Leptoproduction of pions and the pion-induced exclusive Drell-Yan process

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Outline:

- **Introduction: handbag approach, GPDs, subprocess amplitudes**
- **Summary of analyses of hard exclusive processes**
- **Analysis of pion leptoproduction**

  (the pion pole and transversity)
- **The exclusive Drell-Yan process -** $\pi^- p \to l^+ l^- n$
- **Summary**

Hard exclusive scattering within the handbag approach

rigorous proofs of collinear factorization in generalized Bjorken regime:

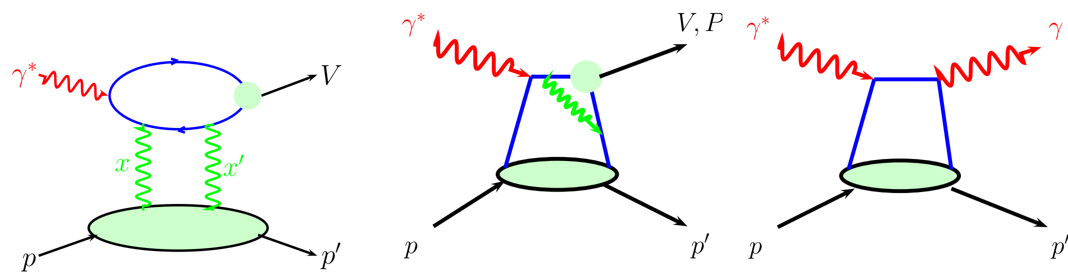
for $\gamma^*_L \to V_L(P)$ and $\gamma^*_T \to \gamma_T$ amplitudes $\quad\quad$ $(Q^2, W \to \infty,\ x_{Bj}$ fixed$)$

Radyushkin, Collins et al, Ji-Osborne

hard subprocesses

$\gamma^* g \to V g\,,$

$\gamma^* q \to V(P,\gamma) q$



and GPDs and meson w.f.

(encode the soft physics)

$$\mathcal{M} \sim \int_{-1}^{1} dx\, \mathcal{H}(x,\xi,Q^2,t=0) K(x,\xi,t)$$

$$d\sigma/dt \sim \mid \mathcal{M} \mid^2 + \mathcal{O}(1/Q^2)$$

power corrections are theoretical not under control

Exp: strong power corrections from γ^*_T and $\gamma^*_L \to V_L(P)$

GPDs – a reminder

D. Müller et al (94), Ji(97), Radyushkin (97)

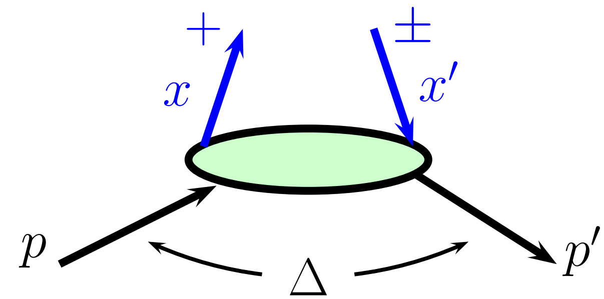
GPDs: $K = K(\bar{x}, \xi, t)$

$$K = H, E, \widetilde{H}, \widetilde{E}, H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \quad x' = \frac{\bar{x}-\xi}{1-\xi}$$

for quarks ($\xi < \bar{x} < 1$) and gluons

(antiquarks for $-1 < \bar{x} < -\xi$, $q\bar{q}$ pairs $-\xi < \bar{x} < \xi$)



properties:

reduction formula $H^q(\bar{x}, \xi = t = 0) = q(\bar{x})$, $\widetilde{H}^q \to \Delta q(\bar{x})$, $H_T^q \to \delta^q(\bar{x})$

sum rules (proton form factors): $F_1^q(t) = \int d\bar{x} H^q(\bar{x}, \xi, t)$, $F_1 = \sum e_q F_1^q$

$E \to F_2$, $\widetilde{H} \to F_A$, $\widetilde{E} \to F_P$

polynomiality, universality, evolution, positivity constraints

Ji's sum rule $\quad J_q = \frac{1}{2} \int_{-1}^{1} d\bar{x}\, \bar{x}\, [H^q(\bar{x}, \xi, t = 0) + E^q(\bar{x}, \xi, t = 0)]$

FT $\mathbf{\Delta} \to \mathbf{b}$ ($\Delta^2 = -t$): information on parton localization in trans. position space

Parametrizing the GPDs

double distribution ansatz (Mueller $et\ al$ (94), Radyushkin (99))

$$K^i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x)\, K^i(\rho,\xi=0,t) w_i(\rho,\eta) + D_i\, \Theta(\xi^2-\bar{x}^2)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2-\eta^2]^{n_i}$ $(n_g = n_{\rm sea} = 2, n_{\rm val} = 1$, generates ξ dep.)
zero-skewness GPD $K^i(\rho,\xi=0,t) = k^i(\rho) \exp\left[(b_{ki} + \alpha'_{ki} \ln(1/\rho))t\right]$
$k = q, \Delta q, \delta^q$ for $H, \widetilde{H}, H_T$ or $N_{ki}\rho^{-\alpha_{ki}(0)}(1-\rho)^{\beta_{ki}}$ for $E, \widetilde{E}, \bar{E}_T$
Regge-like t dep. (for small $-t$ reasonable appr.)

advantages: polynomiality and reduction formulas automatically satisfied
$H_{\rm val}$, $E_{\rm val}$ and $\widetilde{H}_{\rm val}$ at $\xi = 0$ from analysis of form factors (sum rules)
positivity bounds respected Diehl et al(04), Diehl-K (13)

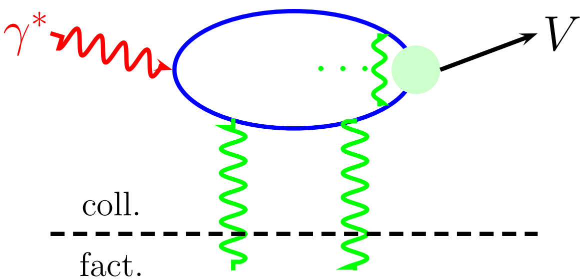
D-term neglected

The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources $\Longrightarrow$ gluon radiation



LO pQCD

$+$ quark trans. mom.

$+$ Sudakov supp.

$\Rightarrow$ asymp. fact. formula (lead. twist) for $Q^2 \to \infty$

Sudakov factor Sterman et al(93)

$$S(\tau, \mathbf{b}_\perp, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b_\perp \Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL $\Rightarrow \exp[-S]$

provides sharp cut-off at $b_\perp = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}^M_{0\lambda,0\lambda} = \int d\tau d^2 b_\perp \, \hat{\Psi}_M(\tau, -\mathbf{b}_\perp) \, e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \mathbf{b}_\perp)$$

$\hat{\Psi}_M \sim \exp[\tau\bar{\tau} b_\perp^2/4a_M^2]$ LC wave fct of meson

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x} + \xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

Sudakov factor generates series of power corr. $\sim (\Lambda^2_{\rm QCD}/Q^2)^n$

from intrinsic $k_\perp$ in wave fct: series $\sim (a_M Q)^{-n}$

What has been done?

- analysis of FF (DFJK04, update: Diehl-K 1302.4604)
  using CTEQ6 (ABM11) PDFs, fixes $H, E, \widetilde{H}$ for valence quarks

- analysis of $d\sigma_L/dt$ for ρ^0 and ϕ production Goloskokov-K, hep-ph/0611290
  data from H1, ZEUS, E665, HERMES
  for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$ and $W \gtrsim 4\,\mathrm{GeV}$ ($\xi \lesssim 0.1$, $-t \lesssim 0.5\,\mathrm{GeV}^2$)
  fixes H for sea quarks and gluons for given H^{val}
  (E negligible, others don't contr.) update with ABM11 required

- analysis of π^+ production, Goloskokov-K, 0906.0460
  $d\sigma/dt$ and A_{UT} data from HERMES ($W \simeq 4\,\mathrm{GeV}$, $Q^2 \simeq 2-5\,\mathrm{GeV}^2$)
  evidence for strong contr. from γ_T^* (H_T)
  fixes $\widetilde{H}$, pion pole and H_T (no clear signal for $\widetilde{E}_{\mathrm{non-pole}}$)

- SDME and A_{UT} for ρ^0 production HERMES,
  π^0 cross section and η/π^0 cross section ratio from CLAS (large skewness!),
  and lattice QCD QCDSF and UKQCD, hep-lat/0612032
  hints at strong contributions from $\bar{E}_T = 2\widetilde{H}_T + E_T$

Applications

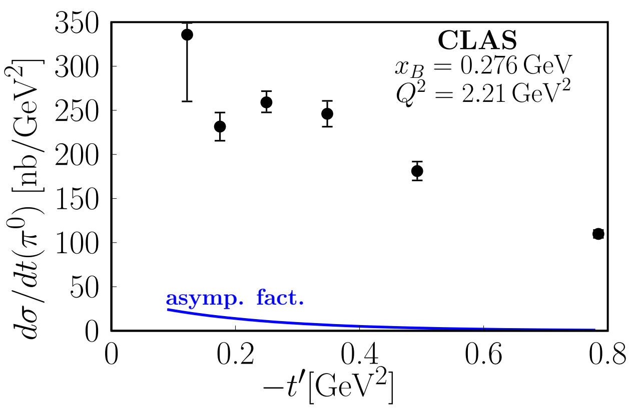
exploiting universality: our set of GPDs allows for parameter free calculations of other hard exclusive reactions (except of possible wave fct effects)

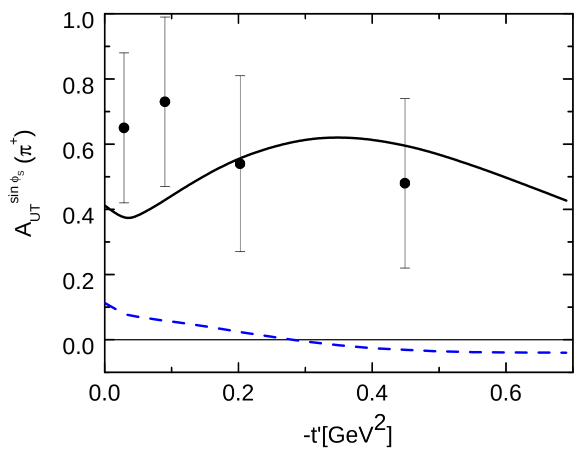
- $\nu_l p \to lPp$ Kopeliovich et al (13)
  V-A structure leads to different combinations of GPDs no data
- timelike DVCS Pire et al (13) no data
- $\gamma^* p \to \omega p$ Goloskokov-K(14)
  compared with SDMEs from HERMES(14) (asymmetries will come)
  prominent role of pion pole
- DVCS K-Moutarde-Sabatie(13)
  compared to data from Jlab, HERMES, H1, ZEUS
  good agreement with small skewness data, less good with Jlab data

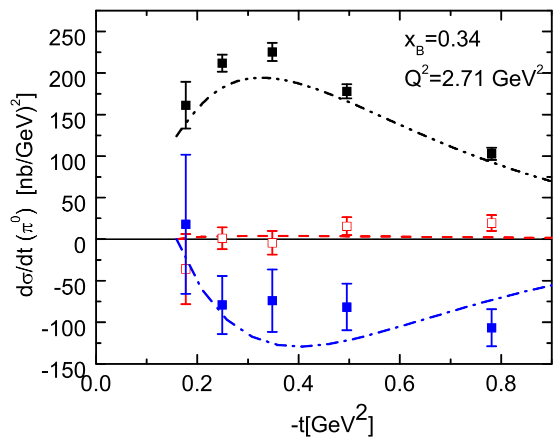
Analysis of pion leptoproduction

leading amplitudes for $Q^2 \to \infty$

$$\mathcal{M}_{0+0+} = \frac{e_0}{2}\sqrt{1-\xi^2}\langle\widetilde{H} - \frac{\xi^2}{1-\xi^2}\widetilde{E}\rangle \qquad \mathcal{M}_{0-0+} = e_0\frac{\sqrt{-t'}}{4m}\xi\langle\widetilde{E}\rangle$$







$\widetilde{H}$ from FF analysis

Diehl-K (13)

$\widetilde{E}$ neglected

(F_P, lattice QCD, π^+)

HERMES(09)

$Q^2 \simeq 2.5\,\mathrm{GeV}^2, W = 3.99\,\mathrm{GeV}$

$\sin\phi_s$ modulation very large

does not vanish for $t' \to 0$

$A_{UT}^{\sin\phi_s} \propto \mathrm{Im}\Big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

CLAS(12)

unsep. cross sec.

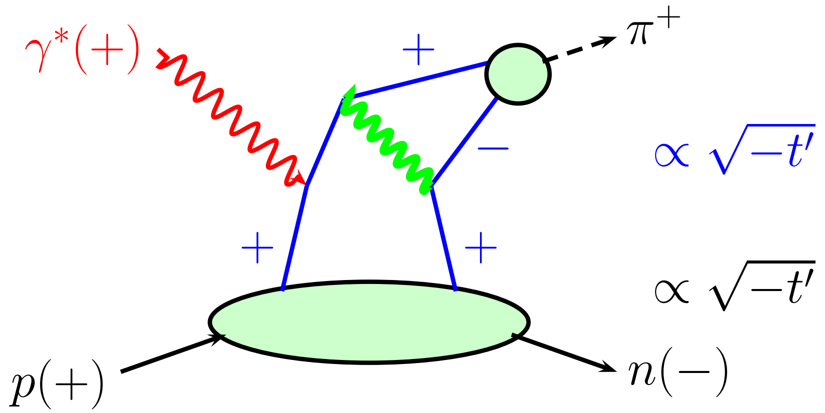
$d\sigma_T + \epsilon d\sigma_L$

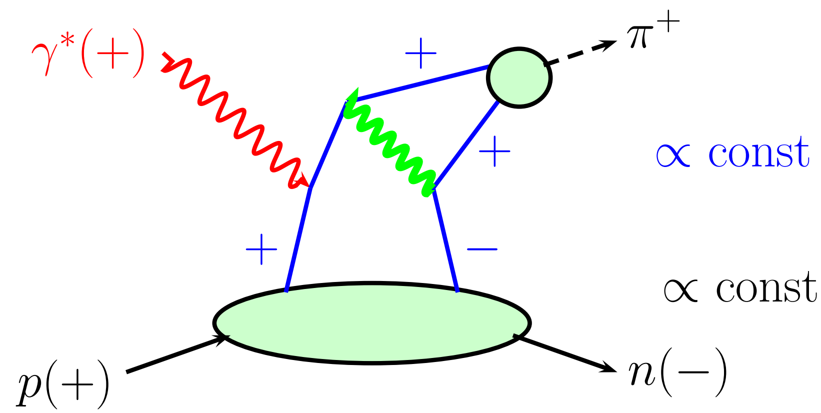
$d\sigma_{LT}$, $d\sigma_{TT}$

How can we model $\mathcal{M}_{0-,++}$ in the handbag?

helicity-non-flip GPDs
$H, E, \widetilde{H}, \widetilde{E}$

helicity-flip (transv.) GPDs
$H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$





lead. twist pion wave fct. $\propto q' \cdot \gamma\gamma_5$
(perhaps including $\mathbf{k}_\perp$)

transversity GPDs required
go along with twist-3 w.f.

$\mathcal{M}_{0-,++} \propto t'$

$\mathcal{M}_{0-,++} \propto$ const

(forced by angular momentum conservation)

The twist-3 pion distr. amplitude

projector $q\bar{q} \to \pi$ (3-part. $q\bar{q}g$ contr. neglected) Beneke-Feldmann (01)

$\sim q' \cdot \gamma\gamma_5\Phi + \mu_\pi\gamma_5\Big[\Phi_P - \imath\sigma_{\mu\nu}(\ldots\Phi'_\sigma + \ldots\Phi_\sigma\partial/\partial\mathbf{k}_{\perp\nu})\Big]$

definition: $\langle\pi^+(q') \mid \bar{d}(x)\gamma_5 u(-x) \mid 0\rangle = f_\pi\mu_\pi\int d\tau \mathrm{e}^{iq'x\tau}\Phi_P(\tau)$

local limit $x \to 0$ related to divergency of axial vector current

$\Longrightarrow \mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\,\mathrm{GeV}$ at scale $2\,\mathrm{GeV}$ (conv. $\int d\tau\Phi_P(\tau) = 1$)

Eq. of motion: $\tau\Phi_P = \Phi_\sigma/N_c - \tau\Phi'_\sigma/(2N_c)$

solution: $\Phi_P = 1, \quad \Phi_\sigma = \Phi_{AS} = 6\tau(1-\tau)$ Braun-Filyanov (90)

$H_{0-,++}^{\mathrm{twist-3}}(t=0) \neq 0$, Φ_P dominant, Φ_σ contr. $\propto t/Q^2$

in coll. appr.: $\mathcal{H}_{0-,++}^{\mathrm{twist-3}}$ singular, in $\mathbf{k}_\perp$ factorization (m.p.a.) regular

$$\mathcal{M}_{0-++} = e_0\sqrt{1-\xi^2}\int dx\mathcal{H}_{0-++}^{\mathrm{twist-3}}H_T\,, \qquad \mathcal{M}_{0+\pm+} = -e_0\frac{\sqrt{-t'}}{4m}\int dx\mathcal{H}_{0-++}^{\mathrm{twist-3}}\bar{E}_T$$

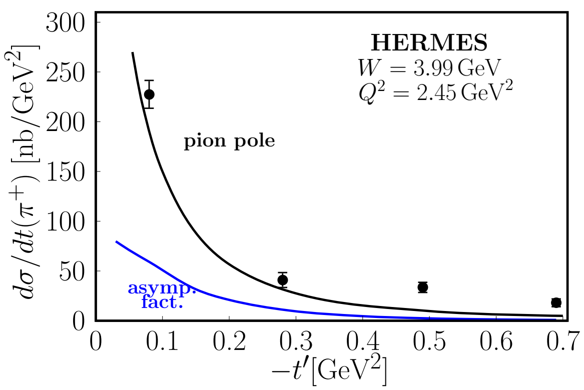
(suppressed by μ_π/Q as compared to $L \to L$ amplitudes)

The pion pole

$$\mathcal{M}_{0+0+} = \frac{e_0}{2}\sqrt{1-\xi^2}\langle\widetilde{H} - \frac{\xi^2}{1-\xi^2}\widetilde{E}\rangle \qquad \mathcal{M}_{0-0+} = e_0\frac{\sqrt{-t'}}{4m}\xi\langle\widetilde{E}\rangle$$

leading amplitudes for $Q^2 \to \infty$

For π^+ production - pion pole: (Mankiewicz et al (98), Penttinen et al (99))



$$\widetilde{E}^u_{\rm pole} = -\widetilde{E}^d_{\rm pole} = \Theta(|x|\le\xi)\frac{mf_\pi g_{\pi NN}}{\sqrt{2}\xi}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}\Phi_\pi(\frac{x+\xi}{2\xi})$$

$$\Longrightarrow \frac{d\sigma_L^{\rm pole}}{dt} \sim \frac{-t}{Q^2}\left[\sqrt{2}e_0 g_{\pi NN}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}Q^2 F_\pi^{\rm pert}(Q^2)\right]^2$$

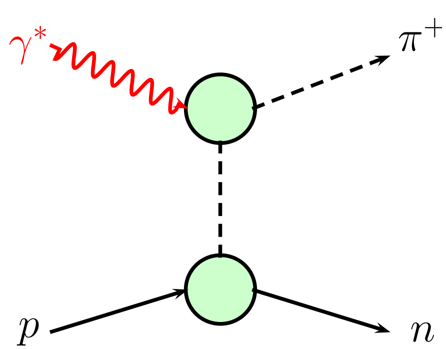
understimates c.s.(blue l.) $\qquad F_\pi^{\rm pert.} \simeq 0.3{-}0.5 F_\pi^{\rm exp.}$

(F_π measured in π^+ electroproduction at Jlab)

Goloskokov-K(09): $F_\pi^{\rm pert} \to F_\pi^{\rm exp}$

knowledge of the sixties suffices to explain

π^+ data at small $-t$



Parametrization of H_T and $\bar{E}_T$

H_T: transversity PDFs Anselmino et al(09)

$\delta^q(x) = N^q_{H_T}\sqrt{x}(1-x)\big[q(x)+\Delta q(x)\big]$ DD ansatz

parameters: $\alpha'_{H_T} = 0.45\,\mathrm{GeV}^{-2}$, $b_{H_T} = 0$, $N^{u(d)}_{H_T} = 0.78(-1.01)$

opposite sign for u and d quarks but u larger than d

Alternative (favored): normalize to lattice moments QCDSF-UKQCD(05)

$\bar{E}_T$: only available lattice result for moments: QCDSF-UKQCD(06)

Large, same sign and almost same size for u and d quarks

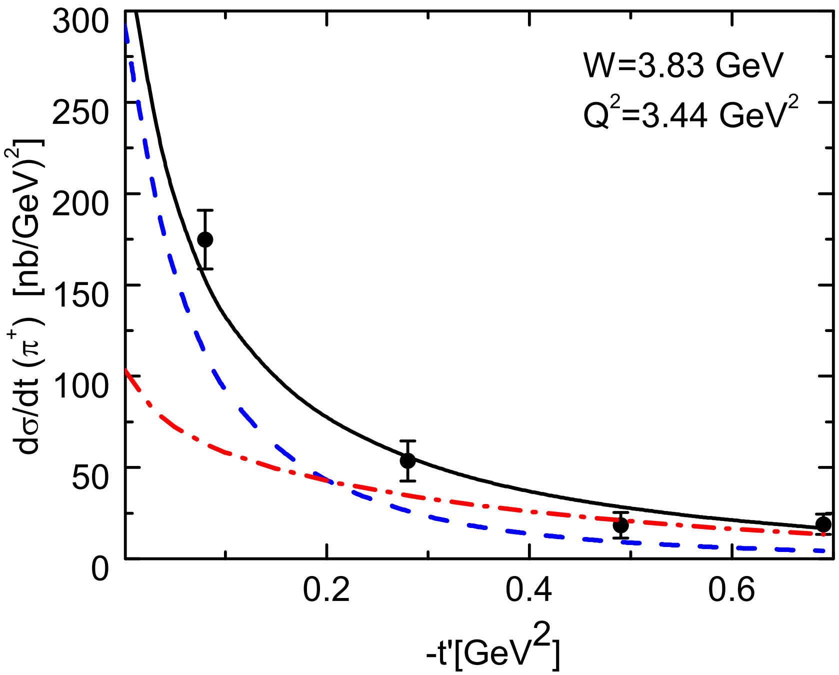
$\bar{E}_T$ parameterization: $e^a_T = \bar{N}^a_{e_T} e^{b_{e_T} t} x^{-\alpha^{e_T}(t)}(1-x)^{\beta^a_{e_T}}$

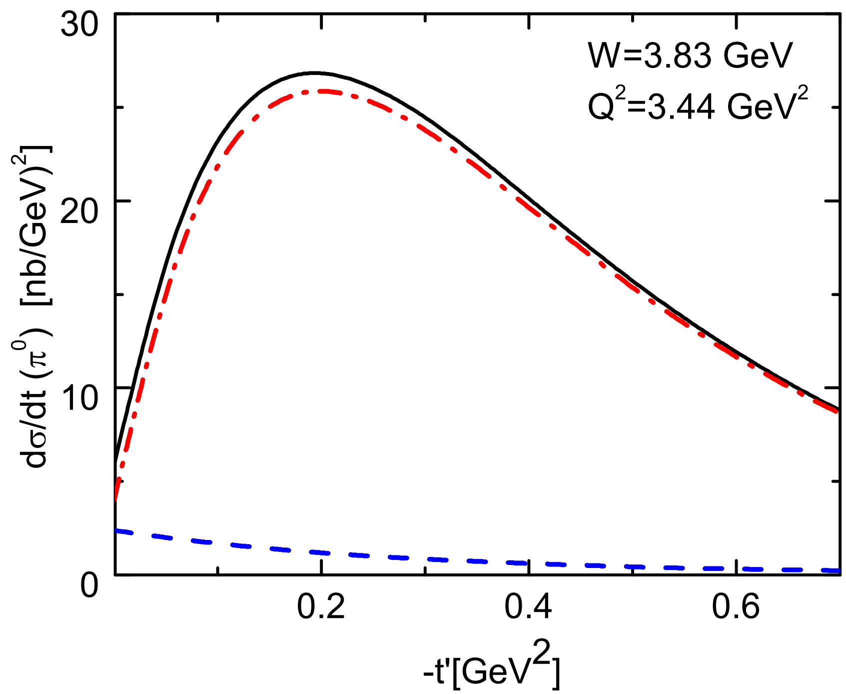
parameters: $\alpha_{e_T}(0) = 0.3$, $\alpha'_{e_T} = 0.45\,\mathrm{GeV}^{-2}$, $b_{e_T} = 0.5\,\mathrm{GeV}^{-2}$,$\beta^{u(d)}_{e_T} = 4(5)$,
$\bar{N}^{u(d)}_{e_T} = 6.83(5.05)$,

adjusted to lattice results

Burkardt: related to Boer-Mulders fct $\langle\cos(2\phi)\rangle$ in SIDIS $-$ same pattern

H_T and $\bar{E}_T$ in pion electroproduction





unseparated (longitudinal, transverse) cross sections

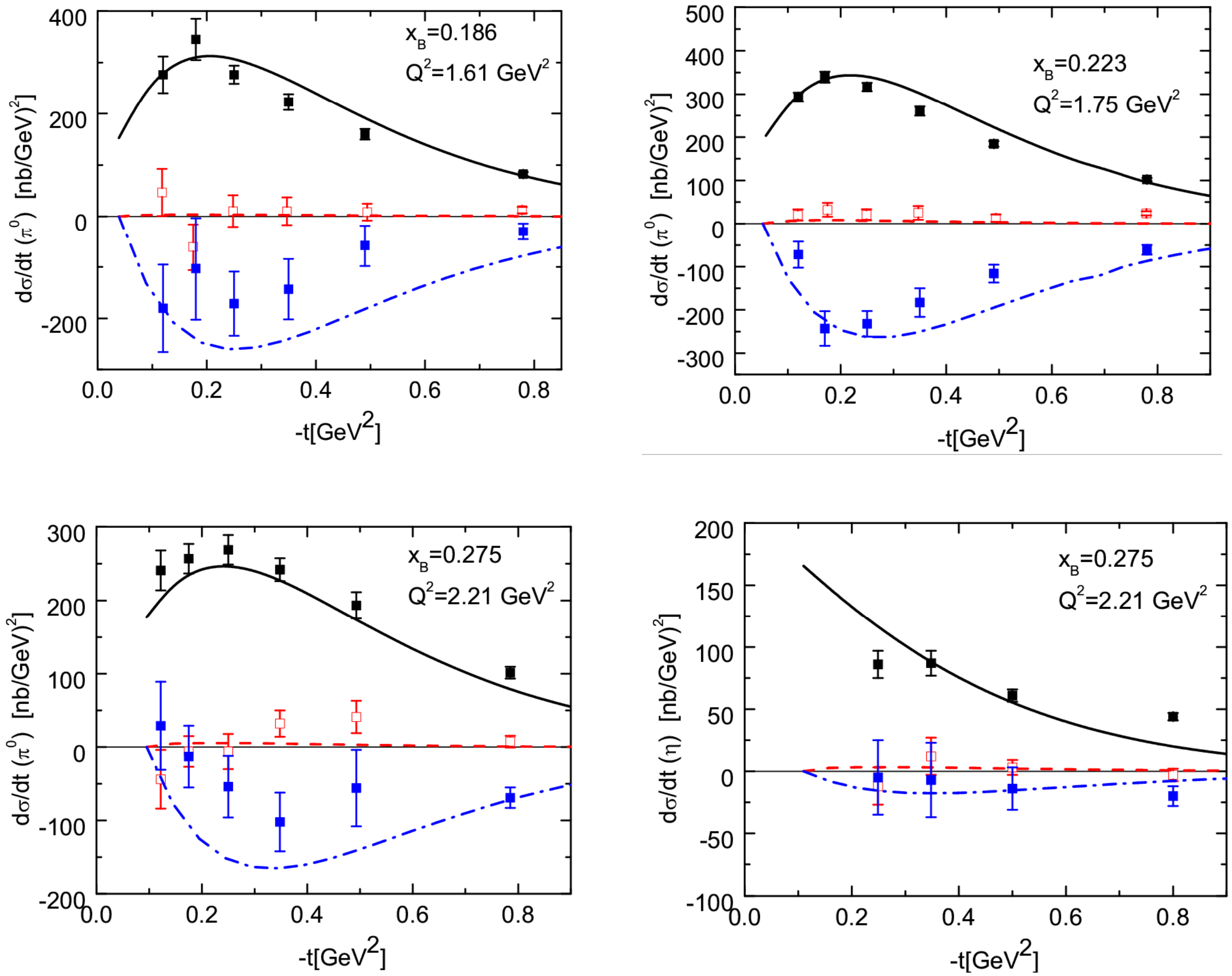
π^+: pion pole and $\propto K^u - K^d$

π^0: no pion pole and $\propto e_u K^u - e_d K^d$

consider $u-d$ signs: $\bar{E}_T$ same, $\widetilde{H}, H_T$ opposite sign

$\Longrightarrow$ $\widetilde{H}$ and H_T large for π^+, small for π^0

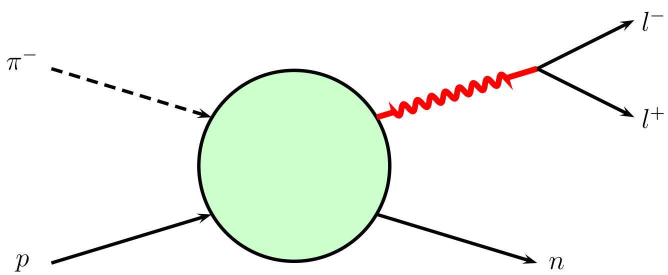
$\bar{E}_T$ small for π^+, large for π^0



data: Bedlinsky et al (12)

(Large x_B (ξ), only estimates)

$\pi^- p \to l^- l^+ n$



the exclusive limit of the Drell-Yan process
directly related to leptoproduction of pions
- same GPDs
- $\hat{s} - \hat{u}$ crossed subprocess

$$\mathcal{H}^{\pi^- \to \gamma^*}(\hat{u}, \hat{s}) \;=\; -\mathcal{H}^{\gamma^* \to \pi^+}(\hat{s}, \hat{u})$$

or for hard scattering kernel $\mathcal{F}^{t.l.}(x, \xi, \mathbf{k}_\perp^2, \bar{z}) \;=\; \mathcal{F}^{s.l.*}(x, \xi, -\mathbf{k}_\perp^2, z)$
equivalent to $Q^2 \to -Q'^2$

Berger-Diehl-Pire (01): leading-twist, LO analysis of long. cross section
(i.e. exploiting asymp. factorization formula)

we know that leading-twist analysis of π^+ production fails with HERMES data by order of magnitude

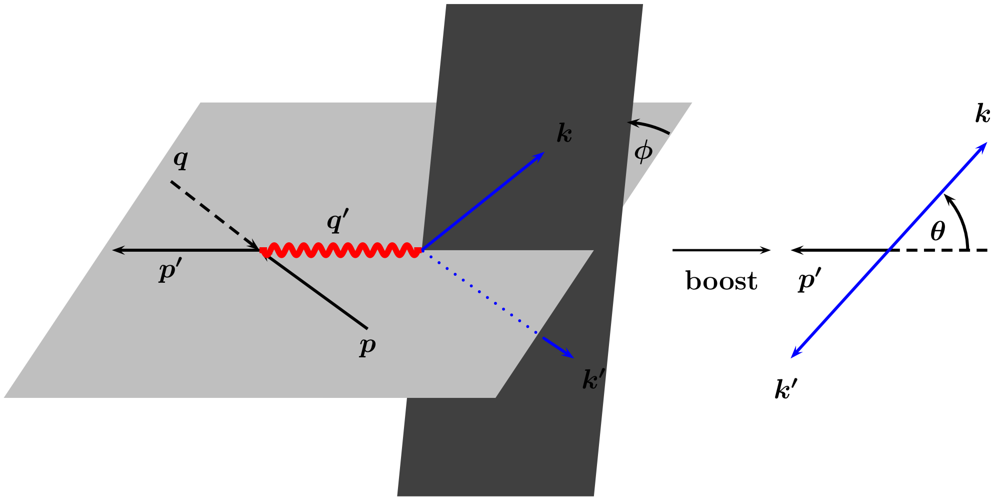
Therefore . . .

a reanalyis of the exclusive Drell-Yan process seems appropriate
making use of what we have learned from analysis of pion production

- retaining quark transverse momenta in the subprocess (the MPA)
- treating pion-pole contribution as an OPE term
- take into account transverse photons and transversity GPDs

Goloskokov-K(15) - in preparation

Cross section



k momentum of l^-

$\tau = Q'^2/(s-m^2)$

$$\frac{d\sigma}{dt\,dQ'^2\,d\cos\theta\,d\phi} = \frac{3}{8\pi}\left\{\sin^2\theta\,\frac{d\sigma_L}{dt\,dQ'^2} + \frac{1+\cos^2\theta}{2}\,\frac{d\sigma_T}{dt\,dQ'^2}\right.$$
$$\left. + \frac{1}{\sqrt{2}}\sin(2\theta)\cos\phi\,\frac{d\sigma_{LT}}{dt\,dQ'^2} + \sin^2\theta\cos(2\phi)\,\frac{d\sigma_{TT}}{dt\,dQ'^2}\right\}$$

$$\frac{d\sigma_L}{dtdQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2}\frac{\tau^2}{Q'^6}\sum_{\nu'}|\mathcal{M}_{0\nu',0+}|^2 \qquad \frac{d\sigma_T}{dtdQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2}\frac{\tau^2}{Q'^6}\sum_{\mu=\pm1,\nu'}|\mathcal{M}_{\mu\nu',0+}|^2$$

$$\frac{d\sigma_{LT}}{dtdQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2}\frac{\tau^2}{Q'^2}\,\text{Re}\sum_{\nu'}\mathcal{M}^*_{0\nu'0+}(\mathcal{M}_{+\nu'0+} - \mathcal{M}_{-\nu'0+})$$

$$\frac{d\sigma_{TT}}{dtdQ'^2} = \frac{\alpha_{\text{elm}}}{48\pi^2}\frac{\tau^2}{Q'^2}\,\text{Re}\sum_{\nu'}\mathcal{M}^*_{+\nu'0+}\mathcal{M}_{-\nu'0+}$$

ϕ integration

$$\frac{d\sigma}{dtdQ'^2 d\cos\theta} = \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L}{dtdQ'^2} + \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_T}{dtdQ'^2}$$

θ integration

$$\frac{d\sigma}{dtdQ'^2} = \frac{d\sigma_L}{dtdQ'^2} + \frac{d\sigma_T}{dtdQ'^2}$$

The time-like Sudakov factor

Sterman et al(93): Sudakov factor in space-like region
with sharp cut-off at $b = 1/\Lambda_{\rm QCD}$

time-like Sudakov factor unknown

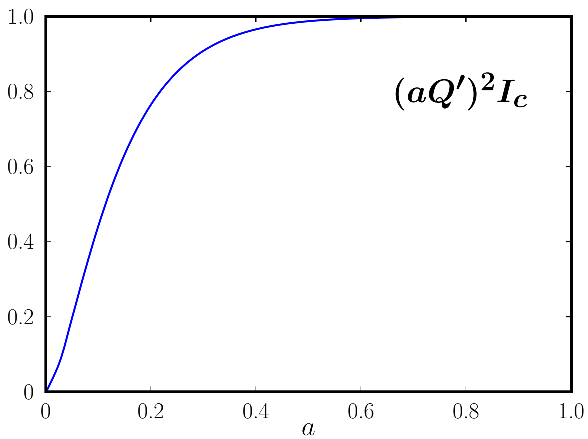
replacement $Q^2 \to -Q'^2$ leads to unplausible oscillations Magnea-Sterman(90)

Gousset-Pire (95): use $Q^2 \to Q'^2$ (s.l.=t.l.)

alternative: use $\Theta(\Lambda_{\rm QCD} - b)$ since, for Q'^2 of interest,

wave function $\Psi \sim \exp\left[\tau\bar{\tau}b^2/(4a_\pi^2)\right]$ is more important

difference considered as part of uncertainties



role of cut-off

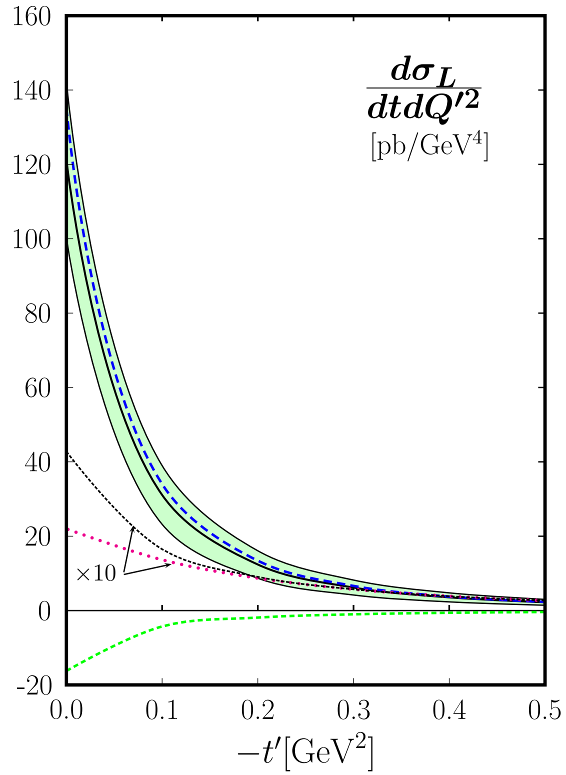
$I = 2\pi \int_0^{b_0} bdbK_0(\sqrt{a}Q'b)$

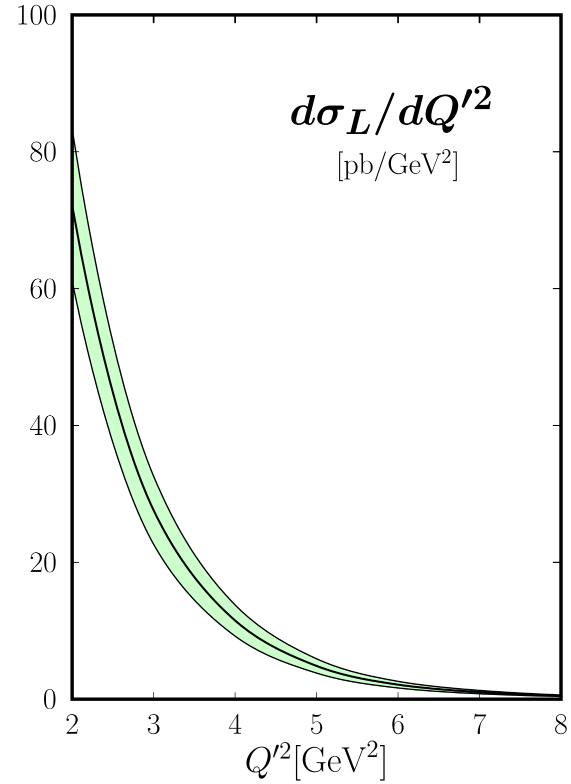
$b_0 \to \infty$: $I \to lim_{\mathbf{k}_\perp \to 0}[aQ'^2 \pm \mathbf{k}_\perp^2 + i\epsilon]^{-1}$

b_0 finite:

$I = \frac{1}{aQ'^2}\left(1 - \sqrt{a}Q'K_1(\sqrt{a}Q'b)\right)$

Results on the longitudinal cross section





$Q'^2 = 4\,\mathrm{GeV}^2$ and $s = 20\,\mathrm{GeV}^2$ solid lines with error bands: full result

pion pole, $|\langle\widetilde{H}^{(3)}\rangle|^2$, interference, short dashed: leading-twist contribution

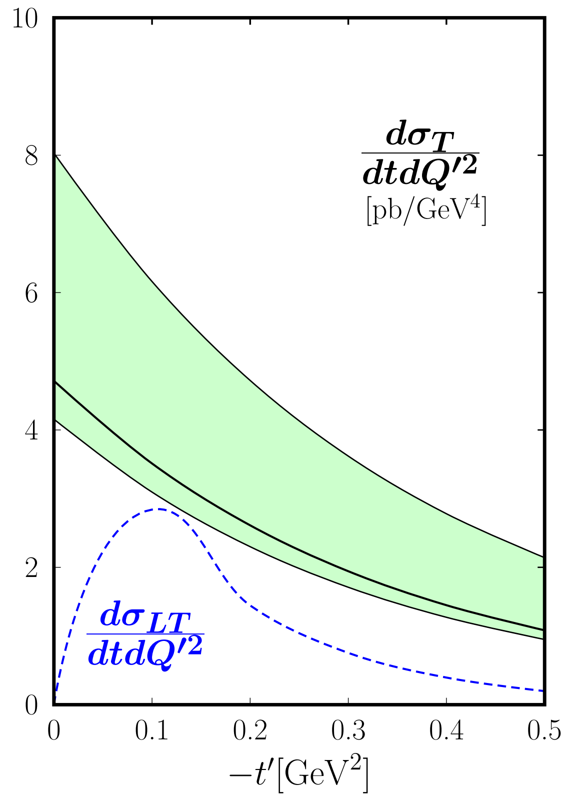
time-like pion FF: $Q'^2|F_\pi(Q'^2)| = 0.88 \pm 0.04\,\mathrm{GeV}^2$ (CLEO, BaBar, $J/\Psi \to \pi^+\pi^-$)

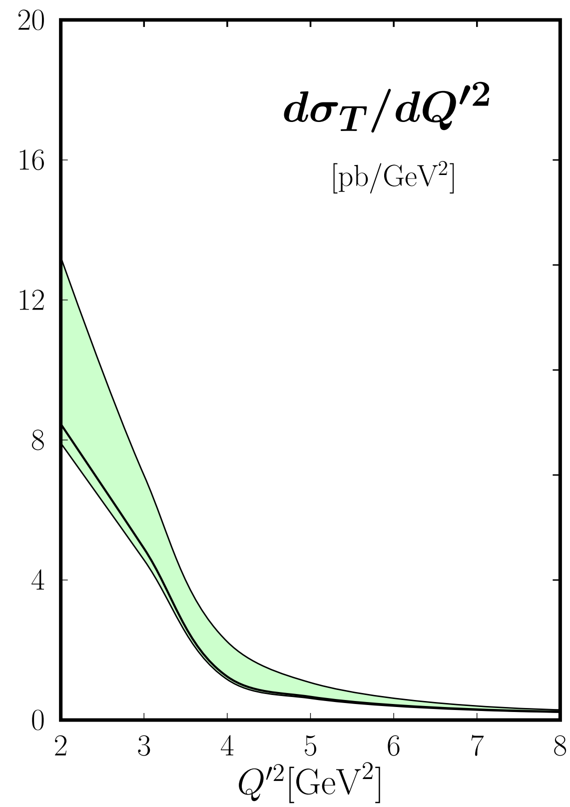
phase from disp. rel. Belicka et al(11) for $Q'^2 < 7\,\mathrm{GeV}^2$

$\delta = 182.6^\circ + 11.2^\circ(Q'^2 - 2\,\mathrm{GeV}^2) - 1.67^\circ(Q'^2 - 2\,\mathrm{GeV}^2)^2$

for $Q'^2 \geq 7\,\mathrm{GeV}^2$: $\delta = 180^\circ$, pQCD result

Results on the transverse cross section





dominated by H_T $\bar{E}_T^u - \bar{E}_T^d$ small

$d\sigma_{TT} \leq 0.1\,\mathrm{pb/GeV}^4 \Longrightarrow d\sigma_T \sim |<H_T>|^2$

Remarks on processes with time-like virtual photons

- time-like excl. processes difficult to understand theoretically
  e.g. no satisfactory explanation of time-like elm form factors
  within pert. QCD
- Drell-Yan process $\pi^- p \to l^+ l^- X$
  large K-factor needed (larger than NLO corr. Sutton et al (92))
  now understood as 'threshold logs' $(Q'^2/(x_1 x_2 s) \to 1)$
  (gluon radiation resummed to NLL Sterman(87), Catani-Trentadue(89))
  leading finally to reasonable fits of data and extraction of PDFs for the
  pion with plausible behavior for $x \to 1$ Aicher-Schäfer-Vogelsang (11)
- hard exclusive scattering processes with time-like virtual photons
  no data as yet but predictions
  time-like DVCS (Pire et al (13)) and $\pi^- p \to l^+ l^- n$ (in progress)
  experimental verification of predictions important

Summary

- asymptotia is far away
  interpretation of data on pion leptoproduction requires strong power corrections from the pion pole and from transverse photons
- within handbag approach $\gamma_T^* \to \pi$ transitions are related to transversity (helicity-flip) GPDs accompanied by a twist-3 pion wave fct.
- making use of what we have learned from pion leptoproduction we are evaluating the long. and transverse cross sections for the exclusive Drell-Yan process
- long. cross section dominated by the pion pole
  transverse cross section fed by H_T ($\bar{E}_T$ small)
  Φ-dependence: various interference terms
- t.l. π FF: $l^+l^- \to \pi^+\pi^-$ (CLEO, BaBar) versus $\pi^-\pi^{+*} \to l^+l^-$