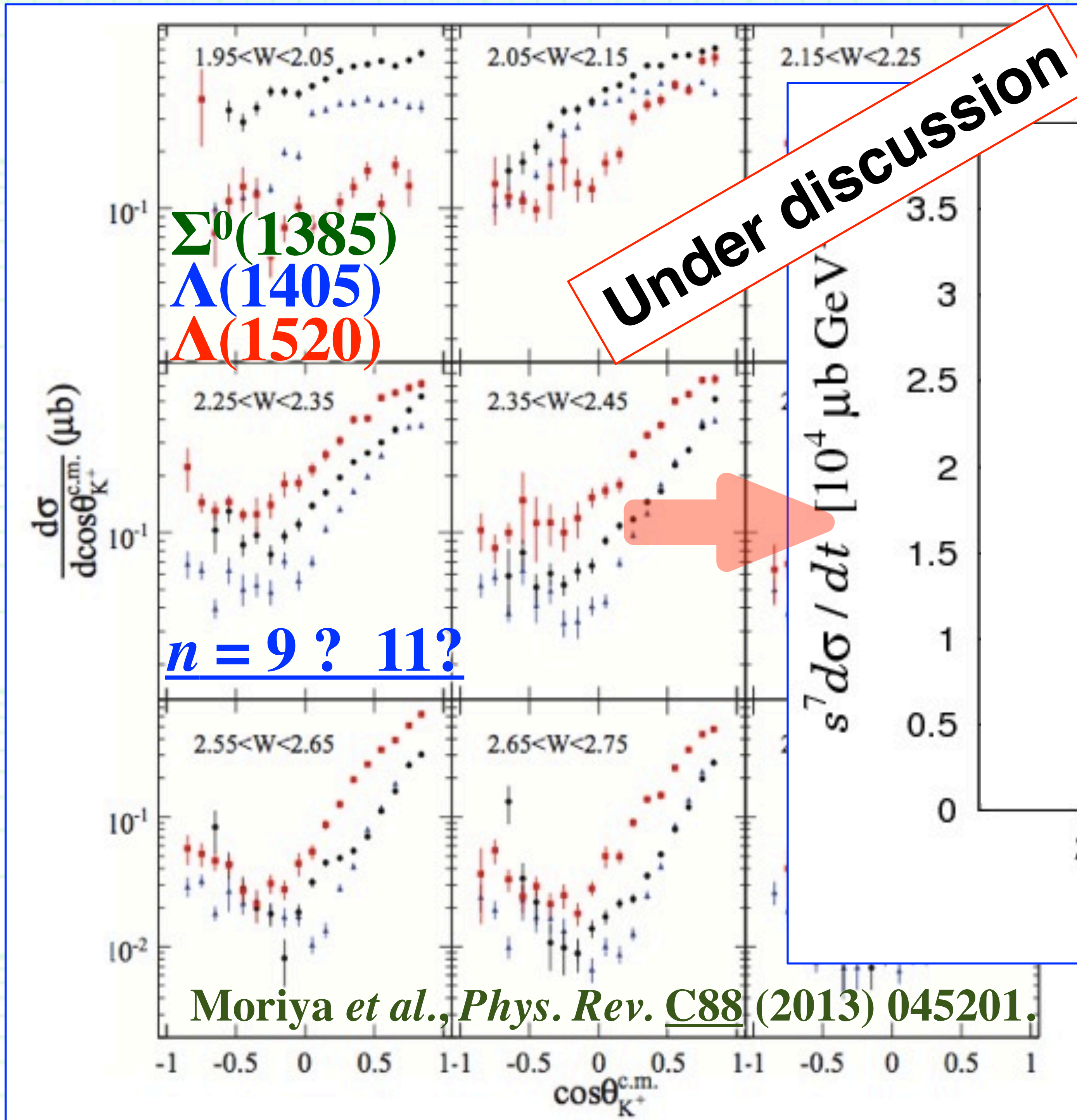
- The scaling from Exp. data of $\gamma\, p \to K^+\, \Sigma^0(1385), \Lambda(1405), \Lambda(1520)$ taken by CLAS collaboration.

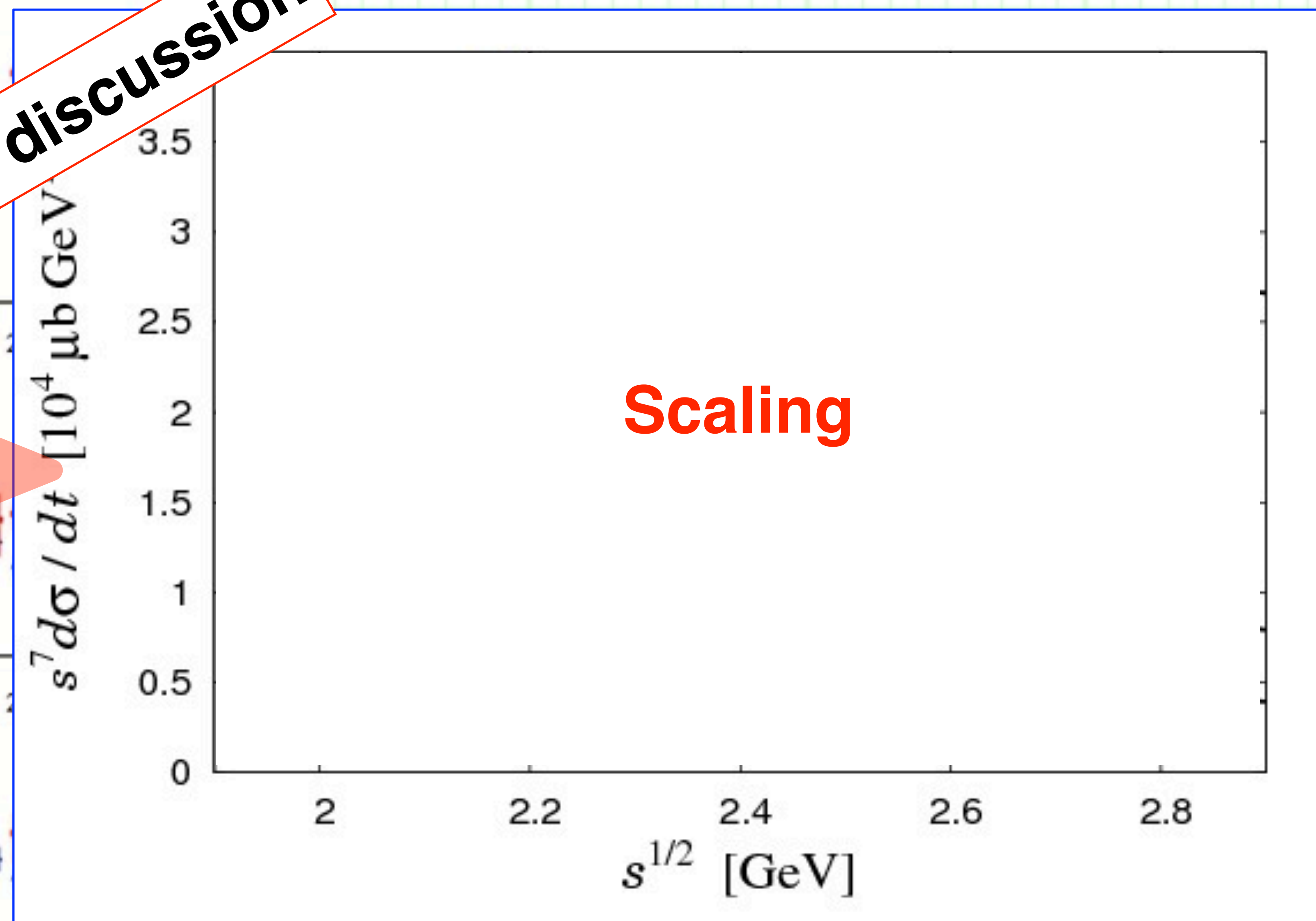


2. Hard exclusive process

++ How about $\Lambda(1405)$ photoproduction? ++

- The scaling from **Exp. data of $\gamma\, p \to K^+\, \Sigma^0(1385), \Lambda(1405), \Lambda(1520)$ taken by CLAS collaboration.**





Under discussion

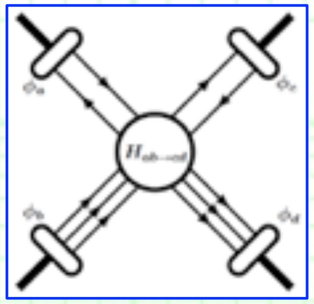
--- But the energy where the scaling is valid are unknown.

2. Hard exclusive process

++ Summary of the counting rule ++

- **The constituent counting rule** **in exclusive reactions at high energy with high momentum transfer** **may elucidate hadron structure.**

$$\left(\frac{d\sigma}{dt}\right)_{ab\to cd} \sim s^{2-n} \times f(\theta_{\rm cm}), \quad n \equiv n_a + n_b + n_c + n_d$$



- **Ground N and Λ productions indicates**
  a scaling law with $n_q(N) = n_q(\Lambda) = 3$.

- **We estimate high-energy cross section $\pi^- p \to K^0\,\Lambda(1405)$ at $\theta_{\rm cm} = 90^{\rm o}$ from resonance region.**

--> For $\Lambda(1405)$, cross section for $3q$ ($5q$) $\Lambda(1405)$ is ~ 10 nb (1 nb) at $\sqrt{s}$ = 3 GeV and the deviation gets larger at higher energies.

- **The "scaling" from CLAS seems to imply $5q$ for $\Lambda(1405)$.**

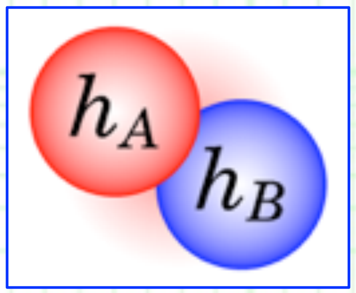
--- However, we need both theoretical and experimental improvement more to determine the $\Lambda(1405)$ structure. --> J-PARC etc.

3. Hadronic molecules and compositeness

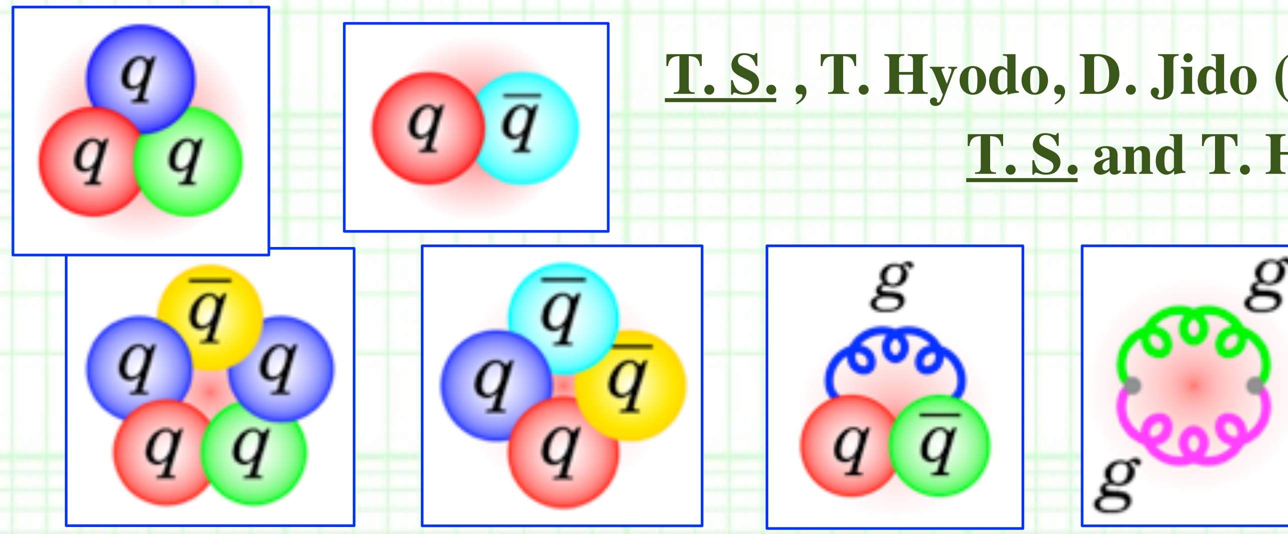
3. Compositeness

++ Uniqueness of hadronic molecules ++

- **Hadronic molecules** should be **unique**, because they would have large spatial size compared to other (compact) hadrons.



Hadronic molecules



...

T. S. , T. Hyodo, D. Jido (2008), (2011);
T. S. and T. Hyodo, (2013).

- The uniqueness comes from the fact that hadronic molecules are composed of color-singlet hadrons themselves.

  - Actually the deuteron was proved to be a proton-neutron bound state by considering general wave equations (not QCD !).

--- Field renormalization const. Z in the weak binding: Weinberg (1965).

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(m_\pi^{-1}), \quad r_e = -\frac{Z}{1-Z} R + \mathcal{O}(m_\pi^{-1}), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

$$a = 5.419 \pm 0.007 \text{ fm}, \quad r_e = 1.7513 \pm 0.008 \text{ fm}$$

--> Consistent with $Z \approx 0$!

3. Compositeness

++ Identifying hadronic molecules ++

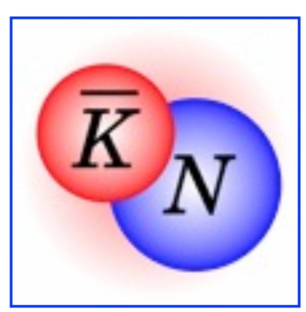
- The Weinberg's study indicates that:
  - Hadronic molecules may be able to be identified without relying directly upon QCD, since constituents are color singlet.
  - In the weak binding, Z can be determined model independently.
- In this context, the compositeness was recently introduced to observe the two-body components inside a resonance.

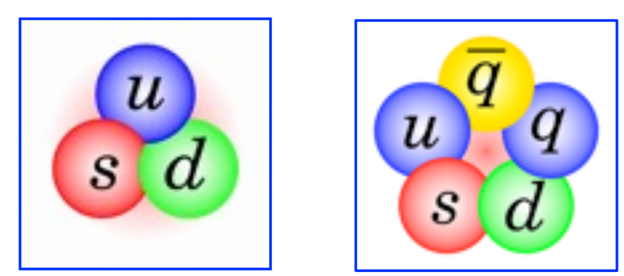
Hyodo, Jido, Hosaka (2012), Aceti-Oset (2012), Hyodo (2013), Nagahiro-Hosaka (2014), ...

See also T. S. , Hyodo and Jido arXiv:1411.2308.

  - Compositeness can be defined as the contribution of the two-body component to the normalization of the total wave function.

$$\langle \Lambda(1405) | \Lambda(1405) \rangle = X_{\bar{K}N} + X_{\pi\Sigma} + \cdots + Z = 1$$





...

3. Compositeness

++ Compositeness in experiments ++

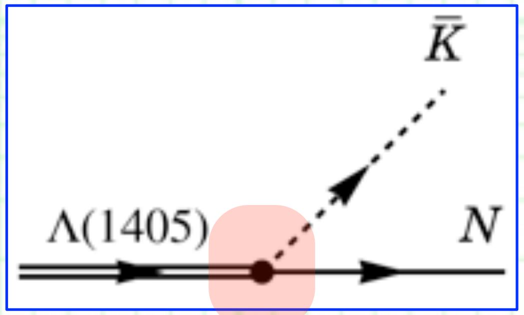
- We want to determine **compositeness in experiments** !

--- However, **compositeness is not observable** but **model-dependent parameter**, since they are calculated directly from wave functions. (cf. probability of s- and d-wave components in deuteron)

- In this study, we **employ the separable interaction model**, in which the wave function and compositeness are expressed as follows:

$$\tilde{\Psi}(\vec{q}) = \frac{g}{s_{\rm pole} - [\omega(\vec{q}) + \omega'(\vec{q})]^2}$$

$$X \equiv \int \mathcal{D}q \left[\tilde{\Psi}(\vec{q}) \right]^2 = -g^2 \left[\frac{dG}{ds} \right]_{s = s_{\rm pole}}$$



--- In this model we can evaluate compositeness from the pole position $s_{\rm pole}$ and the coupling constant g_i .

1. The pole position can be estimated **from PDG**: $\sqrt{s_{\rm pole}} = M - i\,\Gamma / 2$.

2. The coupling constant affects **reactions which are sensitive to it**.

--> **The radiative decay of $\Lambda(1405)$** for its $\bar{K}N$ compositeness.

--> **The $a_0(980)$-$f_0(980)$ mixing** for their $K\bar{K}$ compositeness.

3. Compositeness

++ The $\Lambda(1405)$ radiative decay ++

- There is an "experimental" value of the $\Lambda(1405)$ radiative decay:

  $\Gamma(\Lambda(1405) \to \Lambda\gamma) = 27 \pm 8$ keV, PDG; Burkhardt and Lowe, *Phys. Rev.* C44 (1991) 607.

  $\Gamma(\Lambda(1405) \to \Sigma^0\gamma) = 10 \pm 4$ keV or 23 ± 7 keV.

- There are also several theoretical studies on the radiative decay:

  Geng, Oset and Döring, *Eur. Phys. J.* A32 (2007) 201.

Table 3. The radiative decay widths of the $\Lambda(1405)$ predicted by different theoretical models, in units of keV. The values denoted by "UχPT" are the results obtained in the present study. The widths calculated for the low-energy pole and high-energy pole are separated by a comma.

Decay channel	UχPT	χQM [35]	BonnCQM [36]	NRQM	RCQM [39]
$\gamma\Lambda$	16.1, 64.8	168	912	143 [37], 200, 154 [38]	118
$\gamma\Sigma^0$	73.5, 33.5	103	233	91 [37], 72, 72 [38]	46

Decay channel	MIT bag [38]	Chiral bag [40]	Soliton [41]	Algebraic model [42]	Isobar fit [23]
$\gamma\Lambda$	60, 17	75	44,40	116.9	27 ± 8
$\gamma\Sigma^0$	18, 2.7	1.9	13,17	155.7	10 ± 4 or 23 ± 7

--- Structure of $\Lambda(1405)$ has been discussed in these models,
but the $\bar{K}N$ compositeness for $\Lambda(1405)$ has not been discussed.

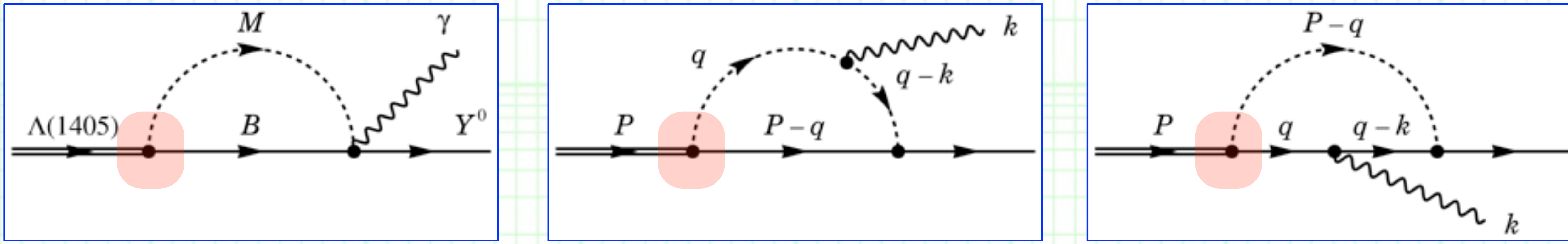
--> Discuss the $\bar{K}N$ compositeness from the $\Lambda(1405)$ radiative decay !

3. Compositeness

++ The Λ(1405) radiative decay ++

- We calculate **the radiative decay width from following diagrams**:

Geng, Oset and Döring, *Eur. Phys. J.* A32 (2007) 201.



--- Photon emission from meson-baryon components inside Λ(1405).

- **Λ(1405) pole position** from PDG values.

- **The coupling constant g_{KN}** is determined from the compositeness relation **as a function of X_{KN}**:

$$|X_{\bar{K}N}| = |g_{\bar{K}N}|^2 \left| \frac{dG_{K^-p}}{d\sqrt{s}} + \frac{dG_{\bar{K}^0 n}}{d\sqrt{s}} \right|_{\sqrt{s}=W_{\text{pole}}}$$

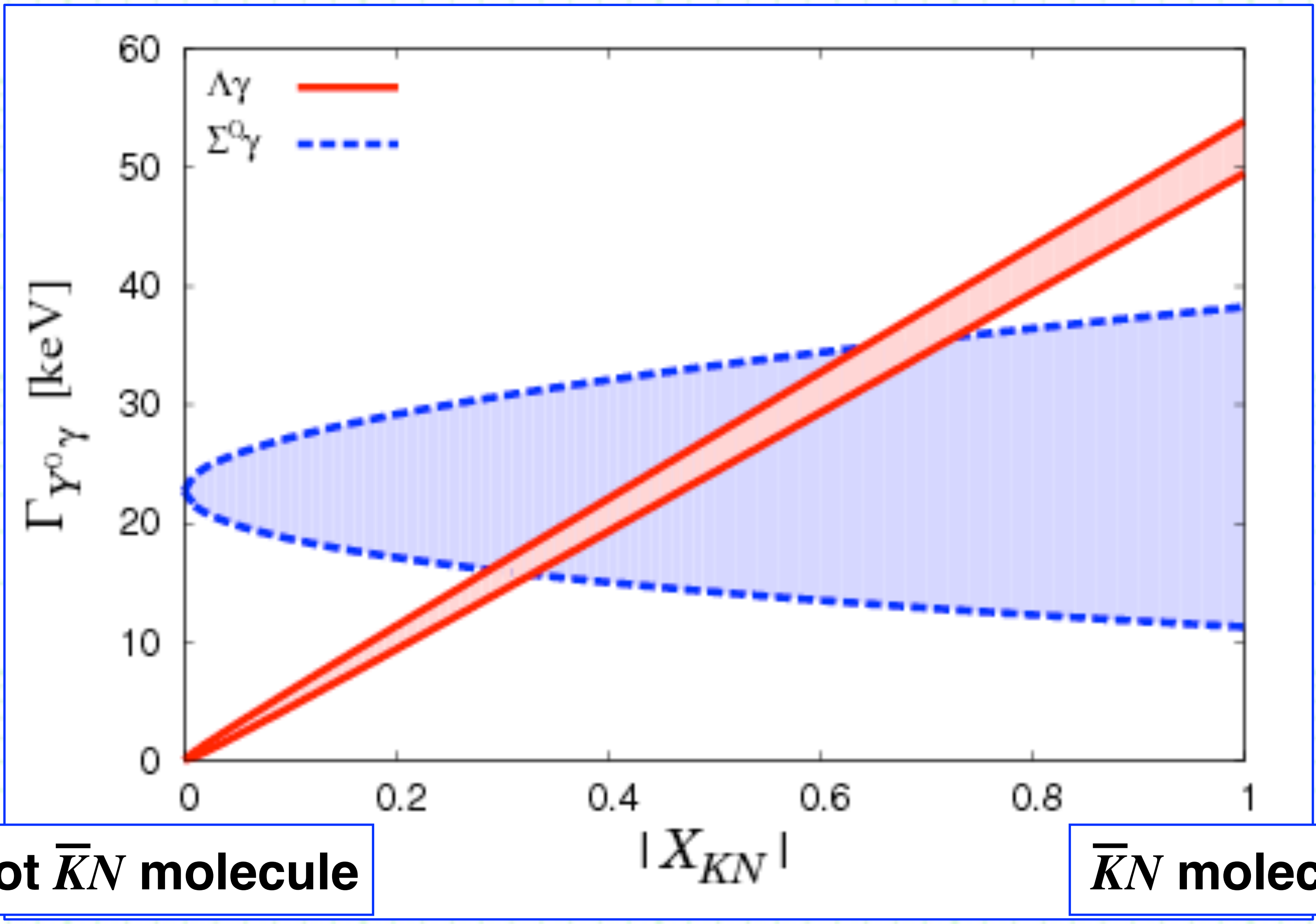
- The coupling constant $g_{\pi\Sigma}$ from Λ(1405) --> πΣ decay width, and we do not fix the relative phase between g_{KN} and $g_{\pi\Sigma}$ but calculate both maximally constructive / destructive interferences.

3. Compositeness

++ The $\Lambda(1405)$ radiative decay ++

- We obtain allowed region of the $\Lambda(1405)$ radiative decay width with respect to the absolute value of the $\bar{K}N$ compositeness $|X_{\bar{K}N}|$.

T. S. and S. Kumano, *Phys. Rev.* C89 (2014) 025202.



Not $\bar{K}N$ molecule

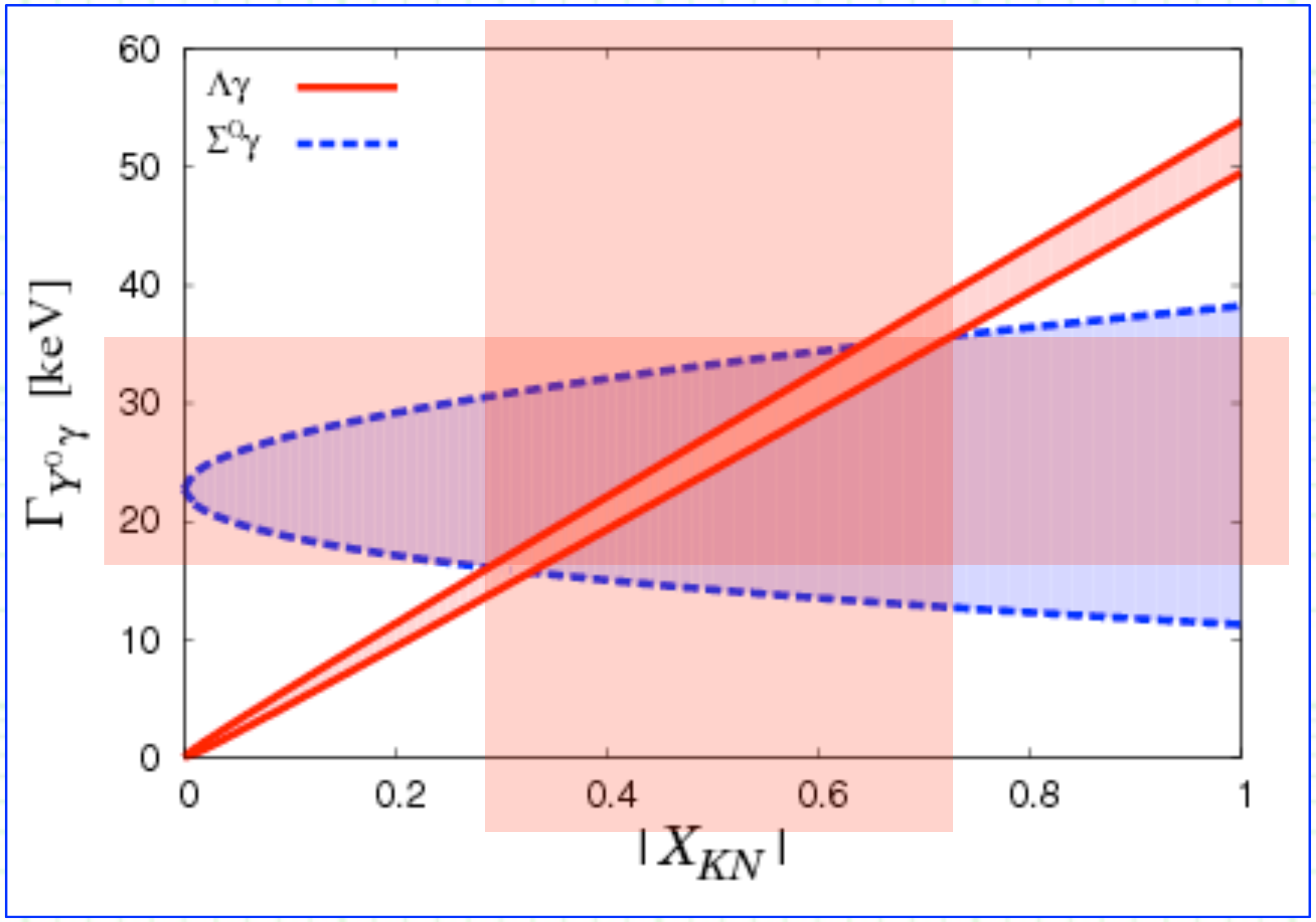
$\bar{K}N$ molecule

- Due to the large cancellation of the $\pi^{\pm}\Sigma^{\mp}$ contributions, the $\Lambda\gamma$ decay mode is suited to observe the $\bar{K}N$ component inside $\Lambda(1405)$.

3. Compositeness

++ The Λ(1405) radiative decay ++

- There is an "experimental" value of the Λ(1405) radiative decay:
  $\Gamma(\Lambda(1405) \to \Lambda\gamma) = 27 \pm 8$ keV, PDG; Burkhardt and Lowe, *Phys. Rev.* C44 (1991) 607.



- Especially, from $\Gamma(\Lambda(1405) \to \Lambda\gamma) = 27 \pm 8$ keV: $|X_{KN}| = 0.5 \pm 0.2$.

--- $\bar{K}N$ seems to be the largest component inside Λ(1405) !

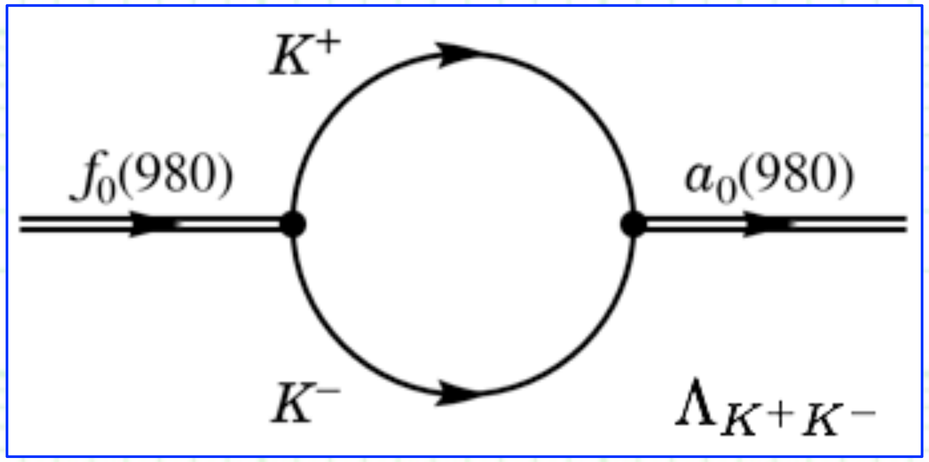
<-- However, the "experimental" value depends on a model analysis.

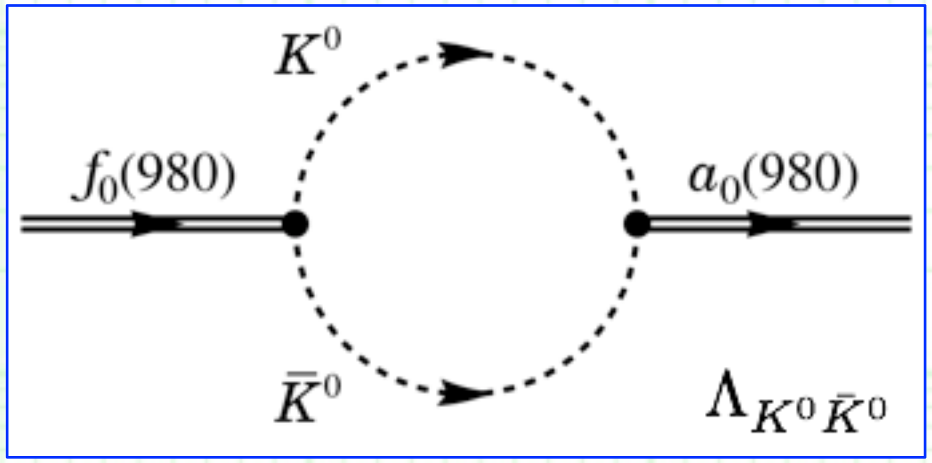
3. Compositeness

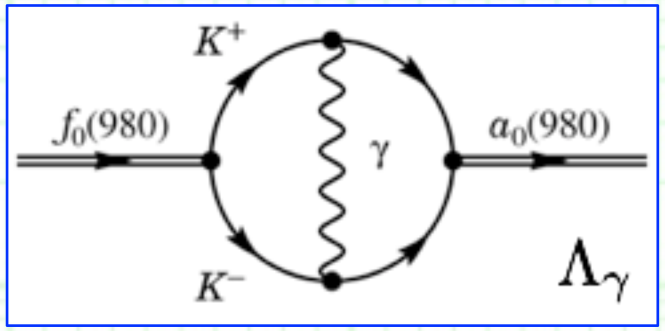
++ The $a_0(980)$-$f_0(980)$ mixing ++

- **The $a_0(980)$-$f_0(980)$ mixing** was predicted as a phenomenon caused by **the threshold difference between charged and neutral $K\bar{K}$ loops**.

Achasov, Devyanin and Shestakov (1979).







--- Namely, in the energy between the K^+K^- and $K^0\bar{K}^0$ thresholds (987 ~ 995 MeV) **the mixing effect is unusually enhanced**:

$$\Lambda_{K^+K^-} + \Lambda_{K^0\bar{K}^0} = \mathcal{O}\left(\sqrt{\frac{m_{K^0}^2 - m_{K^+}^2}{m_{K^0}^2 + m_{K^+}^2}}\right)$$

<--> Natural size: $\mathcal{O}[(m_{K^0}^2 - m_{K^+}^2)/(m_{K^0}^2 + m_{K^+}^2)]$ [cf. ϱ(770)-ω(782) mixing]

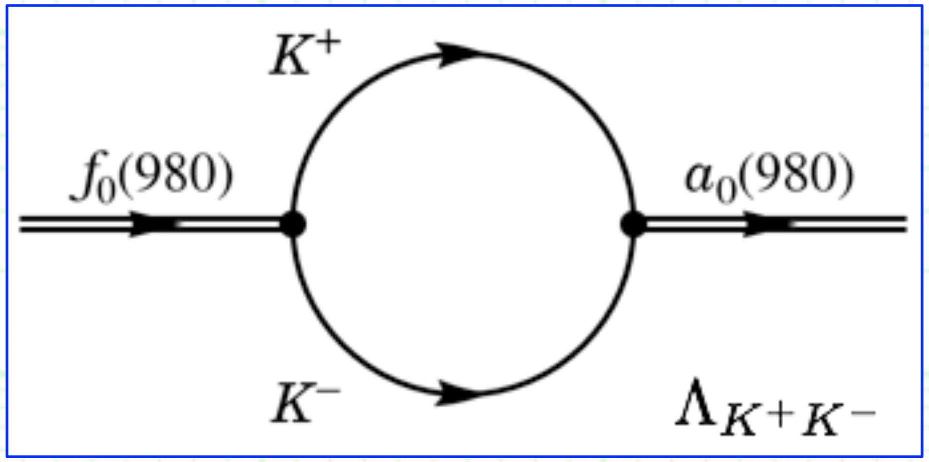
- **The $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants** are the model parameters of the mixing amplitude.

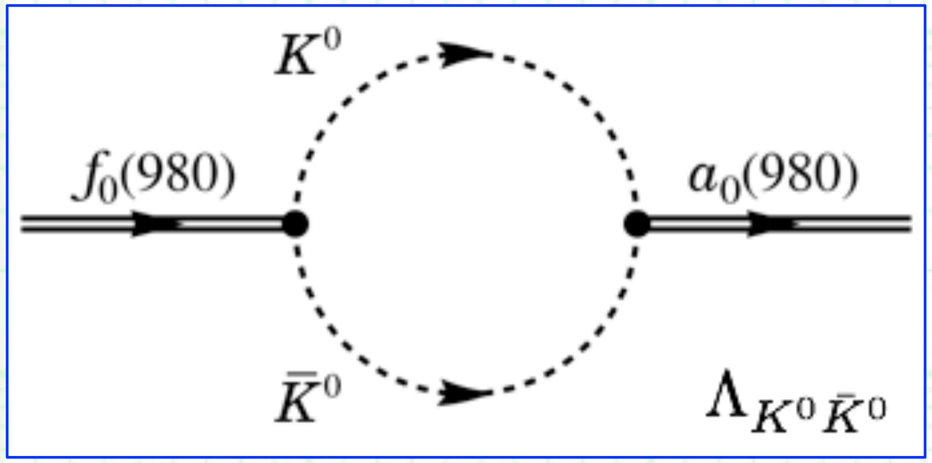
3. Compositeness

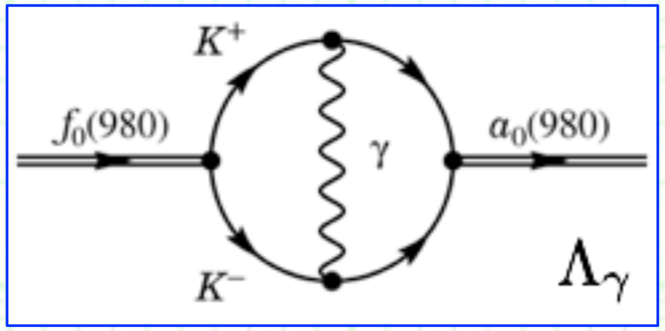
++ The $a_0(980)$-$f_0(980)$ mixing ++

- **The $a_0(980)$-$f_0(980)$ mixing was predicted as a phenomenon caused by the threshold difference between charged and neutral $K\bar{K}$ loops.**

Achasov, Devyanin and Shestakov (1979).



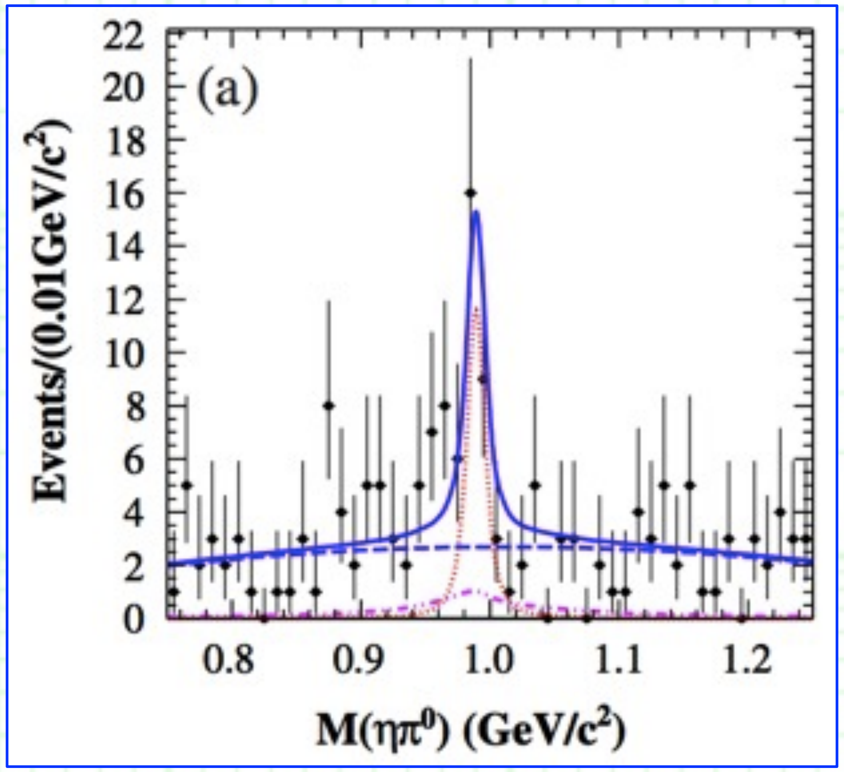




- **Recently the mixing was measured in Exp. by using the J/ψ decay, and its intensity ξ_{fa} is**

Ablikim *et al.* [BES III] (2011).

$$\xi_{fa} \equiv \frac{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \pi^0 \eta)}{\mathrm{Br}(J/\psi \to \phi f_0(980) \to \phi \pi\pi)}$$
$$= 0.60 \pm 0.20(\mathrm{stat}) \pm 0.12(\mathrm{sys}) \pm 0.26(\mathrm{para})\%,$$
$$\xi_{fa}|_{\text{upper limit}} = 1.1\% \quad (90\%\ \mathrm{C.L.})$$

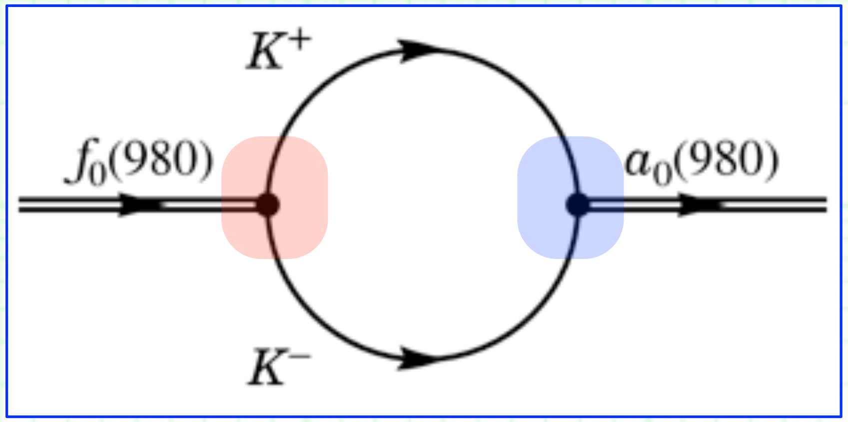


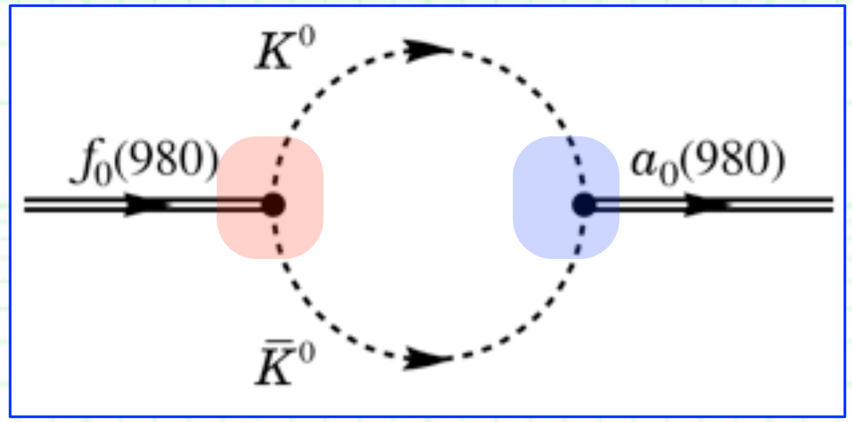
--> Investigate their structure !

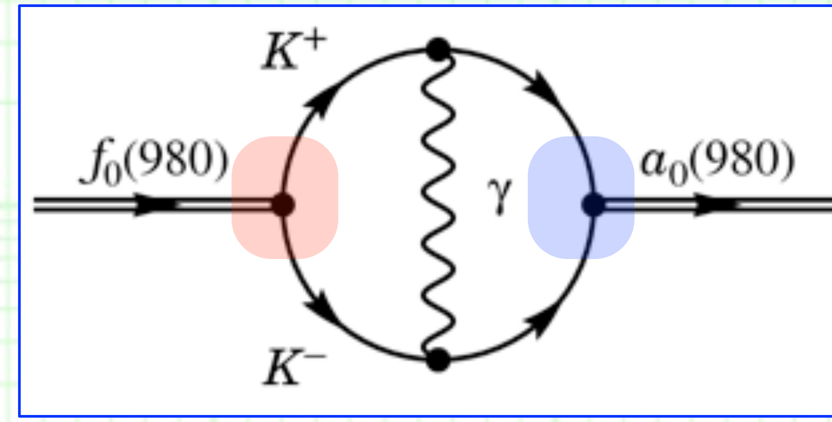
3. Compositeness

++ The $a_0(980)$-$f_0(980)$ mixing ++

- We calculate **the $a_0(980)$-$f_0(980)$ mixing from following diagrams**:







and **the mixing intensity** with

$$\xi_{fa} \equiv \frac{\Gamma(X \to Y f_0(980) \to Y a_0^0(980) \to Y \pi^0 \eta)}{\Gamma(X \to Y f_0(980) \to Y \pi\pi)}$$

- The Flatte parameterization are used and the parameters are:

$$M_a = 990 \text{ MeV}, \quad \bar{g}_{a\pi\eta} = 3.0\text{GeV}, \quad M_f = 970 \text{ MeV}, \quad \bar{g}_{f\pi\pi} = 2.4 \text{ GeV}$$

Flatte (1976).

---Rough average of Exp. params.

- **The coupling constants $a_0(980)$-$K\bar{K}$ and $f_0(980)$-$K\bar{K}$.**

Compositeness

$$X_A = -g_A^2 \left[\frac{dG_{K^+K^-}}{ds} + \frac{dG_{K^0\bar{K}^0}}{ds} \right]_{s=s_A}, \quad A = a_0(980),\ f_0(980)$$

Mixing intensity

Constrain from Exp. !

4. Constraint on their structure

++ Favored | X_a |-| X_f | area ++

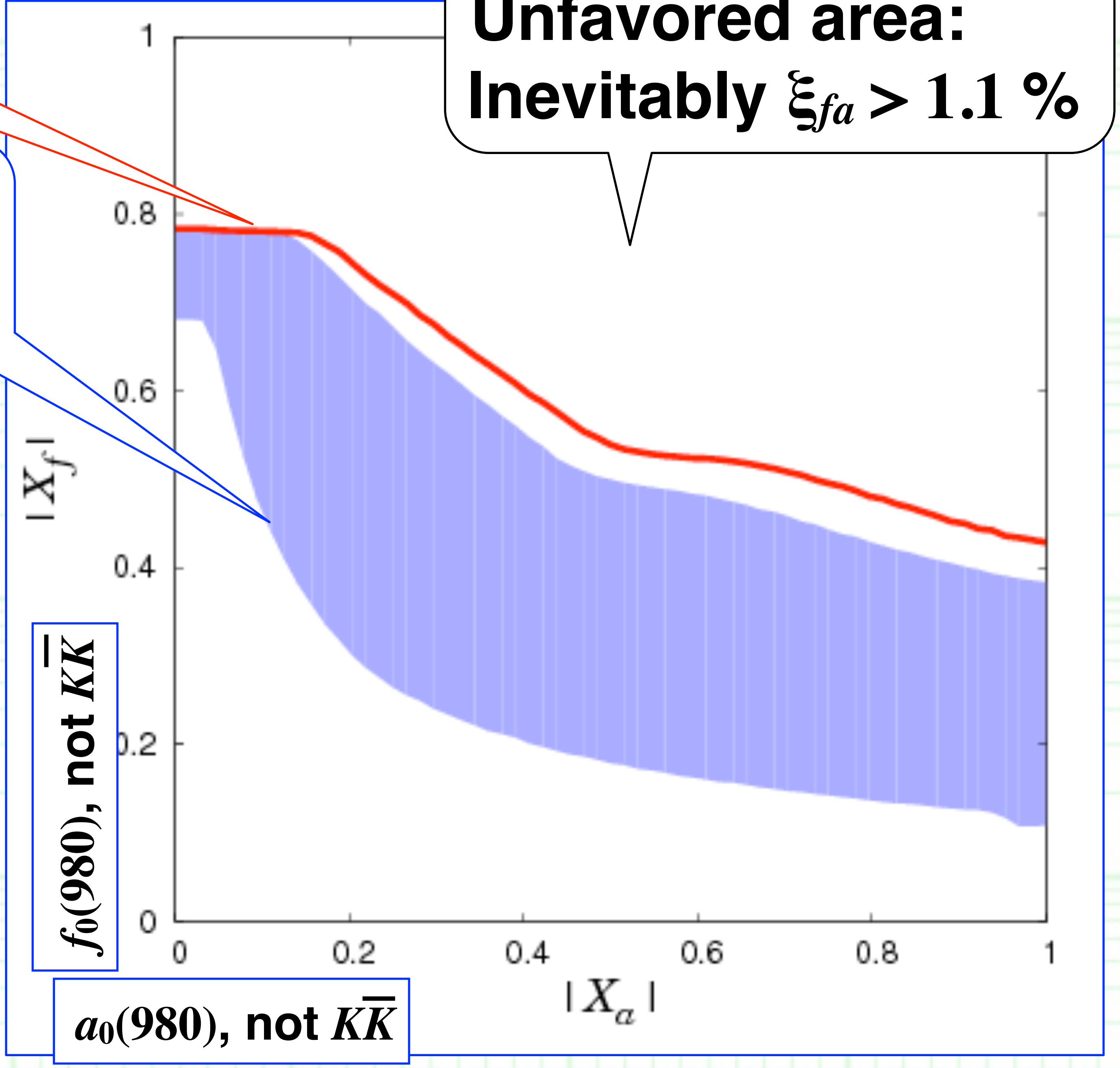
Border line for Exp.:

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

Favored | X_a |-| X_f | area from Exp.:

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})}\%$$

Unfavored area:
Inevitably $\xi_{fa} > 1.1$ %



4. Constraint on their structure

++ Favored | X_a |-| X_f | area ++

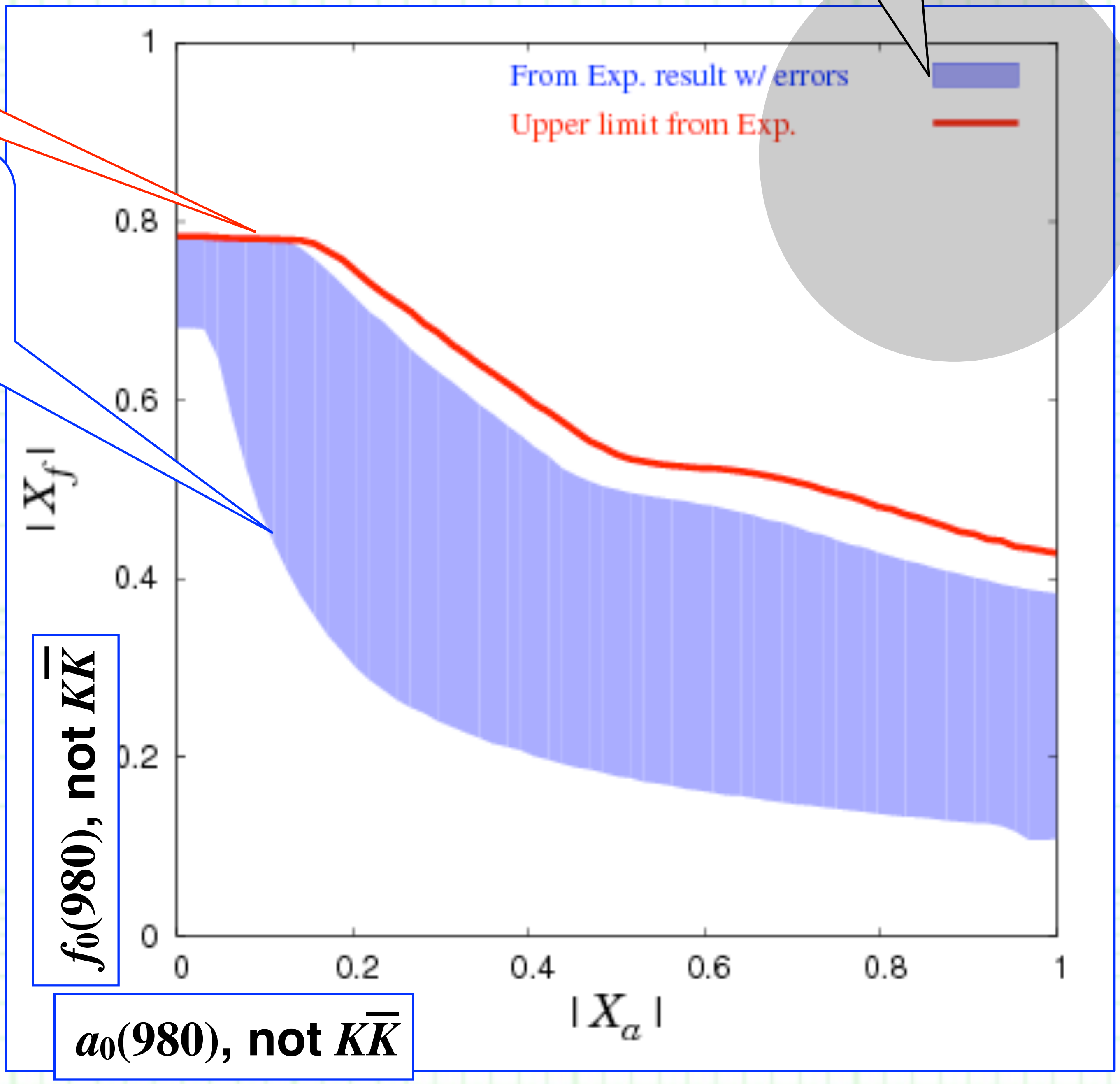
Border line for Exp.:

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

"Both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules" is **not favored**.

Favored | X_a |-| X_f | area from Exp.:

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})}\%$$



- **| X_a | ≈ | X_f | ≈ 1 is not favored.**

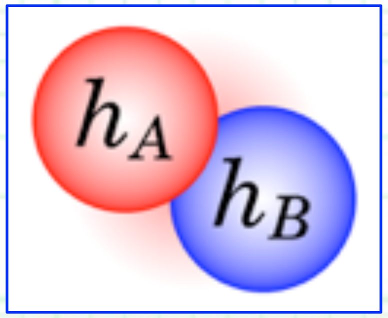
--> We find that "both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules" is questionable.

- "One of them has large $K\bar{K}$ component" is not ruled out.

--- Especially | X_f | $\gtrsim 0.3$ for every | X_a |. --> Not small $K\bar{K}$ in f_0 ?

3. Compositeness

++ Summary of compositeness ++



- **Hadronic molecules are unique because they are composed of color-singlet hadrons themselves.**

--> Various quantitative and qualitative differences.

  - **Two-body wave functions and compositeness.**

- **The two-body wave function for a general separable interaction:**

$$\tilde{\Psi}(\vec{q}) = \frac{g}{s_{\text{pole}} - [\omega(\vec{q}) + \omega'(\vec{q})]^2}$$

Compositeness

$$X \equiv \int \mathcal{D}q \left[\tilde{\Psi}(\vec{q})\right]^2 = -g^2 \left[\frac{dG}{ds}\right]_{s=s_{\text{pole}}}$$

(but not observable = a model dependent quantity, in general).

- **The $\Lambda(1405)$ radiative decay width contains information on its $\bar{K}N$ compositeness via the $\Lambda(1405)$-$\bar{K}N$ coupling constant.**

--> From "Exp." data, $\bar{K}N$ seems to be the largest component.

- **The $a_0(980)$-$f_0(980)$ mixing intensity can constrain their $K\bar{K}$ compositeness via the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants.**

--> From Exp. data, "both are $K\bar{K}$ molecules" is questionable.

4. Summary

4. Summary

- Constituent quark models can <u>support</u> the exotic nature of exotic hadrons = not qqq nor $q\bar{q}$.
  - Mass, width, couplings, etc. of exotic hadrons <u>do not match the predictions from constituent quark models</u>.

--- However, constituent quark models (or, in general, any effective models) cannot provide <u>undoubted evidences</u> of the exotic nature.

--> We need some <u>approaches which do not rely on effective models of QCD</u> to identify the exotic hadrons.

- The scaling law in hard exclusive process can <u>"count" number of constituents inside hadrons</u>.

--> $\Lambda(1405)$, ... in hard exclusive productions.

- Compositeness from general wave equations for two-body systems can <u>identify hadronic molecules</u> (but not observable = a model dependent quantity, in general).

<-- $\Lambda(1405)$ radiative decay width, $a_0(980)$-$f_0(980)$ mixing intensity.

Thank you very much for your kind attention !

Appendix

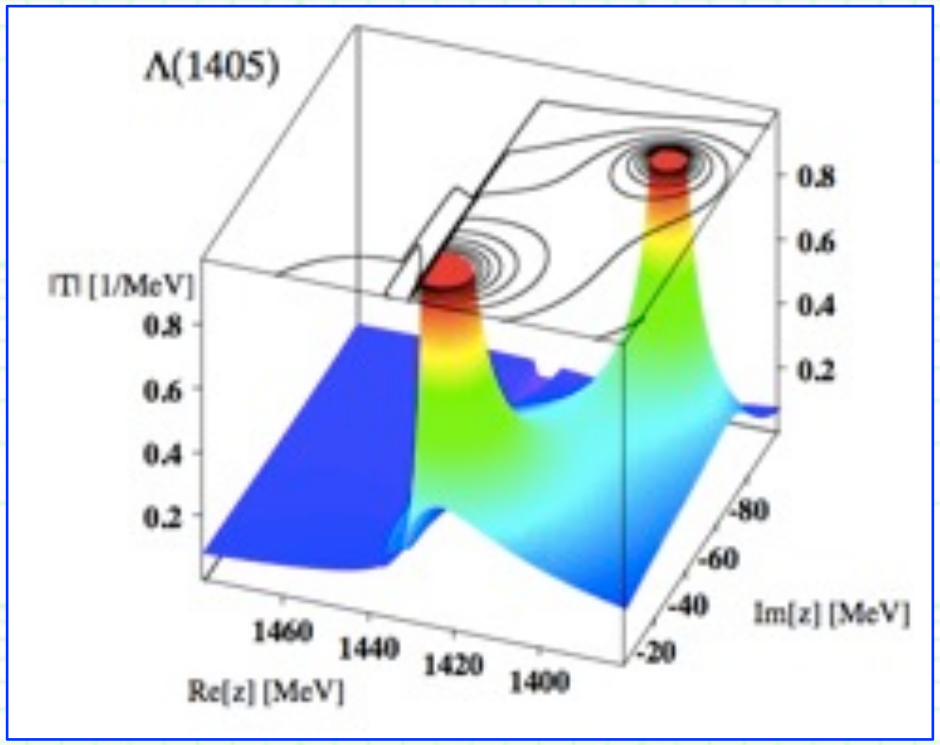
Appendix

++ Model calculation ++

- **Compositeness X and elementariness Z for hadronic resonances in the chiral unitary approach.**

T. S. , T. Hyodo and D. Jido, arXiv:1410.xxxx.

- **Λ(1405) (two poles!).**

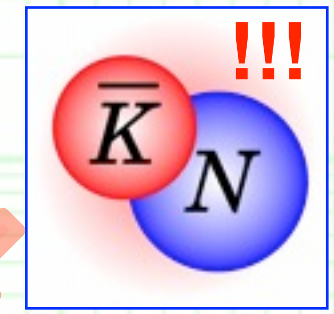


Hyodo and Jido (2012).

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s=s_{\text{pole}}}$$

$$Z = -\sum_{i,j} g_i g_j \left[G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{\text{pole}}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$



	$\Lambda(1405)$, higher pole	$\Lambda(1405)$, lower pole
$\sqrt{s_{\text{pole}}}$	$1424 - 26i$ MeV	$1381 - 81i$ MeV
$X_{\bar{K}N}$	$1.14 + 0.01i$	$-0.39 - 0.07i$
$X_{\pi\Sigma}$	$-0.19 - 0.22i$	$0.66 + 0.52i$
$X_{\eta\Lambda}$	$0.13 + 0.02i$	$-0.04 + 0.01i$
$X_{K\Xi}$	$0.00 + 0.00i$	$-0.00 + 0.00i$
Z	$-0.08 + 0.19i$	$0.77 - 0.46i$

--- **Large $\bar{K}N$ component for (higher) Λ(1405), since X_{KN} is almost unity (not probability but amount).**

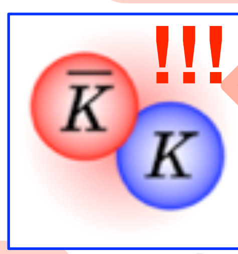
Appendix

++ Model calculation ++

- **Compositeness X and elementariness Z for scalar mesons in the chiral unitary approach. --> Complex values for resonances !**

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s=s_{\rm pole}} \qquad Z = -\sum_{i,j} g_i g_j \left[G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{\rm pole}} \qquad \langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

	$f_0(500)$ or σ	$f_0(980)$	$a_0(980)$	$K_0^*(800)$ or κ
$\sqrt{s_{\rm pole}}$	$471 - 181i$ MeV	$987 - 18i$ MeV	$979 - 53i$ MeV	$750 - 227i$ MeV
$X_{\pi\pi}$	$-0.16 + 0.35i$	$0.01 + 0.01i$	—	—
$X_{K\bar{K}}$	$-0.01 - 0.01i$	$0.74 - 0.11i$	$0.38 - 0.29i$	—
$X_{\pi\eta}$	—	—	$-0.06 + 0.10i$	—
$X_{\pi K}$	—	—	—	$0.32 + 0.36i$
$X_{\eta K}$	—	—	—	$-0.01 + 0.00i$
Z	$1.17 - 0.34i$	$0.25 + 0.10i$	$0.68 + 0.18i$	$0.70 - 0.36i$



!!!

- **We interpret complex compositeness / elementariness on the basis of the similarity to the wave function of the bound state:**

1. Re(X) ~ 1, Im(X) ~ | Z | << 1 <=> Dominated by a molecular state.
2. | X_i | << 1 <=> i-th channel component is very small.

Appendix

++ Model calculation ++

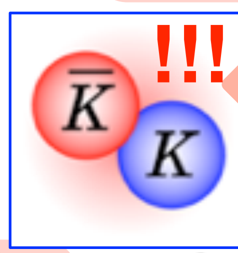
- **Compositeness X and elementariness Z for scalar mesons in the chiral unitary approach. --> Complex values for resonances !**

$$X_i = -g_i^2 \left[\frac{dG_i}{ds} \right]_{s=s_{\rm pole}}$$

$$Z = -\sum_{i,j} g_i g_j \left[G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{\rm pole}}$$

$$\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1$$

	$f_0(500)$ or σ	$f_0(980)$	$a_0(980)$	$K_0^*(800)$ or κ
$\sqrt{s_{\rm pole}}$	$471 - 181i$ MeV	$987 - 18i$ MeV	$979 - 53i$ MeV	$750 - 227i$ MeV
$X_{\pi\pi}$	$-0.16 + 0.35i$	$0.01 + 0.01i$	—	—
$X_{K\bar{K}}$	$-0.01 - 0.01i$	$0.74 - 0.11i$	$0.38 - 0.29i$	—
$X_{\pi\eta}$	—	—	$-0.06 + 0.10i$	—
$X_{\pi K}$	—	—	—	$0.32 + 0.36i$
$X_{\eta K}$	—	—	—	$-0.01 + 0.00i$
Z	$1.17 - 0.34i$	$0.25 + 0.10i$	$0.68 + 0.18i$	$0.70 - 0.36i$



!!!

- **We interpret complex compositeness / elementariness on the basis of the similarity of the wave function of the bound state:**

--> $f_0(980)$ in this model is dominated by the $K\bar{K}$ component.

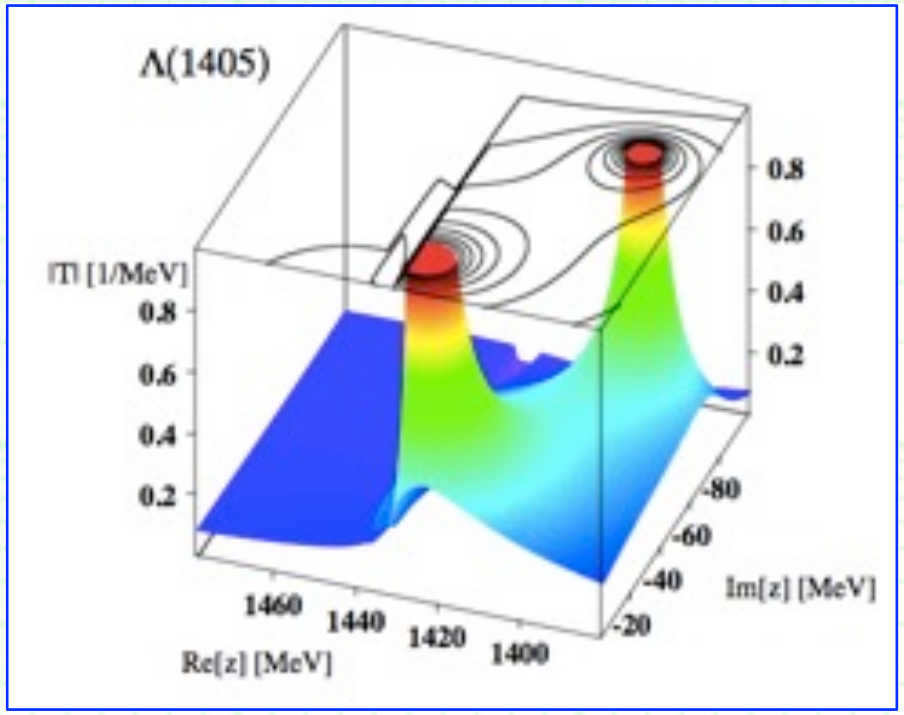
Appendix

++ Pole position dependence ++

- **The Λ(1405) pole position is not well-determined in Exp.**

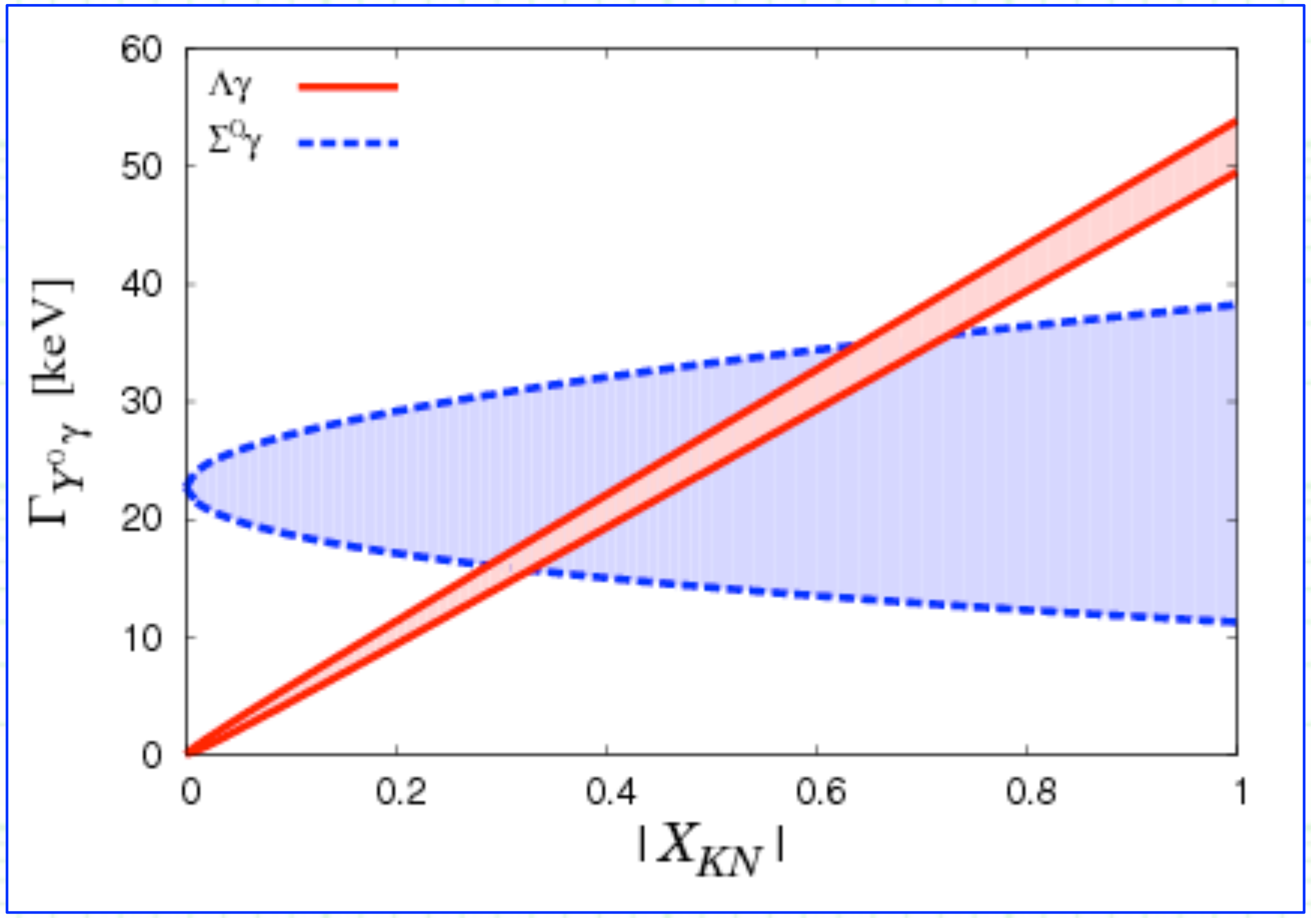
--- Two poles ? 1420 MeV instead of nominal 1405 MeV ?

Braun (1977); D. Jido, E. Oset and T. S. (2009).



Hyodo and Jido, *Prog. Part. Nucl. Phys.* 67 (2012) 55.

$$|X_{\bar{K}N}| = |g_{\bar{K}N}|^2 \left| \frac{dG_{K^-p}}{d\sqrt{s}} + \frac{dG_{\bar{K}^0 n}}{d\sqrt{s}} \right|_{\sqrt{s}=W_{\mathrm{pole}}}$$



- **How the relation between $\Gamma_{\Lambda\gamma}$ $\Gamma_{\Sigma^0\gamma}$ and $|X_{KN}|$ is changed if the pole position is shifted ?**

Pole position from PDG.

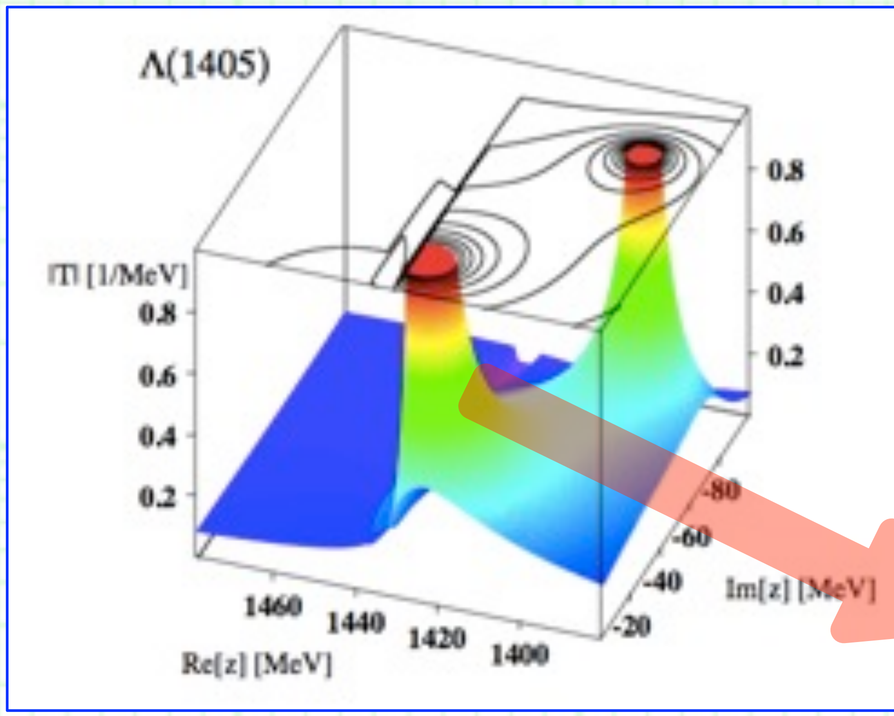
Appendix

++ Pole position dependence ++

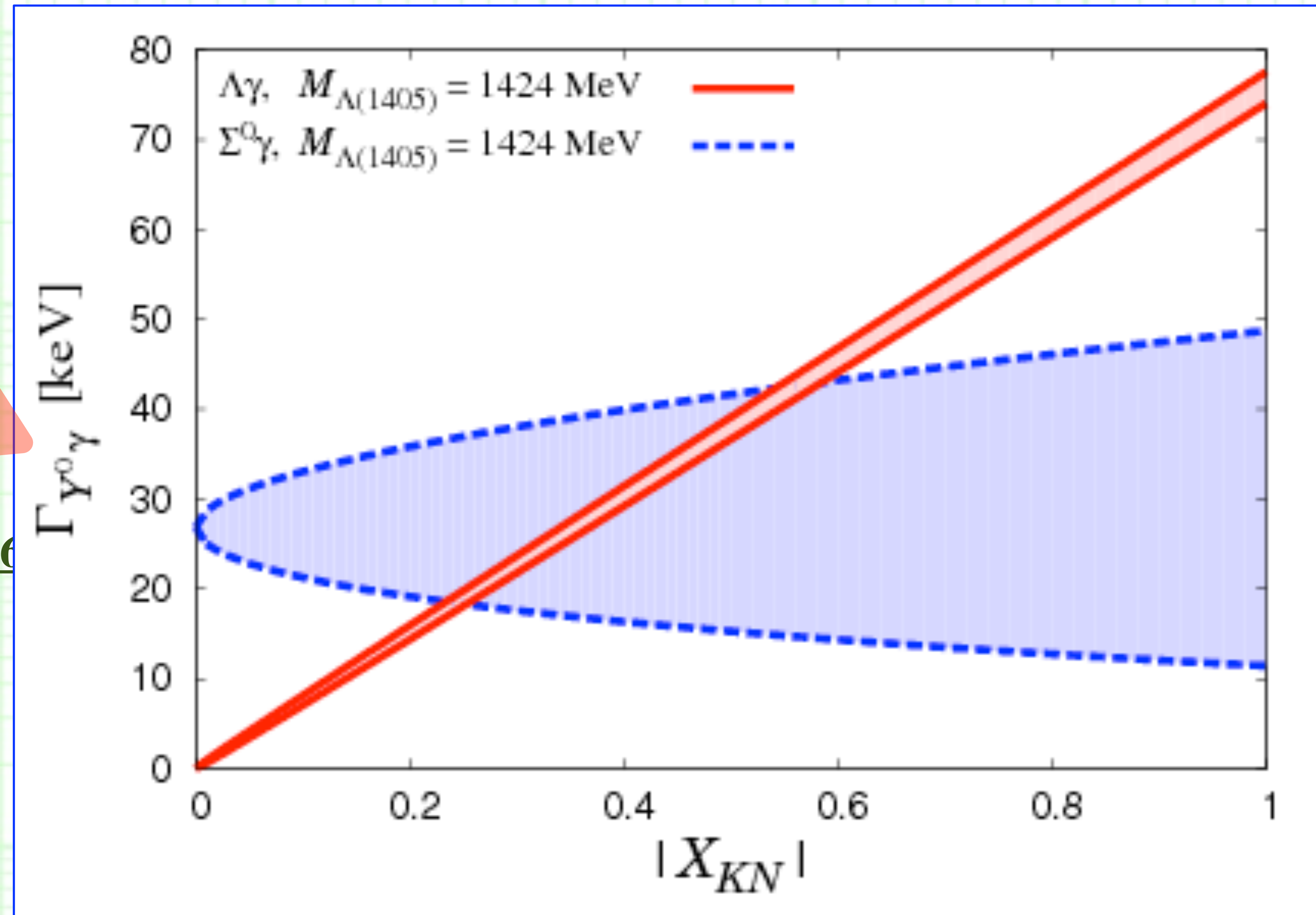
- The $\Lambda(1405)$ pole position is **not well-determined** in Exp.

--- Two poles ? 1420 MeV instead of nominal 1405 MeV ?

Braun (1977); D. Jido, E. Oset and T. S. (2009).



Hyodo and Jido, *Prog. Part. Nucl. Phys.* 6



$$|X_{\bar{K}N}| = |g_{\bar{K}N}|^2 \left| \frac{dG_{K^- p}}{d\sqrt{s}} + \frac{dG_{\bar{K}^0 n}}{d\sqrt{s}} \right|_{\sqrt{s}=W_{\mathrm{pole}}}$$

--- Will be seen in, *e.g.*,

$K^{-} p^*$ --> $\Lambda(1405)$ production.

Higher $\Lambda(1405)$ pole position.

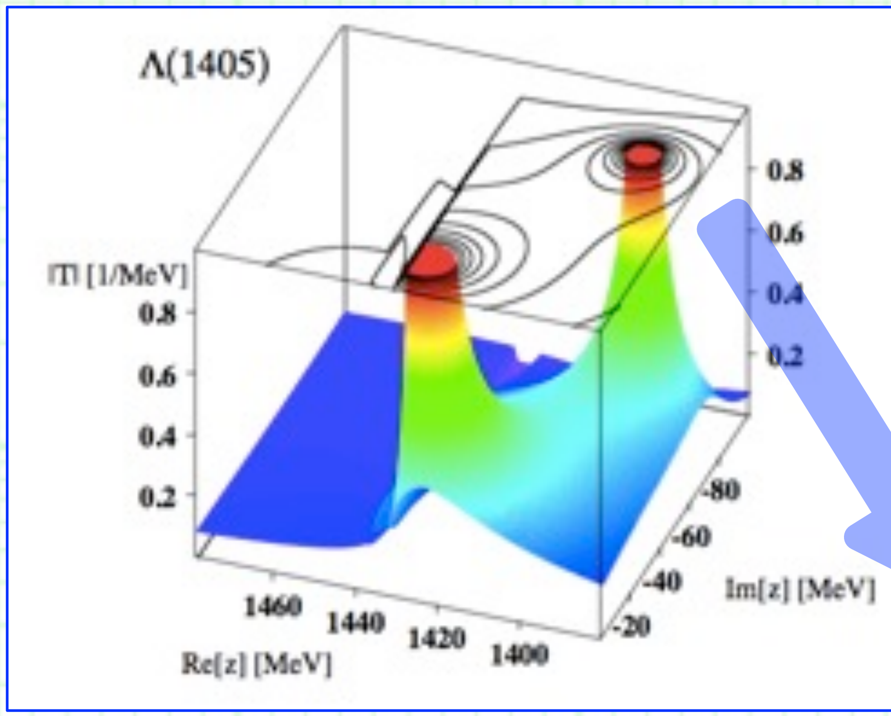
Appendix

++ Pole position dependence ++

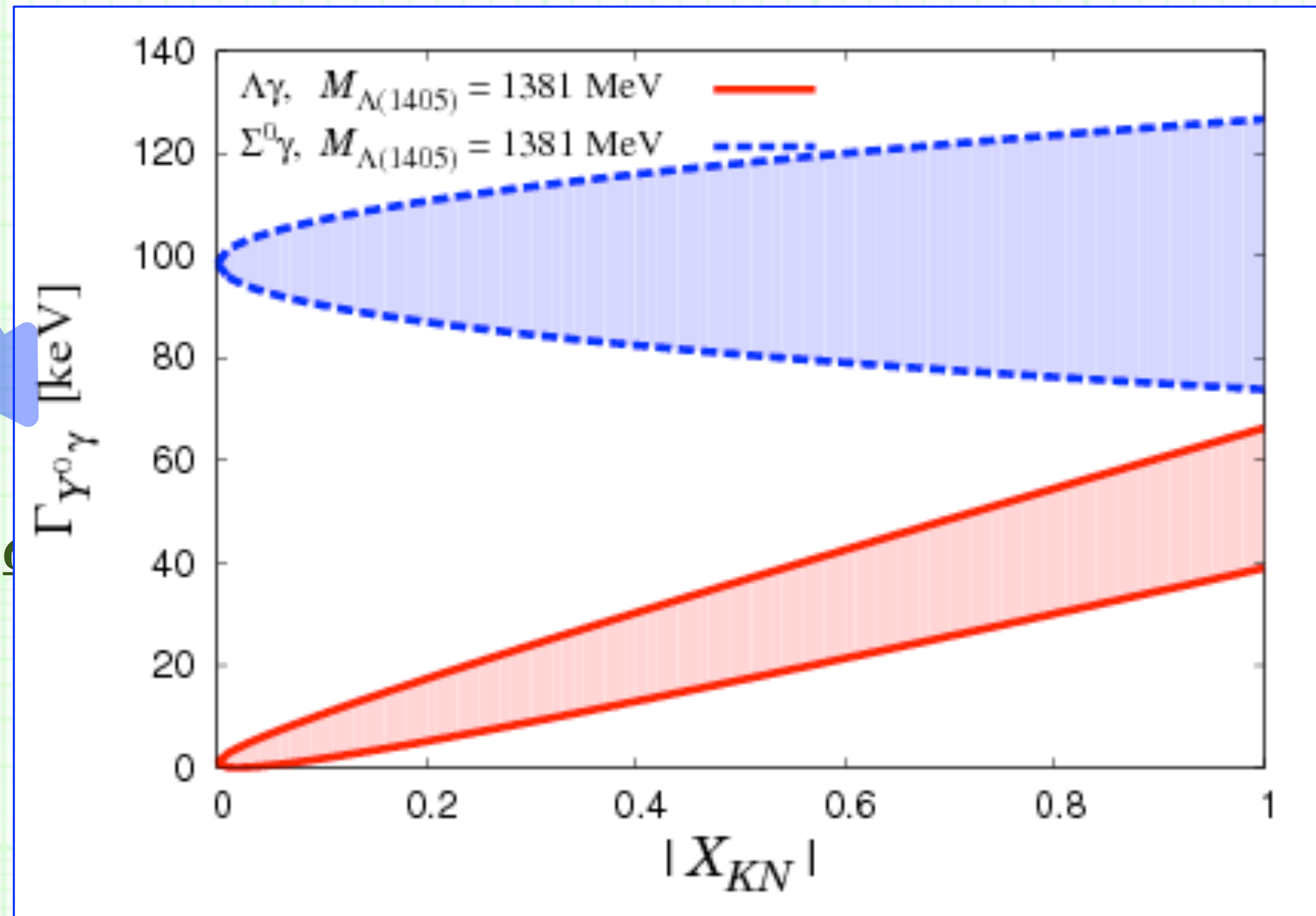
- **The Λ(1405) pole position is not well-determined in Exp.**

--- Two poles ? 1420 MeV instead of nominal 1405 MeV ?

Braun (1977); D. Jido, E. Oset and T. S. (2009).



Hyodo and Jido, *Prog. Part. Nucl. Phys.*



$$|X_{\bar{K}N}| = |g_{\bar{K}N}|^2 \left| \frac{dG_{K^-p}}{d\sqrt{s}} + \frac{dG_{\bar{K}^0 n}}{d\sqrt{s}} \right|_{\sqrt{s}=W_{\rm pole}}$$

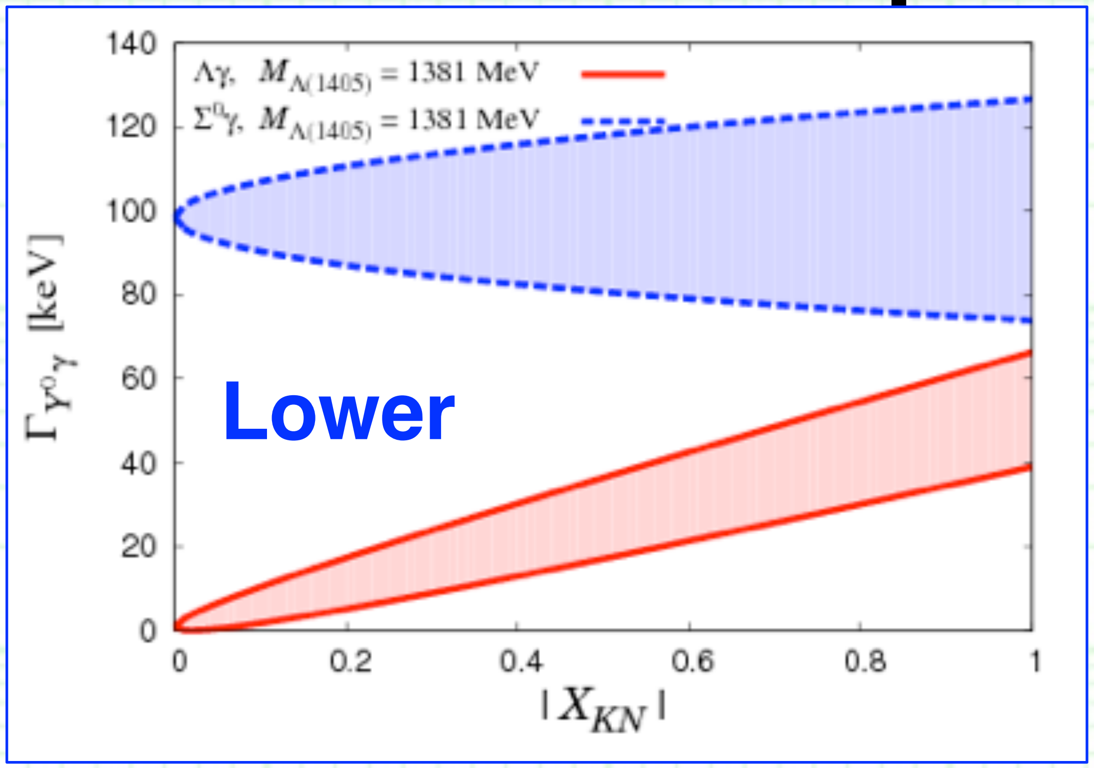
--- Will be seen in, *e.g.*,

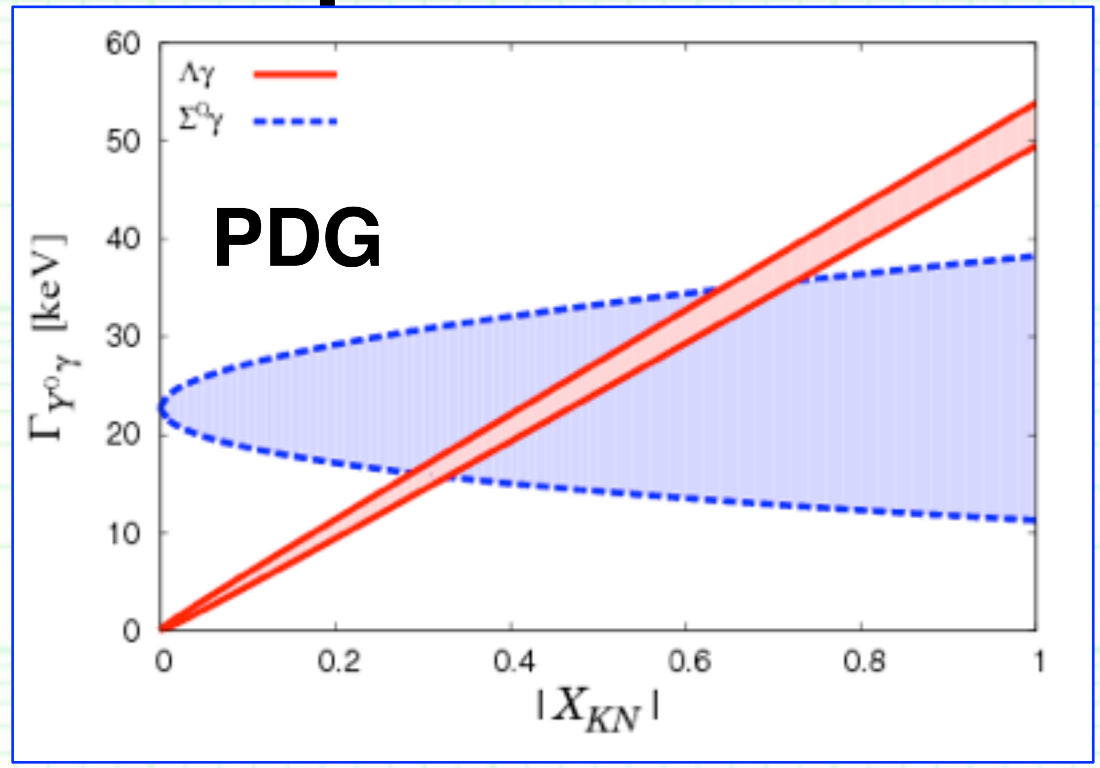
π^- p --> K^0 Λ(1405) production.

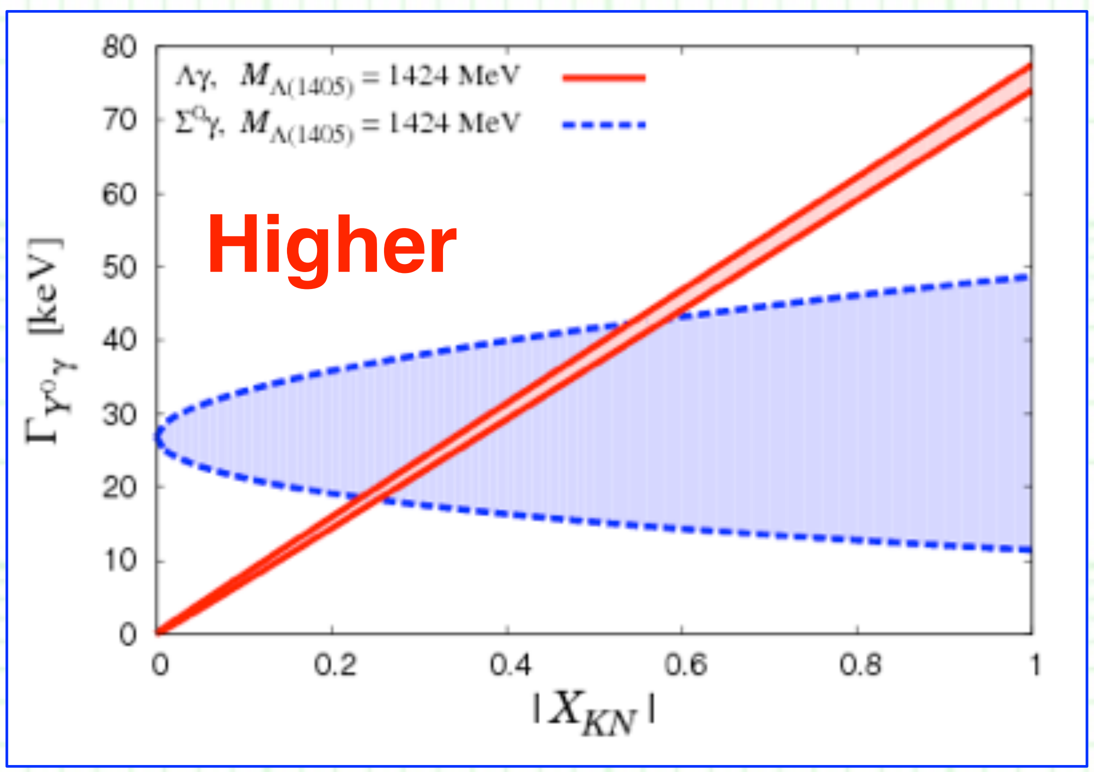
Lower Λ(1405) pole position.

Appendix

++ Pole position dependence ++







- **Pole position dependence is not strong for the Λγ decay mode.**

--- Especially the result of $|X_{KN}|$ from the empirical value of the Λγ decay mode is almost same.

- **Different branching ratio Λγ / $\Sigma^0\gamma$.**

--> Could be evidence of two poles.

Appendix

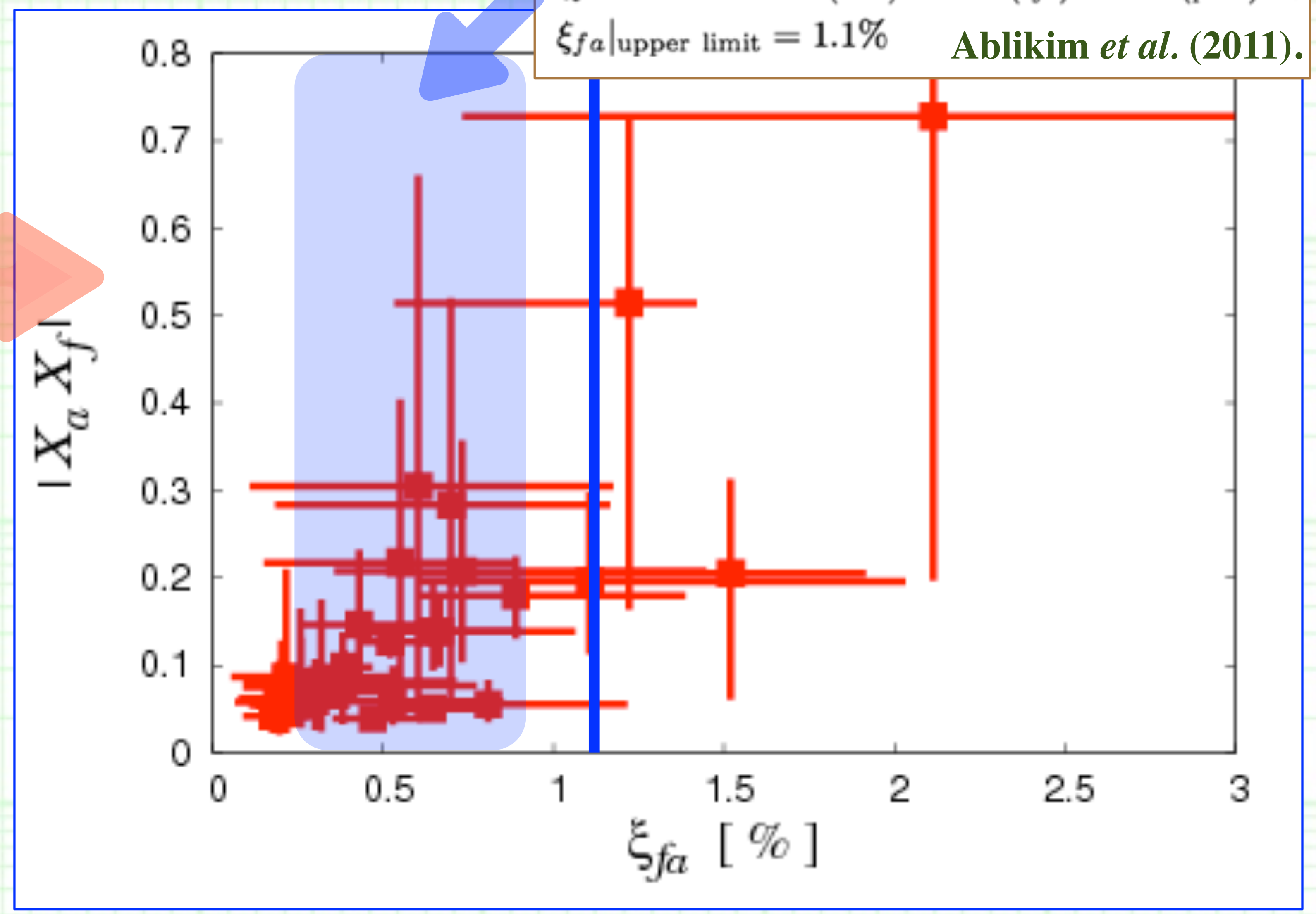
++ Test with Flatte parameters ++

- Now we examine **the relation between the $a_0(980)$-$f_0(980)$ mixing intensity ξ_{fa} and the product of the two $K\bar{K}$ compositeness $|X_a X_f|$**, with the Flatte parameters from Exp. fittings.

Collaboration	M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
$a_0(980)$			
CLEO (2011)	998	3.97 ± 0.77	4.25
KLOE (2009)	982.5	2.84 ± 0.41	2.46
CB (2008)	987.4	2.94 ± 0.12	2.87
SND (2000)	995	$5.93\,^{+10.54}_{-2.39}$	3.11
E852 (1999)	1001	2.36 ± 0.13	2.47

Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
$f_0(980)$			
CDF (2011)	989.6	$4.02\,^{+1.01}_{-1.37}$	2.65
KLOE (2006)	977.3	2.45 ± 0.17	1.21
Belle (2006)	950	$4.07\,^{+0.76}_{-0.95}$	2.28
BES (2005)	965	$5.80\,^{+0.22}_{-0.23}$	2.83
FOCUS (2005)	957	$3.39\,^{+0.62}_{-0.76}$	2.15
SND (2000)	969.8	$7.88\,^{+1.09}_{-0.86}$	3.19

$\xi_{fa} = 0.60 \pm 0.20_{(\mathrm{stat})} \pm 0.12_{(\mathrm{sys})} \pm 0.26_{(\mathrm{para})}\%$,
$\xi_{fa}|_{\mathrm{upper\ limit}} = 1.1\%$ Ablikim *et al.* (2011).



- There is not a clear proportional connection, but there is actually a tendency.

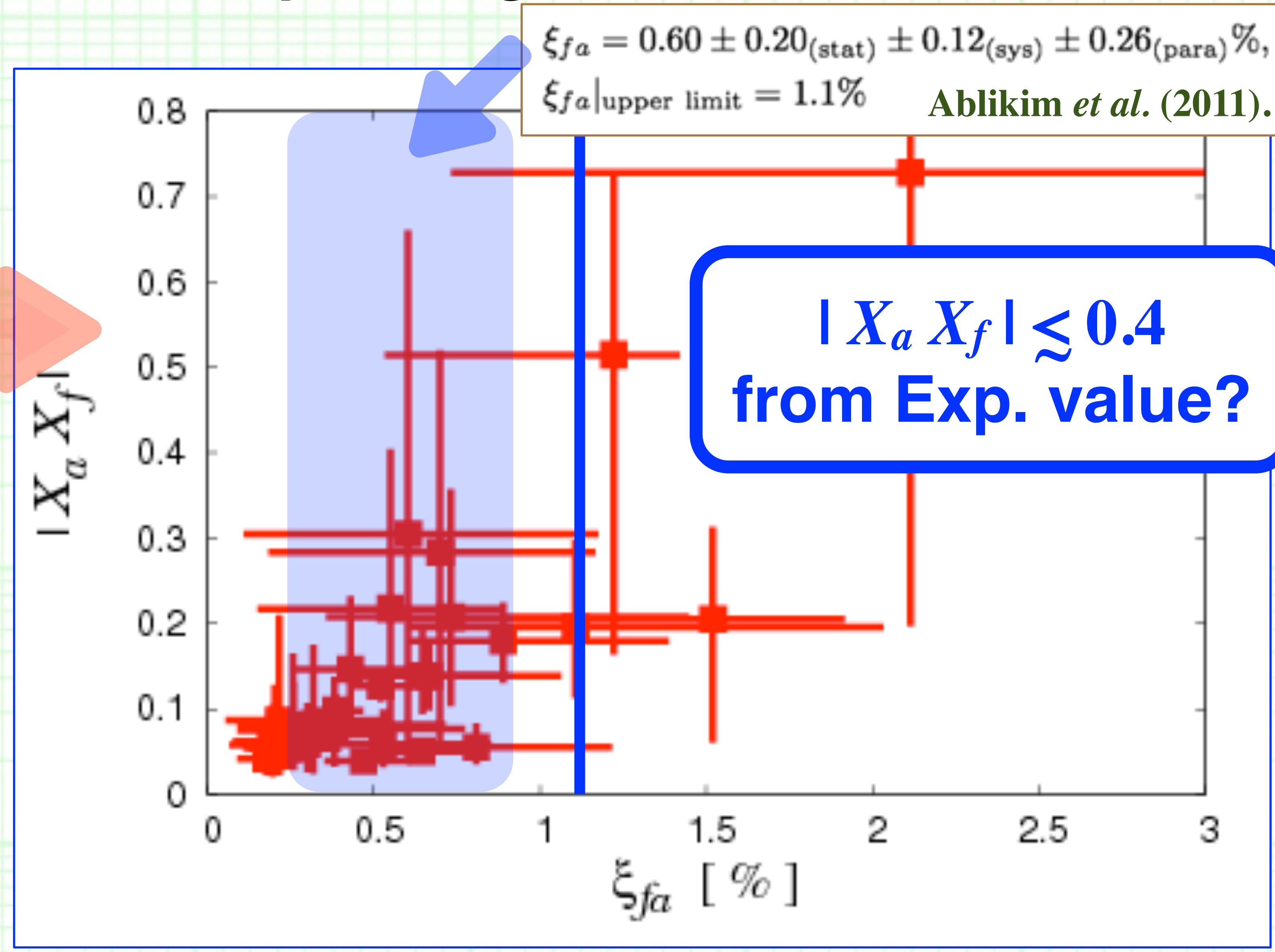
Appendix

++ Test with Flatte parameters ++

- Now we examine **the relation between the $a_0(980)$-$f_0(980)$ mixing intensity ξ_{fa} and the product of the two $K\bar{K}$ compositeness $|X_a X_f|$**, with the Flatte parameters from Exp. fittings.

$a_0(980)$			
Collaboration	M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
CLEO (2011)	998	3.97 ± 0.77	4.25
KLOE (2009)	982.5	2.84 ± 0.41	2.46
CB (2008)	987.4	2.94 ± 0.12	2.87
SND (2000)	995	$5.93\ ^{+10.54}_{-2.39}$	3.11
E852 (1999)	1001	2.36 ± 0.13	2.47

$f_0(980)$			
Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
CDF (2011)	989.6	$4.02\ ^{+1.01}_{-1.37}$	2.65
KLOE (2006)	977.3	2.45 ± 0.17	1.21
Belle (2006)	950	$4.07\ ^{+0.76}_{-0.95}$	2.28
BES (2005)	965	$5.80\ ^{+0.22}_{-0.23}$	2.83
FOCUS (2005)	957	$3.39\ ^{+0.62}_{-0.76}$	2.15
SND (2000)	969.8	$7.88\ ^{+1.09}_{-0.86}$	3.19



- There is not a clear proportional connection, but there is actually a tendency.

Appendix

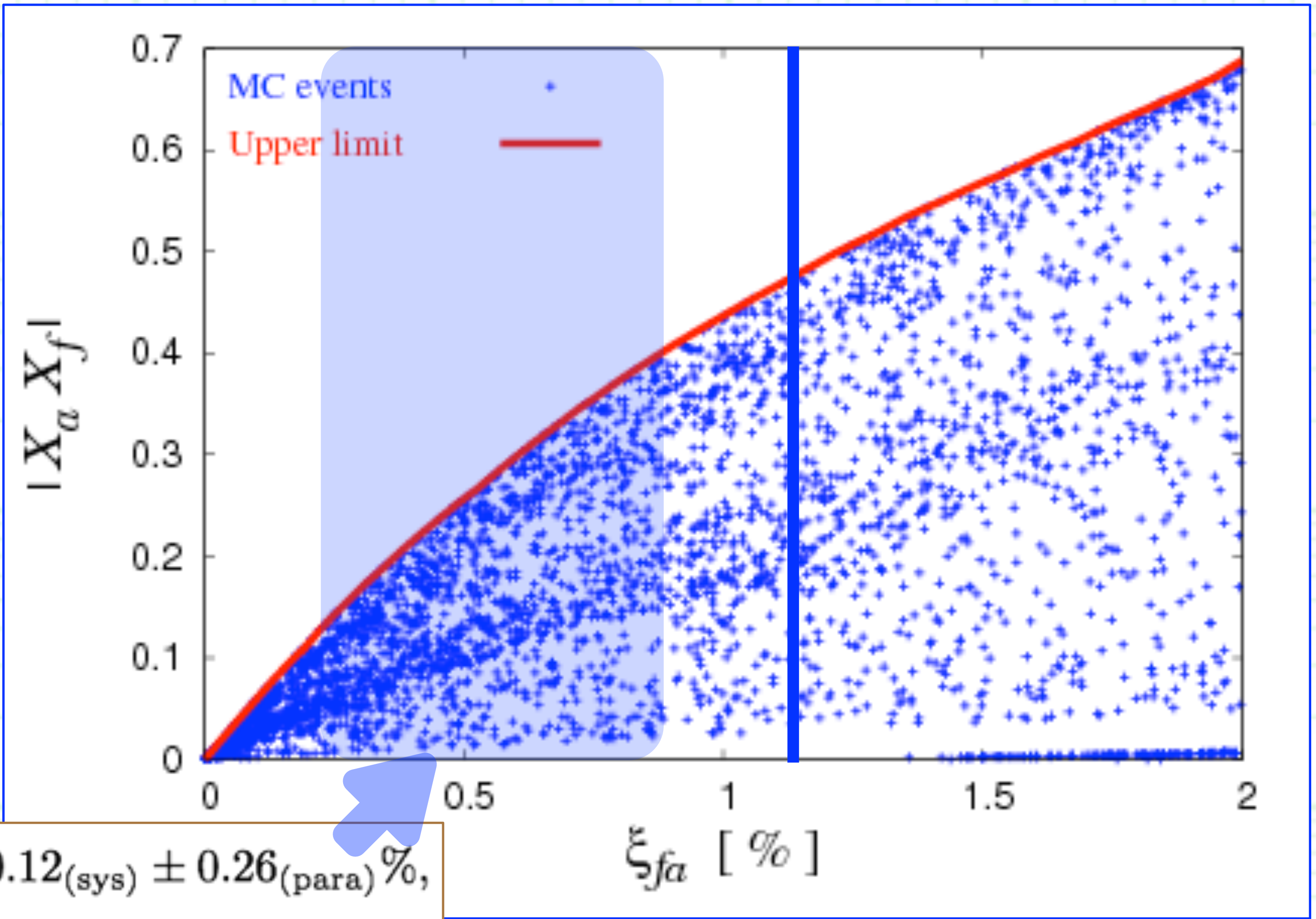
++ In a more general way ++

- We further see **the relation between ξ_{fa} and | $X_a\, X_f$ | in a more general way**. --> 4 of Flatte parameters are fixed as

$$M_a = 990\ \mathrm{MeV}, \quad \bar{g}_{a\pi\eta} = 3.0\mathrm{GeV}, \quad M_f = 970\ \mathrm{MeV}, \quad \bar{g}_{f\pi\pi} = 2.4\ \mathrm{GeV}$$

---Rough average of Exp. params.

while the $a_0(980)$-$K\bar{K}$ and $f_0(980)$-$K\bar{K}$ coupling consts. are allowed to be arbitrary. (generated by random num.)



- There is an upper limit of | $X_a\, X_f$ | for each ξ_{fa} .

--- Especially, from

$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

we have | $X_a\, X_f$ | < 0.47 .

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})}\%,$$
$$\xi_{fa}|_{\text{upper limit}} = 1.1\%$$

Ablikim *et al.* (2011).