Nucleon spin and tomography

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Outline

- Nucleon spin decomposition

  Jaffe-Manohar vs. Ji
  Complete gauge invariant decomposition
  Orbital angular momentum
  Relation to twist-3 GPDs

- 5D tomography of the nucleon

  Wigner distribution
  Husimi distribution

The proton spin problem

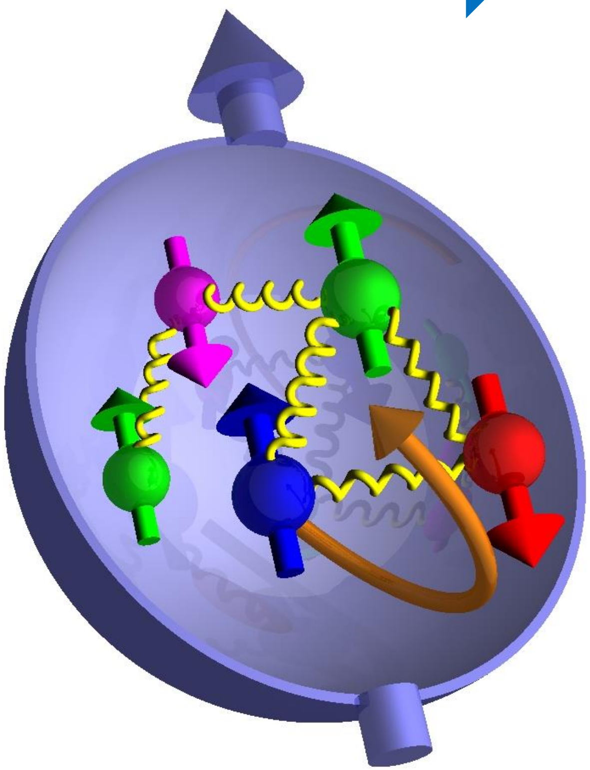
The proton has spin ½.
The proton is not an elementary particle.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z^q + L_z^g$$

Quarks' helicity

Gluons' helicity

Orbital angular Momentum (OAM)



Quark model prediction: $\Delta\Sigma = 1$

$\Delta\Sigma \approx 0.7$ with relativistic effects

`Spin crisis’

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

Latest results from NLO global analysis

$$\Delta\Sigma \approx 0.3 \qquad \int_{0.05}^{1} dx\, \Delta G(x) \approx 0.2$$

DeFlorian, Sassot, Stratmann, Vogelsang (2014)

QCD angular momentum tensor

QCD Lagrangian → Lorentz invariant $x^\mu \longrightarrow x^\mu + \omega^{\mu\nu} x_\nu$

→ Noether current $\partial_\mu M_{can}^{\mu\nu\lambda} = 0$

QCD angular momentum tensor

$$M_{can}^{\mu\nu\lambda} = x^\nu T_{can}^{\mu\lambda} - x^\lambda T_{can}^{\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}\gamma_5\gamma_\rho\psi + F^{\mu\lambda}A^\nu - F^{\mu\nu}A^\lambda$$

quark spin gluon spin

canonical energy momentum tensor

$$T_{can}^{\mu\nu} = \bar{\psi} i \gamma^\mu \overleftrightarrow{\partial}^\nu \psi - F^{\mu\alpha}\partial^\nu A^\alpha - g^{\mu\nu}\mathcal{L}$$

→ Quark OAM → Gluon OAM

Jaffe-Manohar decomposition (1990)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Based on the canonical energy momentum tensor

Operators NOT gauge invariant.

Partonic interpretation in the light-cone gauge $A^+ = 0$

Ji decomposition (1997)

Improved (Belinfante) energy momentum tensor

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu}_{can} + \partial_\rho G^{\rho\mu\nu}$$ ← One can add a total derivative.

$$= \bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi - F^{\mu\rho} F^{\nu}{}_{\rho} - g^{\mu\nu} \mathcal{L}$$

quark part gluon part

$$\frac{1}{2} = J_q + J_g$$

Further decomposition in the quark part

$$\bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi = \bar{\psi} i \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi - \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \partial_\rho (\bar{\psi} \gamma_5 \gamma_\sigma \psi)$$

$$J_q = \frac{1}{2} \Delta\Sigma + L_q$$

Generalized parton distributions (GPD)

Non-forward proton matrix element

$$\int d\lambda e^{i\lambda x} \frac{d\lambda}{2\pi} \langle P'S'|\bar{\psi}(0)\gamma^\mu \psi(\lambda)|PS\rangle$$

$$= \underline{H_q(x)}\bar{u}(P'S')\gamma^\mu u(PS) + \underline{E_q(x)}\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS)$$

Twist-two GPDs

$$J^q = \frac{1}{2}\int dx x(H_q(x) + E_q(x)) \qquad J^g = \frac{1}{4}\int dx (H_g(x) + E_g(x))$$

Two spin communities divided

measured by PHENIX, STAR, COMPASS, HERMES

Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

common and well-known

not measured yet
not even well-defined?

Define rigorously.
Must be related to GPD!

Ji

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

accessible from GPD at JLab, COMPASS, HERMES, J-PARC…
also calculated in lattice QCD

Complete decomposition

Chen, Lu, Sun, Wang, Goldman (2008)
Wakamatsu (2010)
Y.H. (2011)

$$M^{\mu\nu\lambda}_{\text{quark-spin}} = -\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_5\gamma_\sigma\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{quark-orbit}} = \bar{\psi}\gamma^\mu(x^\nu iD^\lambda_{\text{pure}} - x^\lambda iD^\nu_{\text{pure}})\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_a A^{\nu a}_{\text{phys}} - F^{\mu\nu}_a A^{\lambda a}_{\text{phys}}\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-orbit}} = F^{\mu\alpha}_a\Big(x^\nu(D^\lambda_{\text{pure}}A^{\text{phys}}_\alpha)_a - x^\lambda(D^\nu_{\text{pure}}A^{\text{phys}}_\alpha)_a\Big)$$

where
(my choice)

$$A^\mu_{phys} = \frac{1}{D^+}F^{+\mu} \qquad D^\mu_{pure} = D^\mu - iA^\mu_{phys}$$

Gauge invariant completion of Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

OAM from the Wigner distribution

Wigner distribution in QCD   Belitsky, Ji, Yuan (2003)

$$W(\vec{b},\vec{k}) = \int \frac{d^4z}{(2\pi)^4} e^{ikz} \bar{\psi}\left(b - \frac{z}{2}\right) \gamma^\mu \psi\left(b + \frac{z}{2}\right)$$

position   momentum

Need a Wilson line !

Define

$$\vec{L} = \int d^2b d^2k \, \vec{b} \times \vec{k} \, \langle W(\vec{b},\vec{k}) \rangle$$

Lorce, Pasquini (2011)

Which OAM is this??

Canonical OAM from the light-cone Wilson line YH (2011)

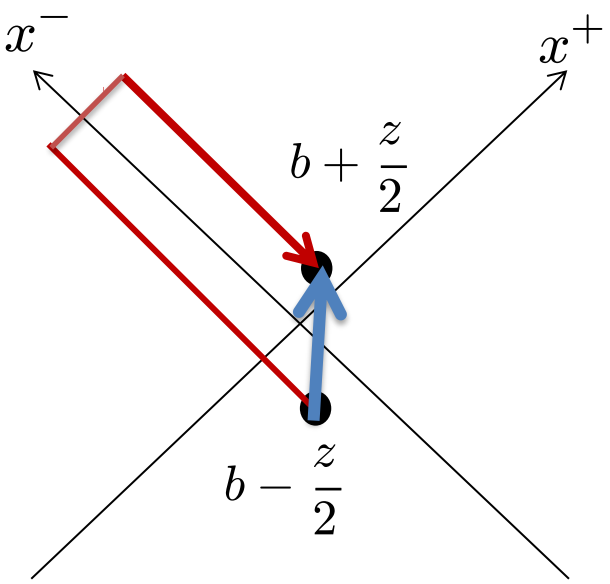
$$\int dq\, \vec{b} \times \vec{k} \langle W_{light-cone}(b,k) \rangle = \langle \bar{\psi} \gamma^{\mu} \vec{b} \times i \overleftrightarrow{D}_{pure} \psi \rangle$$

Kinetic OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

$$\int dk\, \vec{b} \times \vec{k} \langle W_{straight}(b,k) \rangle$$

$$= \langle \bar{\psi} \gamma^{\mu} \vec{b} \times i \overleftrightarrow{D} \psi \rangle$$



Difference between the two OAMs

$$L_{pot} = L - L_{can} = \vec{b} \times \int dx^- \vec{F}$$

` Potential' OAM

Torque acting on a quark

Burkardt (2012)

Twist analysis

YH, Yoshida (2012)

see, also, Ji, Xiong, Yuan (2012)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Understand this relation at the density level

$$\Delta\Sigma = \sum_f \int dx \Delta q_f(x) \qquad \Delta G = \int dx \Delta G(x)$$

$$L^q_{can} = \int dx L^q_{can}(x) \qquad ??$$

c.f. $$\Delta q(x) = \frac{1}{4\pi S^+}\int dz^- e^{ixP^+z^-}\langle PS|\bar{\psi}(z^-)\gamma^+\gamma_5\psi(0)|PS\rangle$$

`Density' of OAM

Ji's OAM — canonical OAM — `potential OAM'

$$\langle \bar{\psi} b \times D\psi \rangle = \langle \bar{\psi} b \times D_{pure}\psi \rangle + ig\langle \bar{\psi} b \times A_{phys}\psi \rangle$$

$$A^{\mu}_{phys} = \frac{1}{D^{+}} F^{+\mu}$$

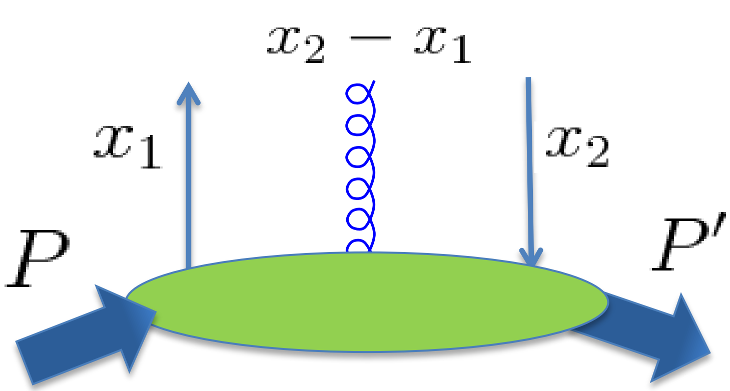
``F-type"

For a 3-body operator, it is natural to define the double density.

$$\int d\lambda d\mu e^{i\frac{\lambda}{2}(x_1+x_2)+i\mu(x_1-x_2)} \langle P'S'|\bar{\psi}(-\lambda/2)D^i(\mu)\psi(\lambda/2)|PS\rangle$$

$$\sim \epsilon^{ij}\Delta_j S^{+}\Phi_D(x_1,x_2)$$

``D-type"



The D-type and F-type correlators are related.

Eguchi, Koike, Tanaka (2006)

$$\langle \bar{\psi} b \times D\psi \rangle = \langle \bar{\psi} b \times D_{pure}\psi \rangle + ig\langle \bar{\psi} b \times A_{phys}\psi \rangle$$

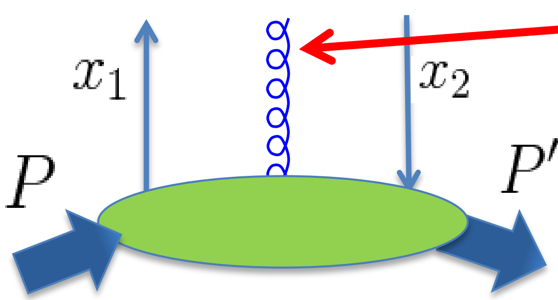
doubly-unintegrate

$$\Phi_D(x_1, x_2) = \delta(x_1 - x_2) L^q_{can}(x_1) + \mathcal{P}\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2)$$

Canonical OAM density

The gluon has zero energy
→ partonic interpretation!

$x_2 - x_1 = 0$



Relation between $L^q_{can}(x)$ and twist-3 GPD

$$\int d\lambda e^{i\lambda x}\frac{d\lambda}{2\pi}\langle P'S'|\bar{\psi}(0)\gamma^\mu\psi(\lambda)|PS\rangle$$

$$= H_q(x)\bar{u}(P'S')\gamma^\mu u(PS) + E_q(x)\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS)$$ twist-2

$$+\underline{G_3(x)}\bar{u}(P'S')\gamma^\mu_\perp u(PS) + \cdots$$ ⬅ twist-3

From the equation of motion,

$$x(H_q(x) + E_q(x) + \underline{G_3(x)}) =$$

$$\Delta q(x) + \underline{L^q_{can}(x)} + \int dx' \mathcal{P}\frac{1}{x - x'}\left(\Phi_F(x,x') + \tilde{\Phi}_F(x,x')\right)$$

⮕ integrate $$\int dx x G_3(x) = -L^q$$

Penttinen, Polyakov, Shuvaev, Strikman (2000)

Quark canonical OAM density

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)} .$$

genuine twist-three

First moment: $J^q = \frac{1}{2}\Delta\Sigma + L^q_{can} + L_{pot}$

The bridge between JM and Ji

Gluon canonical OAM density $L^g_{can}(x)$

$$\frac{1}{2}(H_g(x)+E_g(x)+F_g(x)) - \Delta G(x) + 2\int dX \frac{\Phi_F(X,x)}{x} - 2L^g_{can}(x)$$
$$= -2\int dx' \mathcal{P}\frac{M_F(x,x')}{x(x-x')} - 2\int dx' \mathcal{P}\frac{\tilde{M}_F(x,x')}{x(x-x')}$$

twist-three gluon GPD

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)}\frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)}\frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)}\frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2\tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)}dx_1\int_{-1}^{1}dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

WW part

genuine twist-three

first moment: $J^g + L_{pot} = \Delta G + L^g_{can}$

Complete transverse spin decomposition?

YH, Tanaka, Yoshida (2012)

Longitudinal

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

same!

Transverse

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^{q+g}_{can}$$

cannot be separated in a frame-independent way

Summary so far

- Complete gauge invariant decomposition of nucleon spin now available in QCD, even at the density level.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

- Relation between the two decomposition schemes (JM vs Ji) fully revealed. The connection to twist-3 GPDs clarified.
- Progress towards calculating spin components on a lattice. Ji, Zhang, Zhao (2013,2014); YH, Ji, Zhao (2013)

Husimi distribution for nucleon tomography

Hagiwara and YH, arXiv:1412.4591

Wigner distribution

Phase space distribution in quantum mechanics

$$
\begin{aligned}
f_W(q,p) &= \int_{-\infty}^{\infty} dx e^{-ipx/\hbar} \langle \psi | q - x/2 \rangle \langle q + x/2 | \psi \rangle \\
&= \int_{-\infty}^{\infty} dx e^{-ipx/\hbar} \langle q + x/2 | \hat{\rho} | q - x/2 \rangle
\end{aligned}
$$

Eugene Wigner (1902-1995)

density matrix $\hat{\rho} = |\psi\rangle\langle\psi|$

$$
\int \frac{dq}{2\pi\hbar} f_W(q,p) = |\langle \psi | p \rangle|^2 , \quad \int \frac{dp}{2\pi\hbar} f_W(q,p) = |\langle \psi | q \rangle|^2
$$

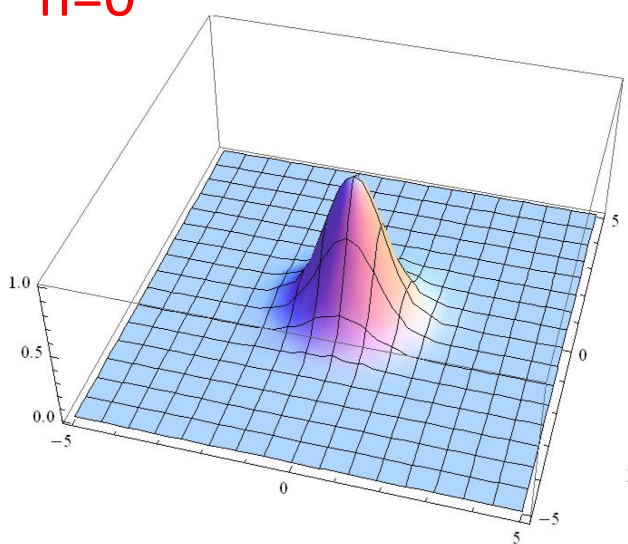
Wigner distribution for the harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

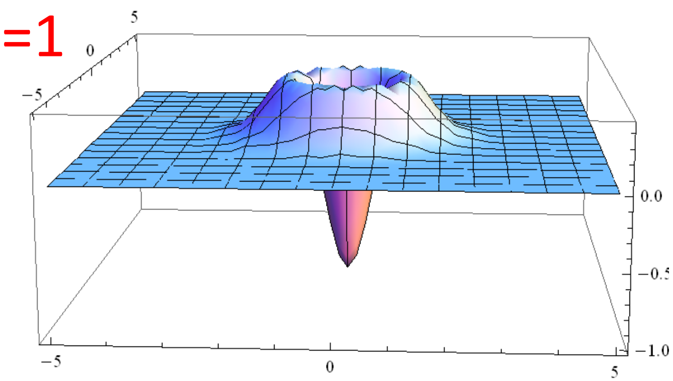
$$f_W(q,p) = 2(-1)^n e^{-2H/\hbar\omega} L_n\left(\frac{4H}{\hbar\omega}\right)$$

Laguerre polynomial

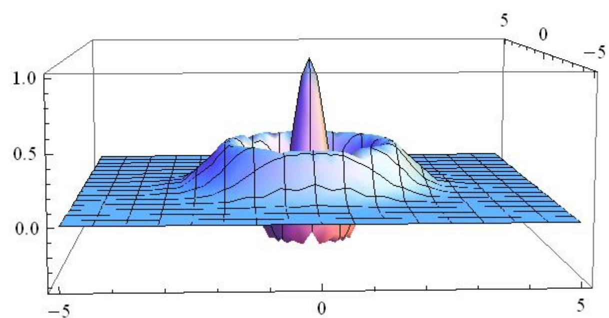
n=0



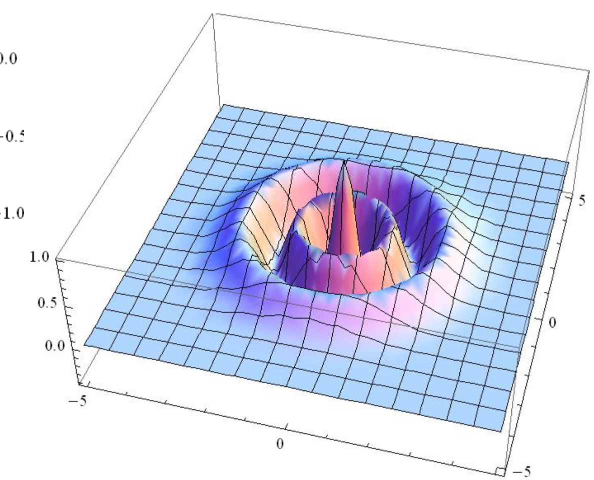
n=1



n=2



n=4



Probability distribution?

No way!

5D imaging of the nucleon:
Wigner distribution in QCD

Ji (2003)
Belitsky, Ji, Yuan (2003)

Wigner distribution of quarks in the nucleon

$$W^{\Gamma}(x,\vec{b}_\perp,\vec{k}_\perp)$$
$$= \int \frac{dz^- d^2 z_\perp}{16\pi^3}\frac{d^2\Delta_\perp}{(2\pi)^2} e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle P+\frac{\Delta}{2}|\bar{\psi}(b-\frac{z}{2})\Gamma\mathcal{L}\psi(b+\frac{z}{2})|P-\frac{\Delta}{2}\rangle$$

\`Mother distribution'

$\vec{b}_\perp$ integral → TMD

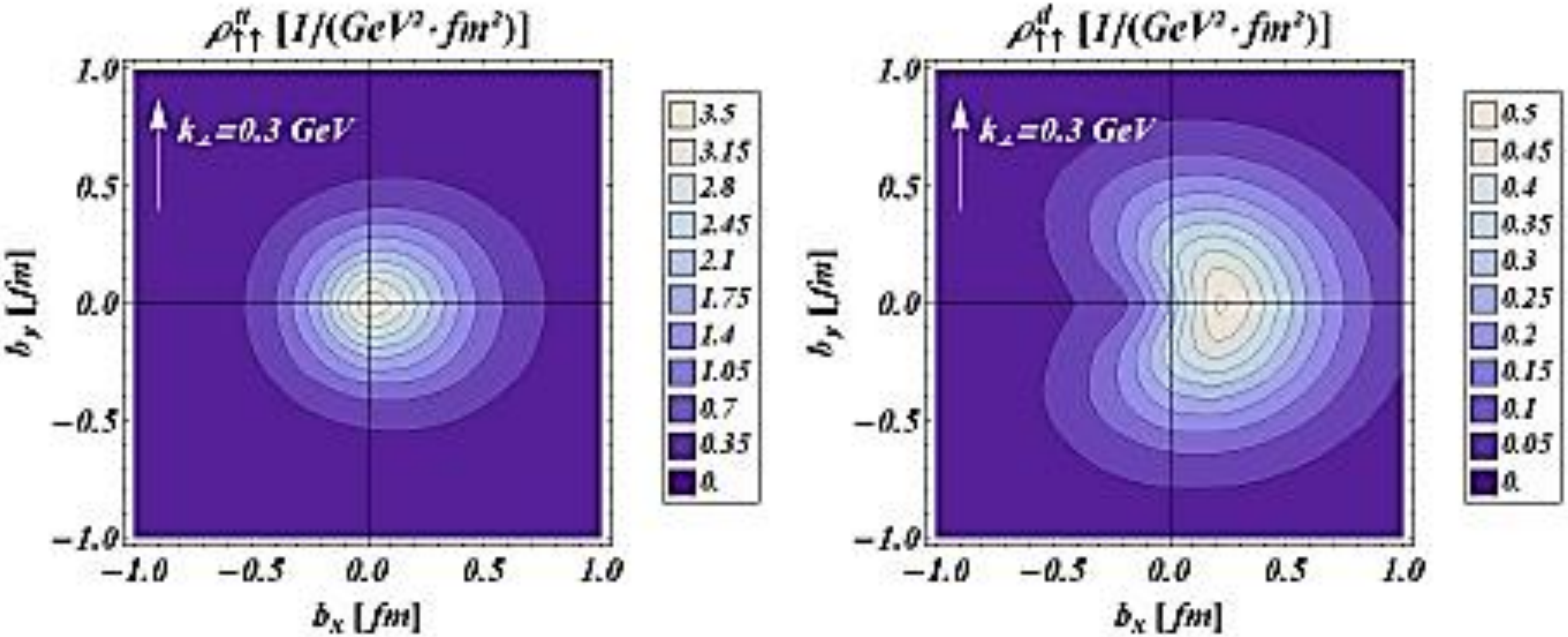
$\vec{k}_\perp$ integral → GPD

$$\Delta^\mu = (0,0,\vec{\Delta}_\perp)$$

momentum recoil
(relativistic effect)

Model calculation

Lorce, Pasquini, (2011)



light-cone quark models
(no gluons included)

Husimi distribution

$$f_H(q,p) = \frac{1}{\pi\hbar}\int dq' dp' e^{-m\omega(q'-q)^2/\hbar-(p'-p)^2/m\omega\hbar} f_W(q',p')$$

Gaussian smearing of the Wigner distribution

$$\Delta q = \sqrt{\hbar/2m\omega} \qquad \Delta p = \sqrt{\hbar m\omega/2}$$

$$\Delta q \Delta p = \hbar/2$$

Positive definite!

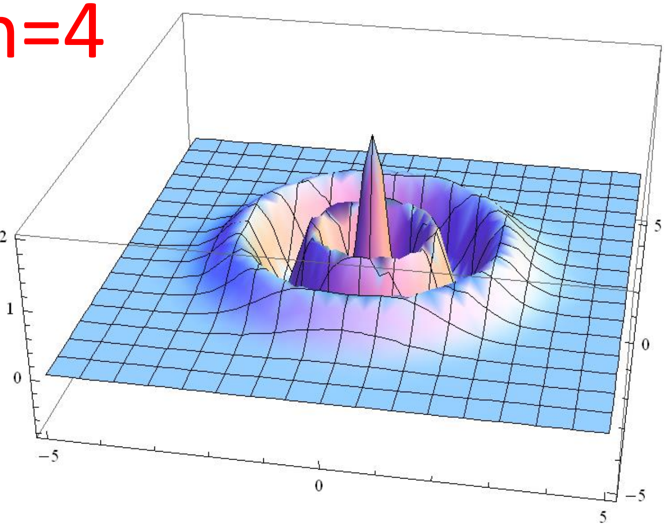
$$f_H(q,p) = \langle\lambda|\hat{\rho}|\lambda\rangle = |\langle\psi|\lambda\rangle|^2 \geq 0$$

$$a|\lambda\rangle = \lambda|\lambda\rangle$$ coherent state

Kodi Husimi (1909ー2008）
伏見康治

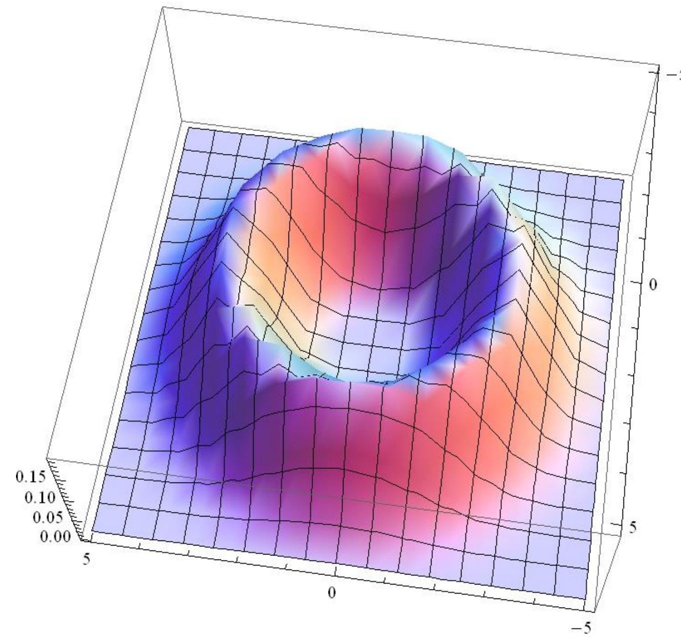
Husimi distribution for the harmonic oscillator

n=4



$$f_W(q,p) = 2(-1)^n e^{-2H/\hbar\omega} L_n\left(\frac{4H}{\hbar\omega}\right)$$

$$f_H(q,p) = \frac{1}{n!} e^{-\frac{H}{\hbar\omega}} \left(\frac{H}{\hbar\omega}\right)^n$$



Localized around the classical orbit

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \approx \hbar\omega(n + \frac{1}{2})$$

Husimi distribution in QCD

Hagiwara and YH, arXiv:1412.4591

Define

$$H^\Gamma(x,\vec{b}_\perp,\vec{k}_\perp) \equiv \frac{1}{\pi^2}\int d^2b'_\perp d^2k'_\perp e^{-\frac{1}{\ell^2}(\vec{b}_\perp-\vec{b}'_\perp)^2-\ell^2(\vec{k}_\perp-\vec{k}'_\perp)^2} W^\Gamma(x,\vec{b}'_\perp,\vec{k}'_\perp)$$

The parameter ℓ is arbitrary, but it is natural to take $\ell \lesssim R_{hadron}$

Positivity?

In the $A^+ = 0$ gauge

$$H \sim \int d^2\Delta_\perp \, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp - \frac{\ell^2\Delta_\perp^2}{4}}$$

$$\times\langle P+\Delta/2|\psi_+^\dagger \delta(K^+ - (1-x)p^+) e^{-\ell^2(\vec{K}_\perp+\vec{k}_\perp)^2}\psi_+|P-\Delta/2\rangle$$

`good component'

Positive definite if it were not for the momentum recoil Δ

However, the Gaussian factor suppresses Δ

Perturbative calculation

Wigner distribution for an on-shell quark

$A^+ = 0$ gauge

$$W(x, \vec{b}_\perp, \vec{k}_\perp)$$
$$= \int \frac{dz^- d^2 z_\perp}{16\pi^3} \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \langle P + \frac{\Delta}{2} | \bar{\psi}(b - \frac{z}{2}) \gamma^+ \mathcal{L} \psi(b + \frac{z}{2}) | P - \frac{\Delta}{2} \rangle$$

Zeroth order

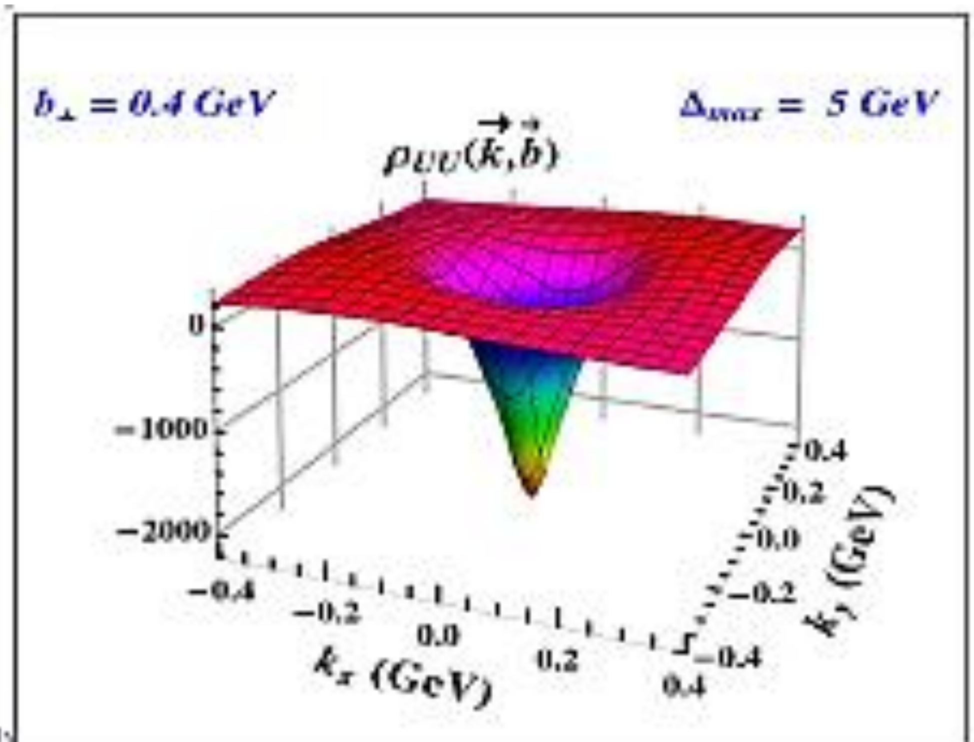
$$W[x, \vec{b}_\perp, \vec{k}_\perp] = \delta(x-1)\delta^{(2)}(\vec{b}_\perp)\delta^{(2)}(\vec{k}_\perp)$$ ⇐ Nonsense

$$\Longrightarrow H(x, \vec{b}_\perp, \vec{k}_\perp) = \delta(1-x) \frac{e^{-b_\perp^2/\ell^2 - \ell^2 k_\perp^2}}{\pi^2}$$ ⇐ Physical

First order in α_s

cf. Mukherjee, Nair, Ojha, 1403.6233

$$W^{\gamma^+}[x,\vec{b}_\perp,\vec{k}_\perp] = \frac{\alpha_s C_F}{2\pi^2}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \times \frac{\left(k_\perp^2 - \frac{\Delta_\perp^2}{4}(1-x)^2\right)P_{qq}(x) + m^2(1-x)^3}{(q_+^2+m^2(1-x)^2)(q_-^2+m^2(1-x)^2)}$$



$$\vec{q}_\pm = \vec{k}_\perp \pm \frac{\vec{\Delta}_\perp}{2}(1-x)$$

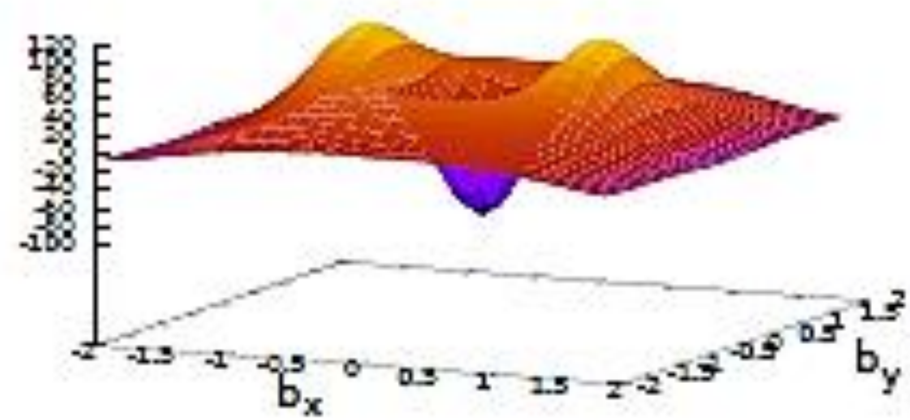
splitting function $P_{qq}(x) = \frac{1+x^2}{1-x}$

Divergent when $\vec{b}_\perp = 0$

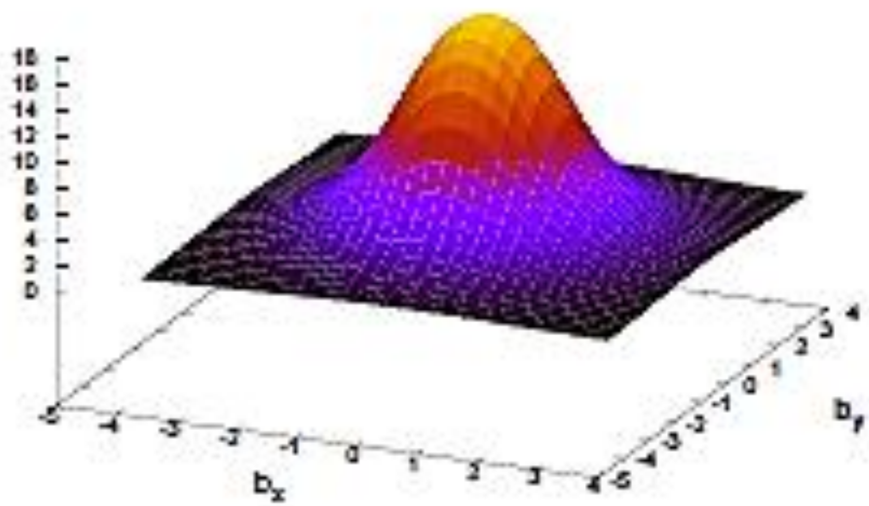
Behaves like crazy

Husimi distribution: Numerical result

Before (Wigner)



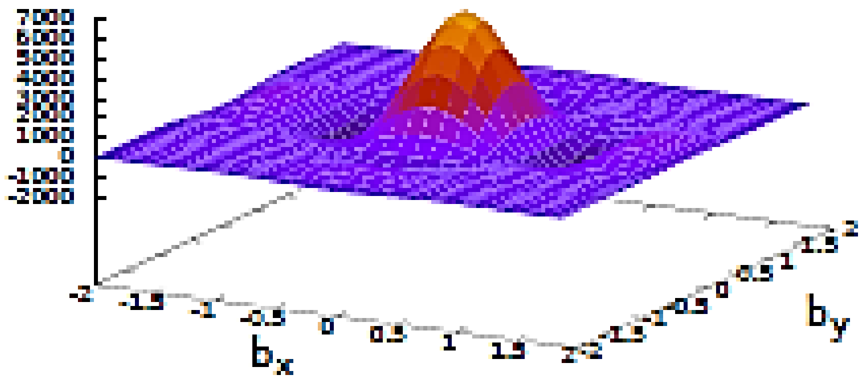
After (Husimi)

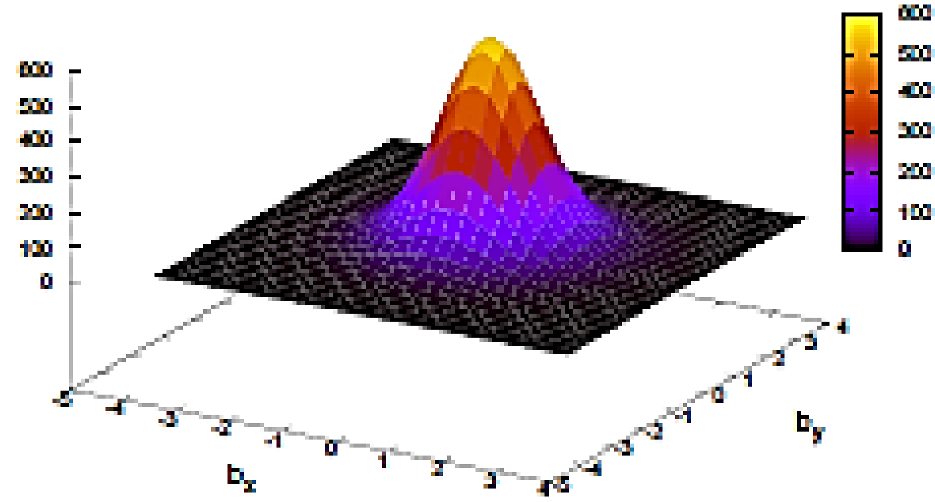


$$x = 0.5,\ m^2 = 0.1\,\mathrm{GeV}^2,\ \ell = 1\,\mathrm{GeV}^{-1}$$

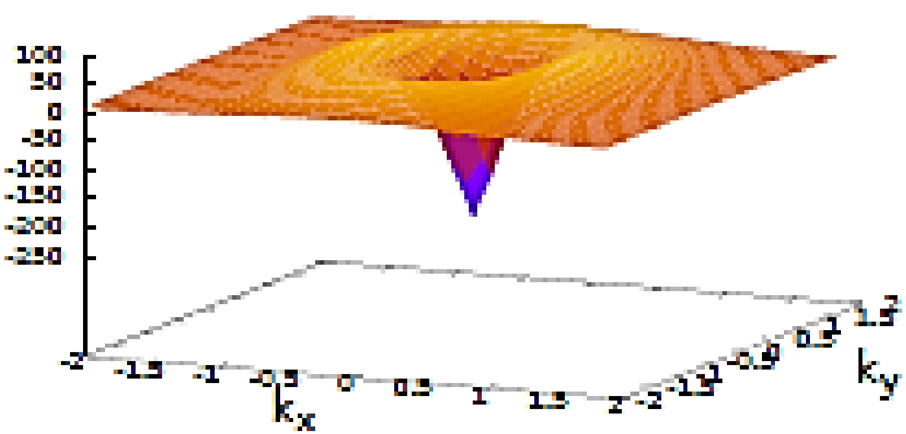
Before (Wigner) | **After (Husimi)**

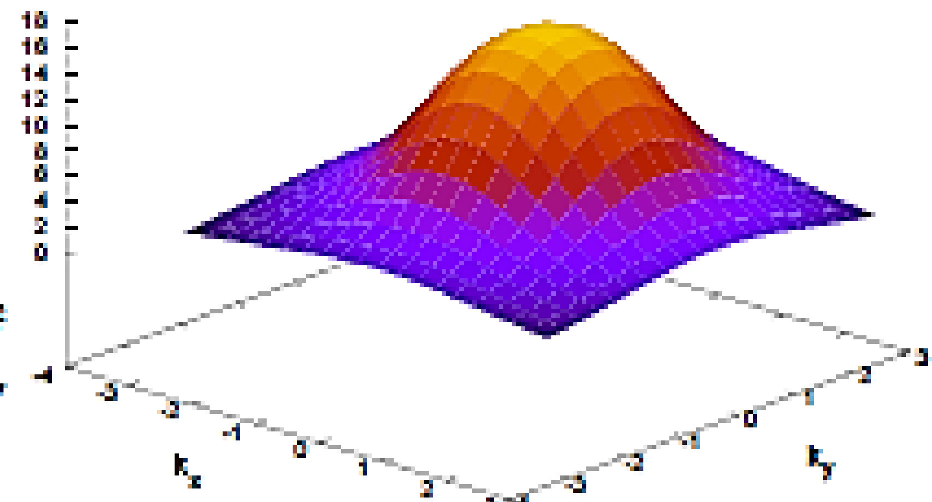
$x = 0.9$





$x = 0.5$



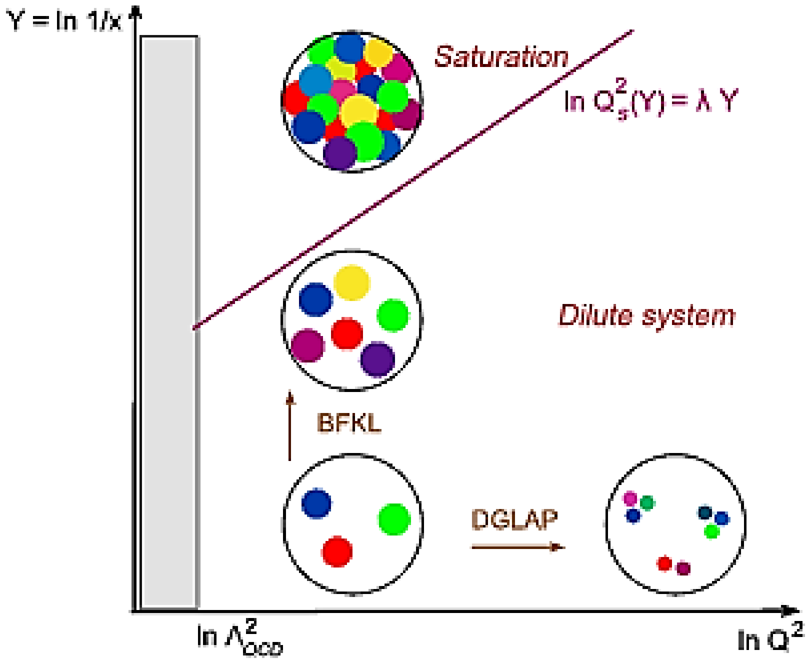


Speculations : relation to Color Glass?

At small-x, the gluons can be treated as a classical coherent state

McLerran, Venugopalan (1993)

Husimi distribution is the coherent state expectation value.



Any relation between the two?

A tantalizing hint

b-moment of the QCD Husimi distribution does not reduce to the TMD.

$$\int d^2b_\perp H^\Gamma(x,\vec{b}_\perp,\vec{k}_\perp)$$
$$= \int \frac{dz^- d^2z_\perp}{16\pi^3} e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)} e^{-\frac{z_\perp^2}{4\ell^2}} \langle P|\bar{\psi}(-z/2)\Gamma\mathcal{L}\psi(z/2)|P\rangle$$

identify $\ell \to \frac{1}{Q_s(x)}$ saturation scale

$$e^{-z_\perp^2/4\ell^2} \to e^{-Q_s^2 z_\perp^2/4}$$ dipole S-matrix

What is computed from classical gluon fields could be interpreted as the Husimi distribution?

Summary of the second part

- Wigner distribution is often unphysical and badly-behaved.
- Husimi distribution much better behaved, can be interpreted as a probability distribution.
- Proof of positivity spoiled by the relativistic kinematical effect, yet a model calculation shows no sign of negative regions.