Spin and Orbital Angular Momenta in Optics and Electromagnetism

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Phys. Rev. A (2010), *New J. Phys.* (2013,2014), *Nat. Commun.* (2014)

Outline

Optical (monochromatic) fields and laboratory experiments

observability and measurements

Quantum (operator) approach and expectation values

spin–orbit interactions

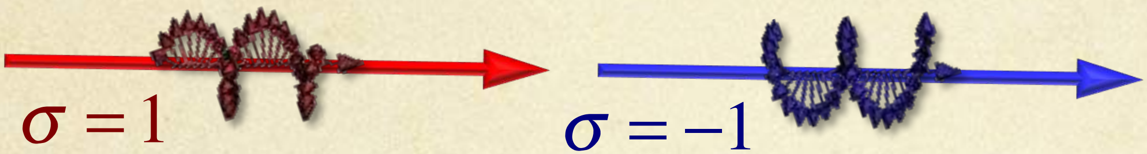
Electromagnetic field theory and local currents

SAM, OAM, helicity, vortices, position, momentum

Spin and orbital AM in optics

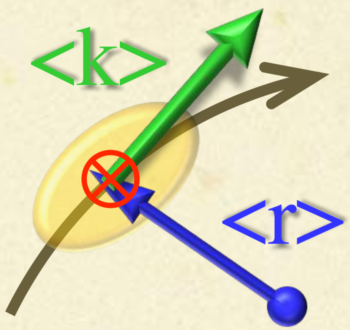
SAM and OAM in paraxial beams

1. Intrinsic spin AM (polarization $\bar{\mathbf{x}} + i\sigma\bar{\mathbf{y}}$)



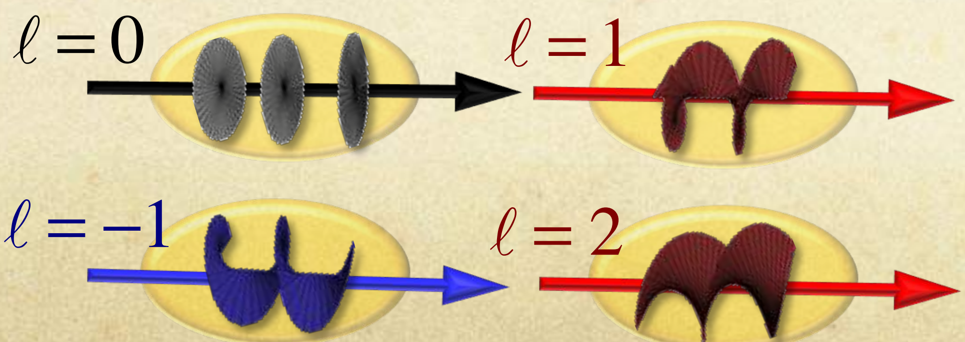
$$\mathbf{S} = \sigma \langle \mathbf{k} \rangle / k$$

2. Extrinsic orbital AM (trajectory)



$$\mathbf{L}_{\text{ext}} = \langle \mathbf{r} \rangle \times \langle \mathbf{k} \rangle$$

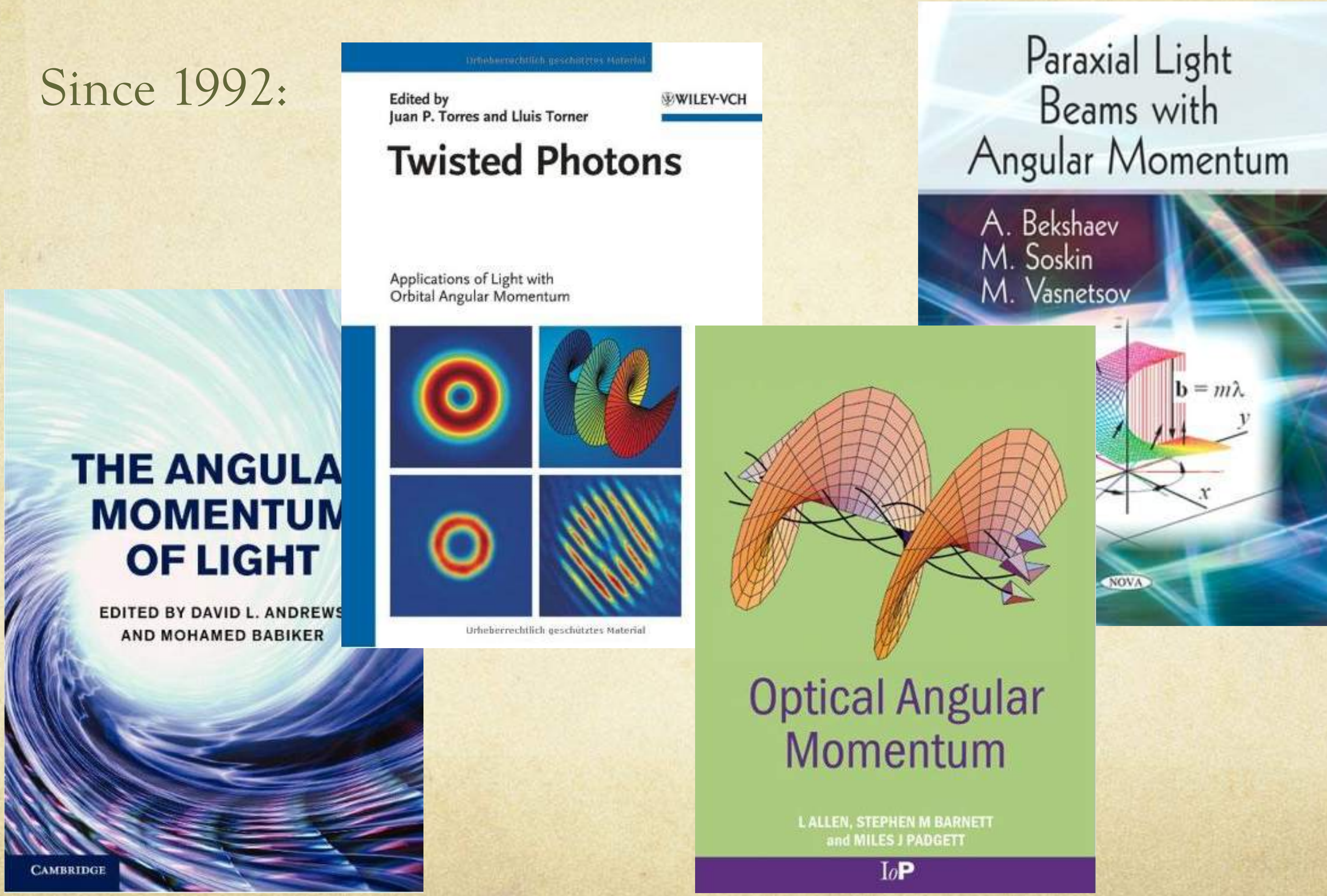
3. Intrinsic orbital AM (vortex $\exp(i\ell\varphi)$)



$$\mathbf{L}_{\text{int}} = \langle \mathbf{r} \times \mathbf{k} \rangle - \mathbf{L}_{\text{ext}} = \ell \langle \mathbf{k} \rangle / k$$

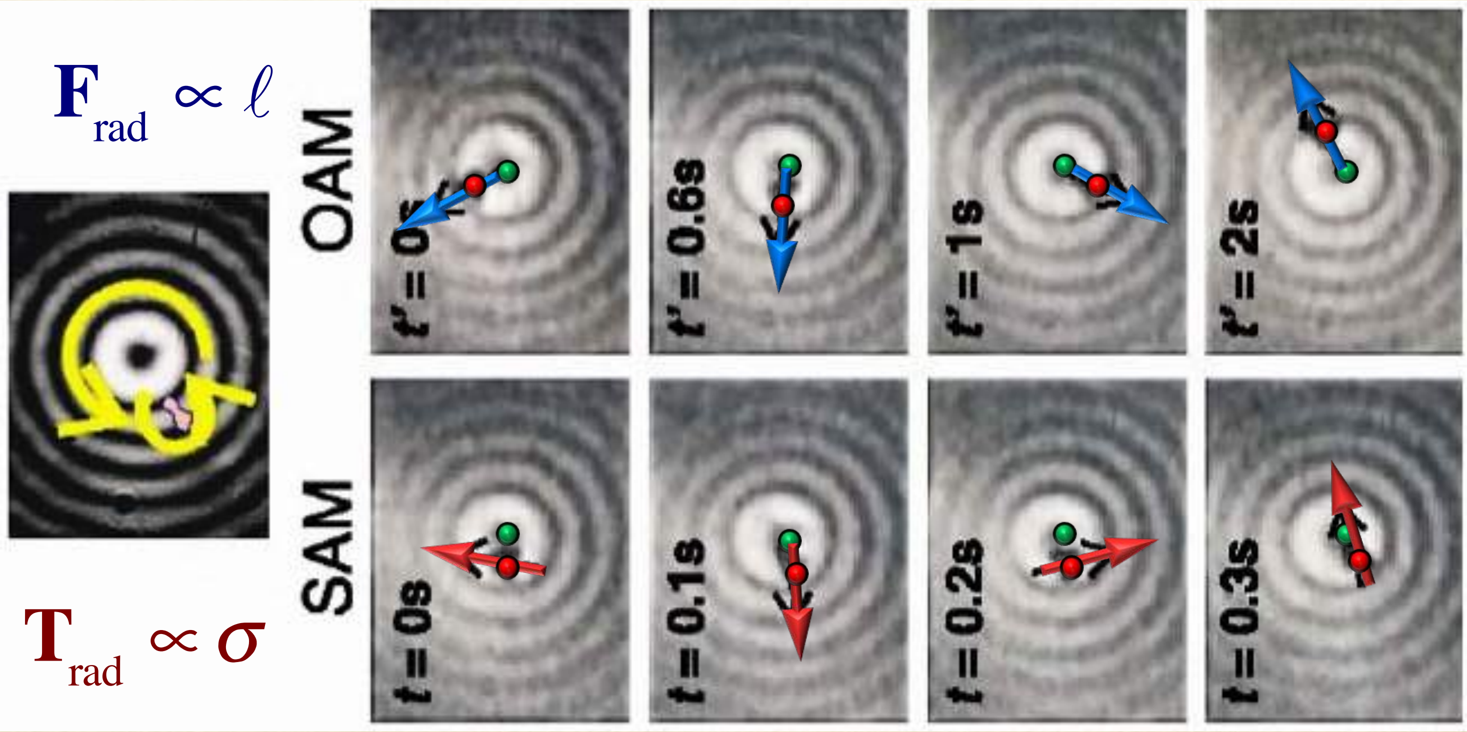
SAM and OAM in paraxial beams

Since 1992:



Observations of the SAM and OAM

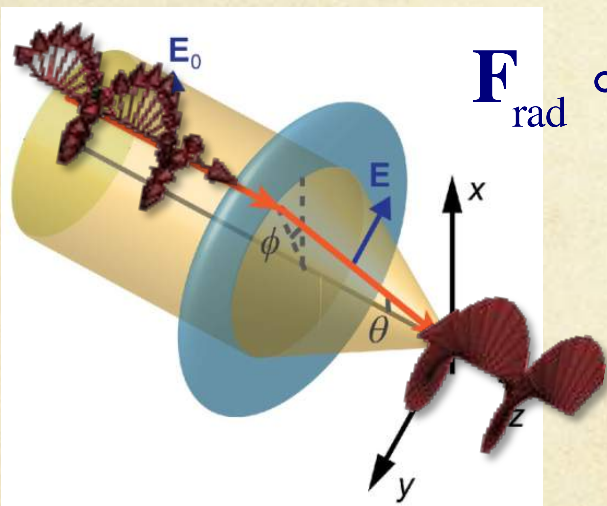
We clearly see different manifestations of the SAM and OAM in paraxial optical beams via spinning and orbital motion of a probe particle, determined by the helicity and vortex quantum numbers:



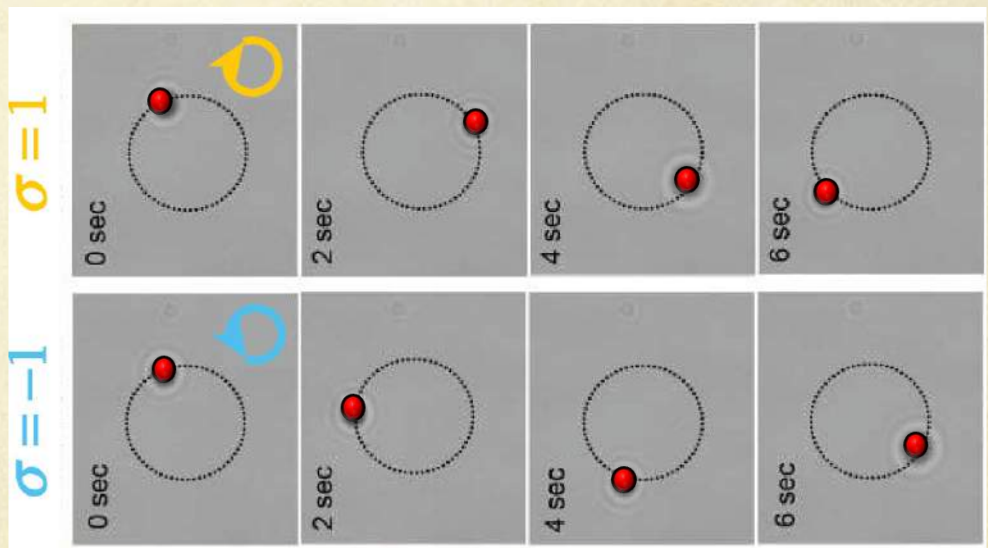
O'Neil *et al.*, PRL (2002); Garces-Chavez *et al.*, PRL (2003)

SAM and OAM in nonparaxial fields

However, in nonparaxial fields, the separation of the SAM and OAM becomes nontrivial. For instance, in a tightly-focused beam, a probe particle shows the presence of a σ–dependent orbital AM:



$$\mathbf{F}_{\mathrm{rad}} \propto \sigma$$



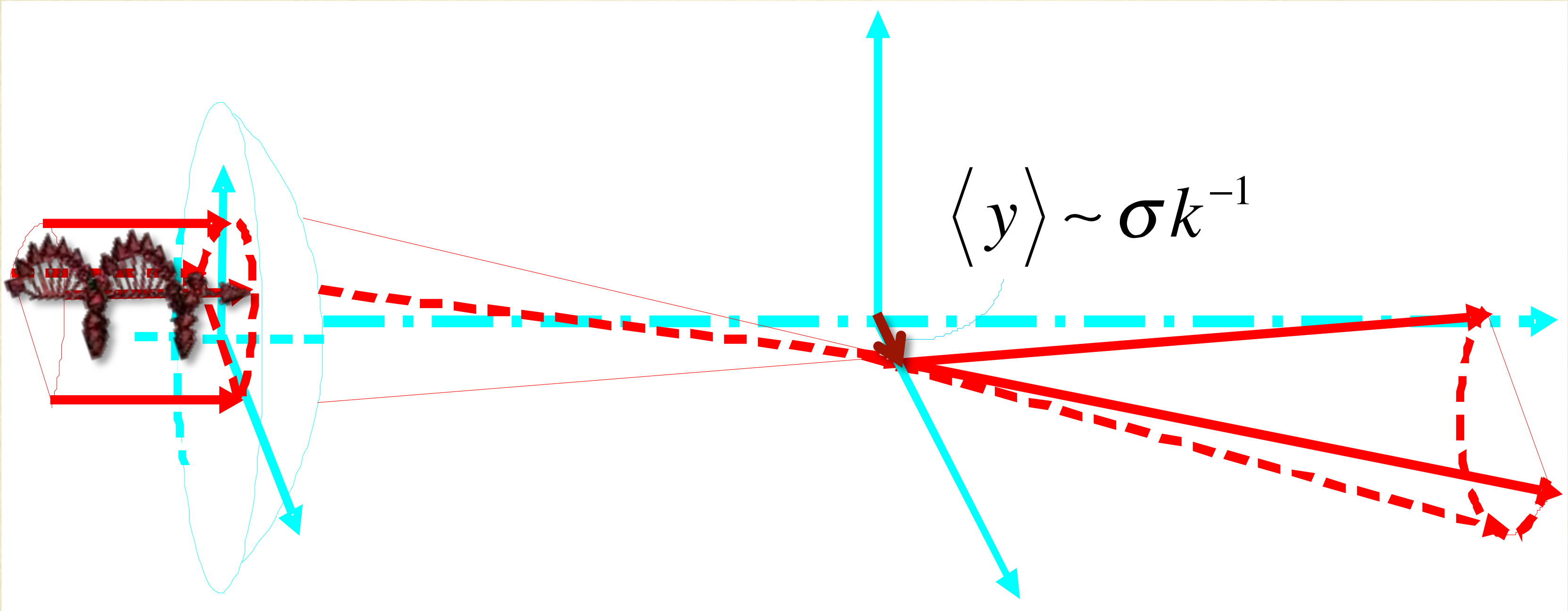
Spin–to–orbital AM conversion

Y. Zhao *et al.*, PRL **99**, 073901 (2007)

SAM and OAM in nonparaxial fields

Furthermore, breaking the cylindrical symmetry reveals nontrivial $\boldsymbol{\sigma}$–dependent positions of light:

$$\phi \in \left(-\pi/2, \pi/2\right)$$



Spin–Hall effect

B. Zel'dovich *et al*., JETP Lett. (1994); K.Y. Bliokh *et al*., PRL (2008)

Quantum approach to SAM and OAM

Quantum approach to SAM and OAM

Akhiezer & Berestetskii, "Quantum Electrodynamics" (1965):

"The separation of the total AM into orbital and spin parts has restricted physical meaning. ... States with definite values of OAM and SAM do not satisfy the condition of transversality in the general case."

Landau & Lifshitz, "Quantum Electrodynamics" (1982):

"Only the total AM of the photon has a meaning. ... It is nevertheless convenient to define 'spin' s and 'orbital AM' l as formal auxiliary quantities."

Barnett & Allen, Opt. Commun. (1994):

"In the general nonparaxial case there is no separation into ℓ–dependent orbital and σ–dependent spin AM"

Canonical SAM and OAM operators

The total AM operator for photon represents a sum of the OAM and SAM operators:

$$\hat{\mathbf{M}} = \hat{\mathbf{r}} \times \hat{\mathbf{p}} + \hat{\mathbf{S}} \equiv \hat{\mathbf{L}} + \hat{\mathbf{S}}$$

$$\hat{\mathbf{r}} = i\partial_{\mathbf{k}}, \quad \hat{\mathbf{p}} = \mathbf{k},$$

$$\left(\hat{S}_a\right)_{ij} = -i\varepsilon_{aij}$$

z-components and their paraxial eigenmodes:

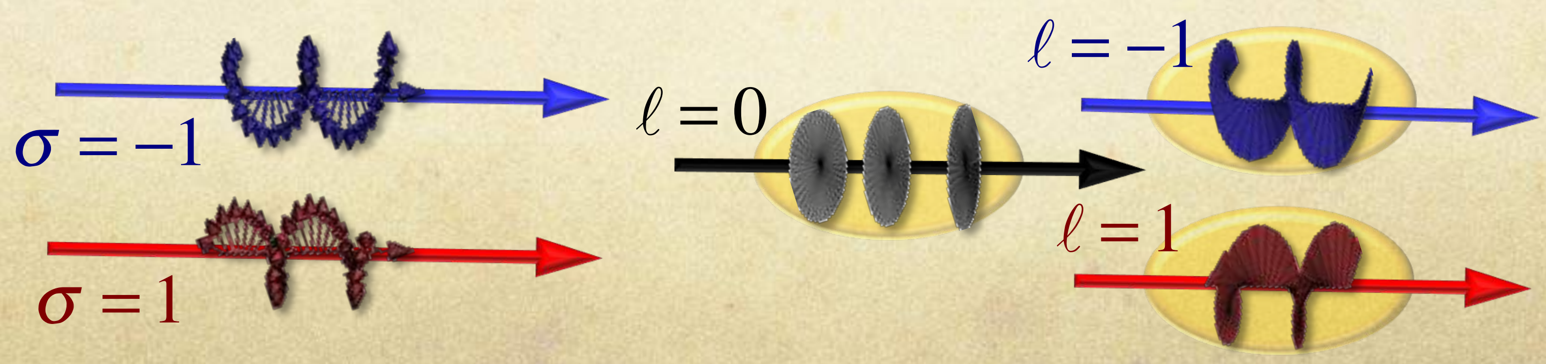
$$\hat{L}_z = -i\partial_\phi,$$

$$\left(\hat{S}_z\right)_{ij} = -i\varepsilon_{zij}$$

$$\mathbf{E}_{\ell\sigma} \propto \left(\bar{\mathbf{x}} + i\sigma\bar{\mathbf{y}}\right)e^{i\ell\phi}$$

$$\sigma = \pm 1$$

$$\ell = 0, \pm 1, \pm 2, \ldots$$

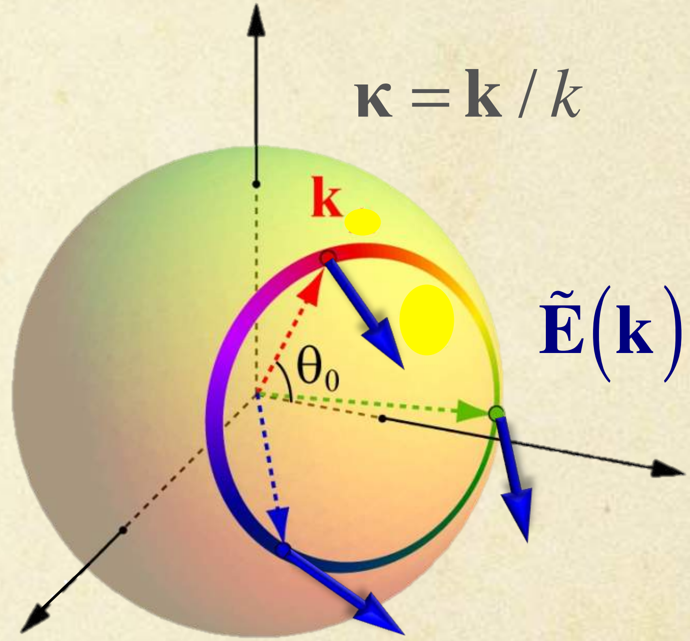


Canonical SAM and OAM operators

In a nonparaxial case, instead of the full 3D geometry, Maxwell fields are subjects to the transversality constraint:

$$\tilde{\mathbf{E}}(\mathbf{k})\cdot\mathbf{k}=0$$

It brings about a 2D spherical geometry in **k**–space (cf. Berry phase):



The SAM and OAM operators conflict with the transversality:

$$\hat{\mathbf{L}}\tilde{\mathbf{E}}\not\perp\boldsymbol{\kappa},\quad \hat{\mathbf{S}}\tilde{\mathbf{E}}\not\perp\boldsymbol{\kappa}\quad \text{although}\quad \hat{\mathbf{M}}\tilde{\mathbf{E}}\perp\boldsymbol{\kappa}$$

Canonical SAM and OAM operators

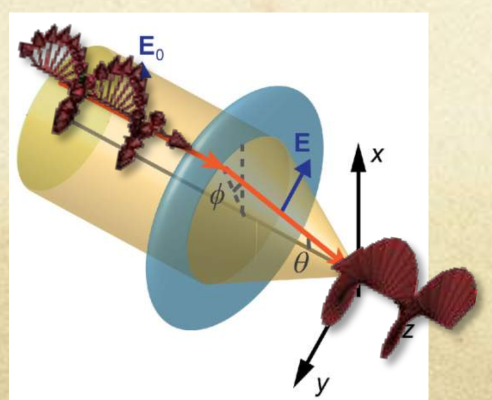
Furthermore, if we construct nonparaxial analogues of the circularly-polarized vortex beams:

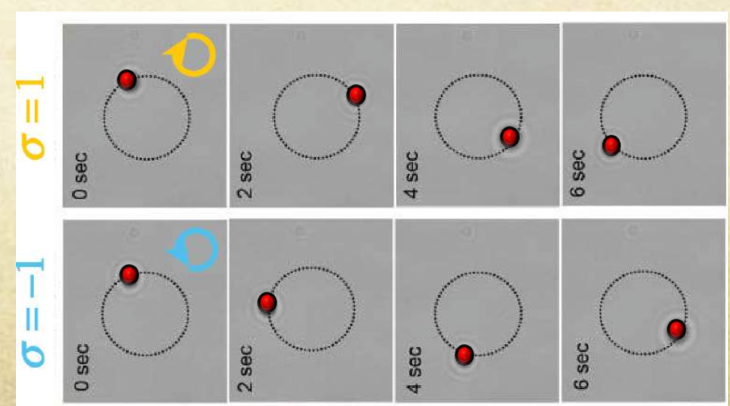
$$\tilde{\mathbf{E}}_{\ell\sigma} \propto \left(\mathbf{e}_\theta + i\sigma\mathbf{e}_\phi\right)e^{i\sigma\phi}e^{i\ell\phi} \equiv \mathbf{e}^\sigma\left(\boldsymbol{\kappa}\right)e^{i\ell\phi},$$

then the SAM and OAM expectation values yield

$$\left\langle L_z \right\rangle = \ell + \gamma\sigma, \quad \left\langle S_z \right\rangle = \left(1-\gamma\right)\sigma \qquad \gamma \propto \theta^2$$

This resembles spin-to-orbital AM conversion:





Modified SAM and OAM operators

To resolve the conflict of the SAM and OAM operators with the transversality, we suggest the modified separation of the SAM and OAM, $\hat{\mathbf{M}} = \hat{\mathbf{L}}' + \hat{\mathbf{S}}'$:

$$\hat{\mathbf{S}}' = \hat{\mathbf{S}} - \hat{\boldsymbol{\Delta}} = \boldsymbol{\kappa}\left(\boldsymbol{\kappa}\cdot\hat{\mathbf{S}}\right) \equiv \boldsymbol{\kappa}\hat{\sigma}$$

$$\hat{\mathbf{L}}' = \hat{\mathbf{L}} + \hat{\boldsymbol{\Delta}} = \hat{\mathbf{r}}' \times \mathbf{k}$$

$$\hat{\boldsymbol{\Delta}} = -\boldsymbol{\kappa}\times\left(\boldsymbol{\kappa}\times\hat{\mathbf{S}}\right)$$

The modified SAM has the natural helicity form, while the modified OAM reveals the Pryce position operator:

$$\hat{\mathbf{r}}' = \hat{\mathbf{r}} + \left(\mathbf{k}\times\hat{\mathbf{S}}\right) / k^2$$

cf. M.H.L. Pryce, *PRSLA* (1948)

K.Y. Bliokh *et al.*, PRA **82**, 063825 (2010)

Modified SAM and OAM operators

These modified operators $\hat{\mathbf{L}}', \hat{\mathbf{S}}', \hat{\mathbf{r}}'$ represent projections of the canonical operators onto the transversality subspace.

1. They are consistent with the transversality:

$$\hat{\mathbf{L}}'\tilde{\mathbf{E}} \perp \boldsymbol{\kappa}, \quad \hat{\mathbf{S}}'\tilde{\mathbf{E}} \perp \boldsymbol{\kappa}, \quad \hat{\mathbf{r}}'\tilde{\mathbf{E}} \perp \boldsymbol{\kappa}$$

2. They have the same expectation values:

$$\left\langle \mathbf{L}' \right\rangle = \left\langle \mathbf{L} \right\rangle, \quad \left\langle \mathbf{S}' \right\rangle = \left\langle \mathbf{S} \right\rangle, \quad \left\langle \mathbf{r}' \right\rangle = \left\langle \mathbf{r} \right\rangle$$

3. They illuminate observable spin-orbit interaction phenomena: Berry phase, spin-to-orbital AM conversion, and spin-Hall effect (shown below).

K.Y. Bliokh *et al.*, PRA **82**, 063825 (2010)

Modified SAM and OAM operators

4. The modified operators possess "strange" commutation relations:

$$\left[\hat{S}'_i, \hat{S}'_j\right] = 0, \qquad \left[\hat{L}'_i, \hat{L}'_j\right] = i\varepsilon_{ijl}\left(\hat{L}'_l - \hat{S}'_l\right),$$

$$\left[\hat{L}'_i, \hat{S}'_j\right] = i\varepsilon_{ijl}\hat{S}'_l, \qquad \left[\hat{r}'_i, \hat{r}'_j\right] = -i\varepsilon_{ijl}\hat{\sigma}\, k_l / k^3$$

cf. van Enk & Nienhuis, JMO (1994); Pryce (1948); Bialynicki-Birula, PRD (1987); Skagerstam (1992); Berard & Mohrbach, PLA (2006)

But they all transformed as vectors: $\left[\hat{M}_i, \hat{O}'_j\right] = i\varepsilon_{ijl}\hat{O}'_l$

Modified commutation relations correspond to rotations/translation "restricted" by the transversality.

S. Barnett, JMO (2010); I. Fernandez–Corbaton et al. (2013)

Modified SAM and OAM operators

Remakably, all modified operators are diagonalized in the helicity basis:

$$\hat{U}(\boldsymbol{\kappa}) = \hat{R}_z(-\phi)\hat{R}_y(-\theta)\hat{R}_z(\phi) : (\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z) \to (\mathbf{e}^+, \mathbf{e}^-, \boldsymbol{\kappa})$$

$$\hat{\mathbf{O}} \to \hat{U}^\dagger \hat{\mathbf{O}} \hat{U}$$

$$\hat{\mathbf{S}}' = \boldsymbol{\kappa}\hat{\sigma}, \quad \hat{\mathbf{L}}' = \hat{\mathbf{L}} - \hat{\sigma}\boldsymbol{\mathcal{A}} \times \mathbf{k}, \quad \hat{\mathbf{r}}' = \hat{\mathbf{r}} - \hat{\sigma}\boldsymbol{\mathcal{A}}$$

$$\hat{\sigma} = \mathrm{diag}(1, -1, 0)$$

$$\boldsymbol{\mathcal{A}} = \frac{1 - \cos\theta}{k \sin\theta}\bar{\boldsymbol{\phi}}$$

$$\boldsymbol{\mathcal{F}} = \partial_{\mathbf{k}} \times \boldsymbol{\mathcal{A}} = \frac{\mathbf{k}}{k^3}$$

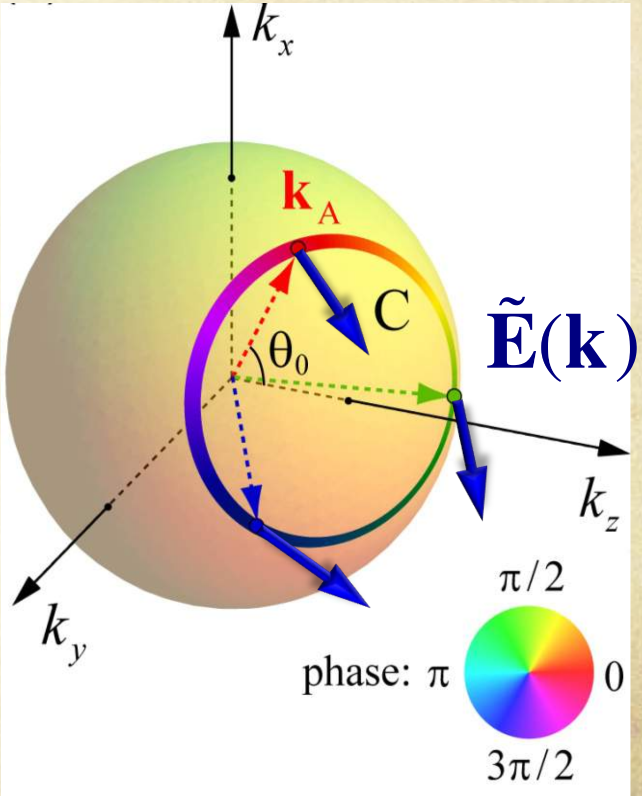
– Berry connection and curvature describing parallel transport of the electric field on the **k**–space sphere.

Application to Bessel beams

We now apply these general results to an example of a nonparaxial electromagnetic field – vector Bessel beams.

They have a very simple spectrum, a circle in **k**–space, and we assume that all waves have the same helicity σ:

$$\tilde{E}_\ell^\sigma \propto \delta\left(\theta - \theta_0\right) e^{i\ell\phi}$$



Real–space field shows nonparaxial σ–dependent corrections:

$$I_\ell^\sigma\left(\rho\right) \propto a^2 J_\ell^2\left(\tilde{\rho}\right) + b^2 J_{\ell+2\sigma}^2\left(\tilde{\rho}\right) + 2ab J_{\ell+\sigma}^2\left(\tilde{\rho}\right)$$

$$b \propto \theta_0^2$$

Application to Bessel beams

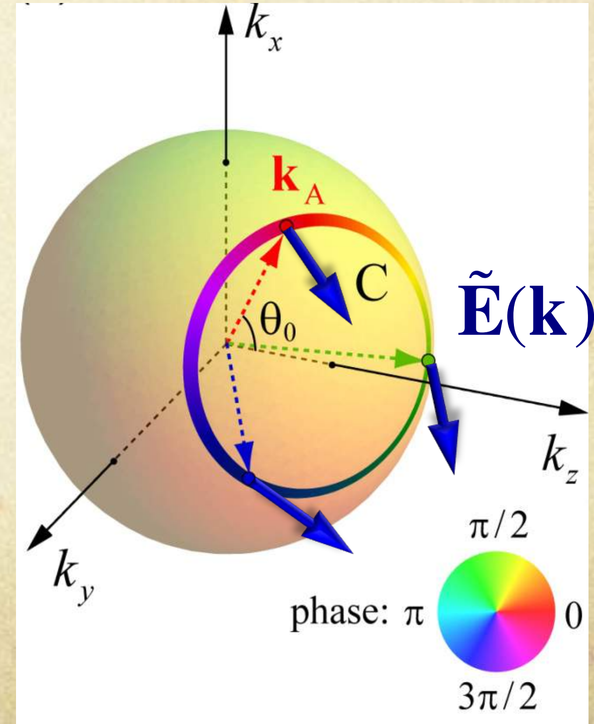
Calculating the OAM and SAM expectation values in the Bessel beam, we obtain:

$$\langle L_z \rangle = \ell + \sigma \Phi_B \,, \quad \langle S_z \rangle = \sigma \left(1 - \Phi_B\right)$$

$$\Phi_B \equiv \frac{\Phi_B}{2\pi} = \frac{1}{2\pi}\oint_C \boldsymbol{\mathcal{A}} \cdot d\mathbf{k} = 1 - \cos\theta_0$$

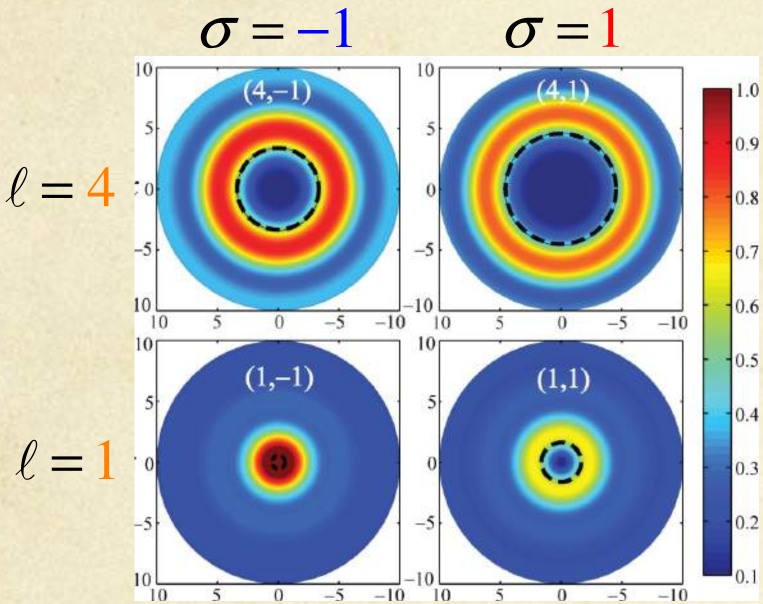
– Berry phase !

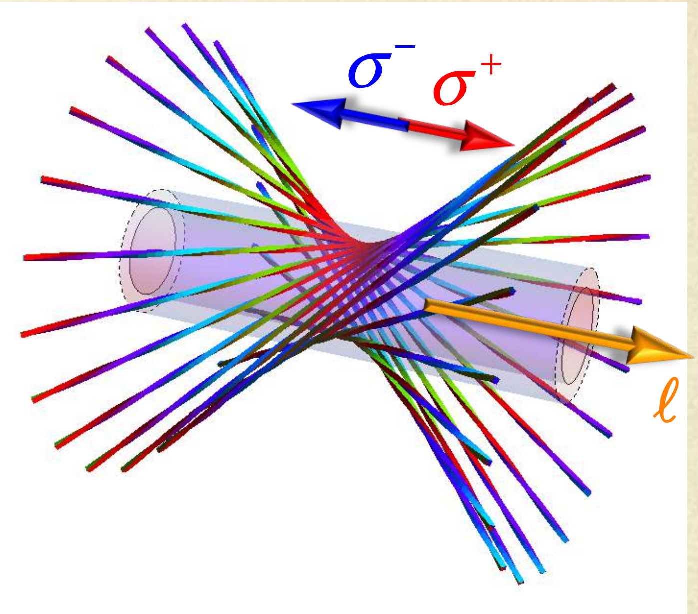
Thus, the spin–to–orbit AM conversion in nonparaxial fields originates from the Berry phase associated with the azimuthal distribution of partial waves.



Application to Bessel beams

The transverse real-space intensity distributions show σ-dependent radii:





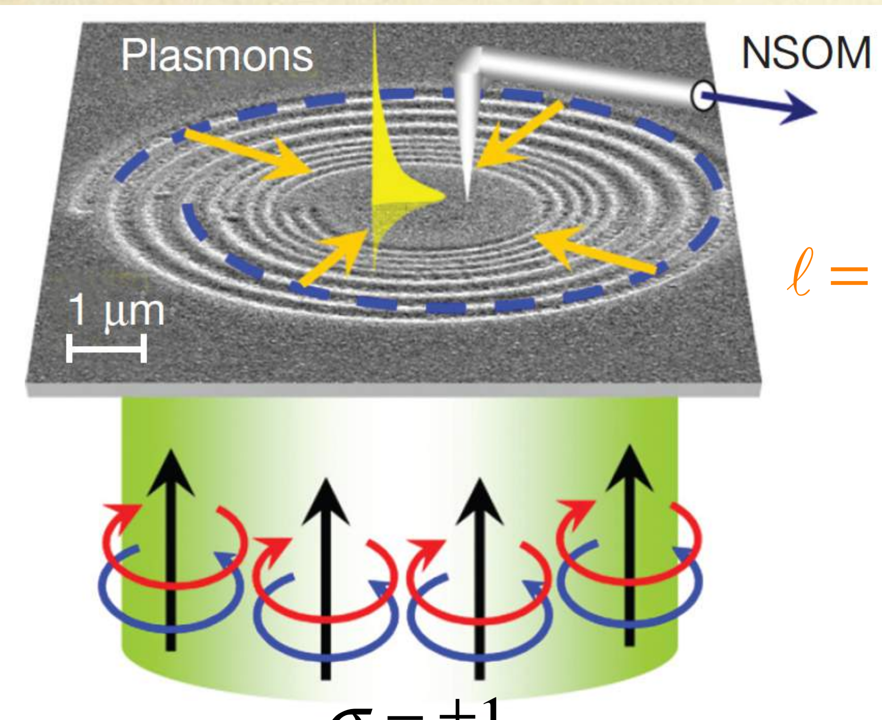
The beam radius is determined by the OAM value and quantization (with Berry phase) and fine SOI splitting of the caustic:

$$k_\perp R_\ell^\sigma = \left| \ell + \sigma \Phi \right| = \left| \langle L_z \rangle \right|$$

Application to Bessel beams

The σ–dependent radius and OAM was demonstrated in a circular plasmonic cavity generating Bessel modes:

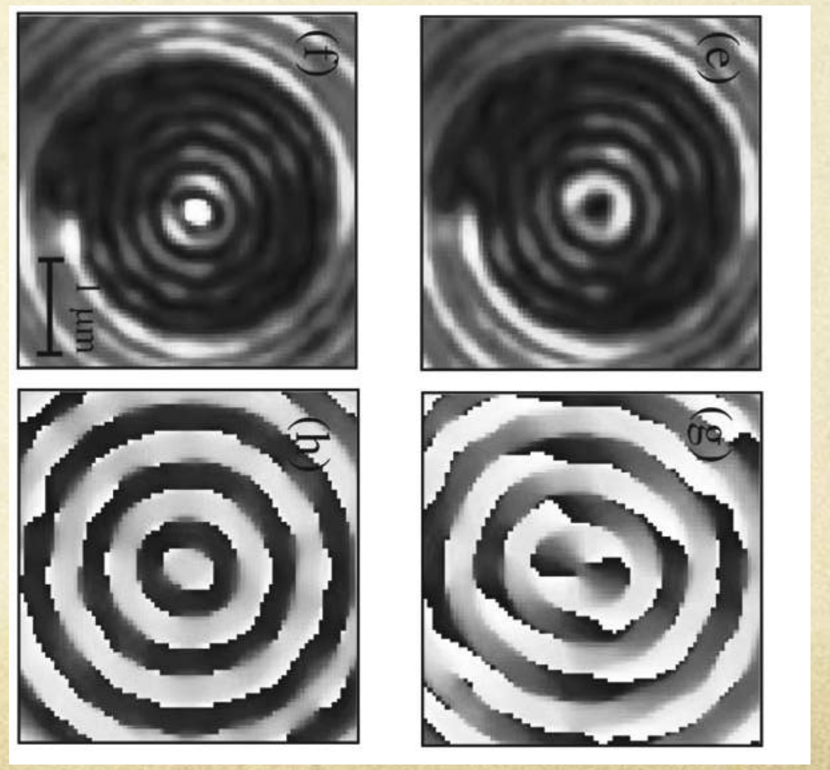
$$\theta_0 = \pi / 2\ ,\ \Phi_B = 2\pi \qquad \ell + \sigma\Phi_B = \ell + \sigma$$



$\sigma = \pm 1$

$\ell = 1$

$\sigma = -1$ $\qquad$ $\sigma = 1$

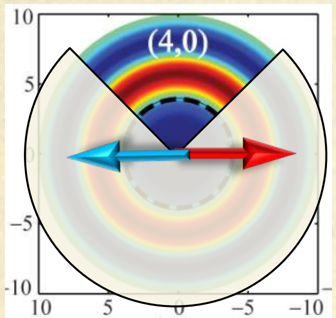


Y. Gorodetski *et al.*, PRL (2008) [cf. QHE in graphene].

Application to Bessel beams

Breaking the cylindrical symmetry unveils the σ–dependent position of light, i.e., the spin–Hall effect:

$$\phi \in (-\delta, \delta)$$

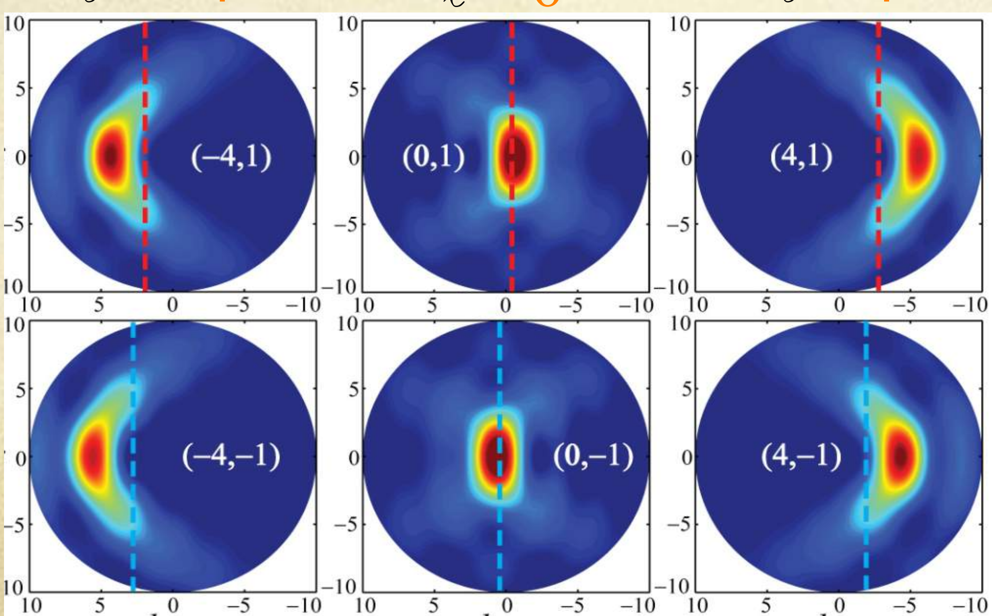


$$\langle y \rangle \propto k_{\perp}^{-1} \left(\ell + \sigma \Phi_B \right)$$

$\ell = -4$ $\quad$ $\ell = 0$ $\quad$ $\ell = 4$

$\sigma = 1$

$\sigma = -1$

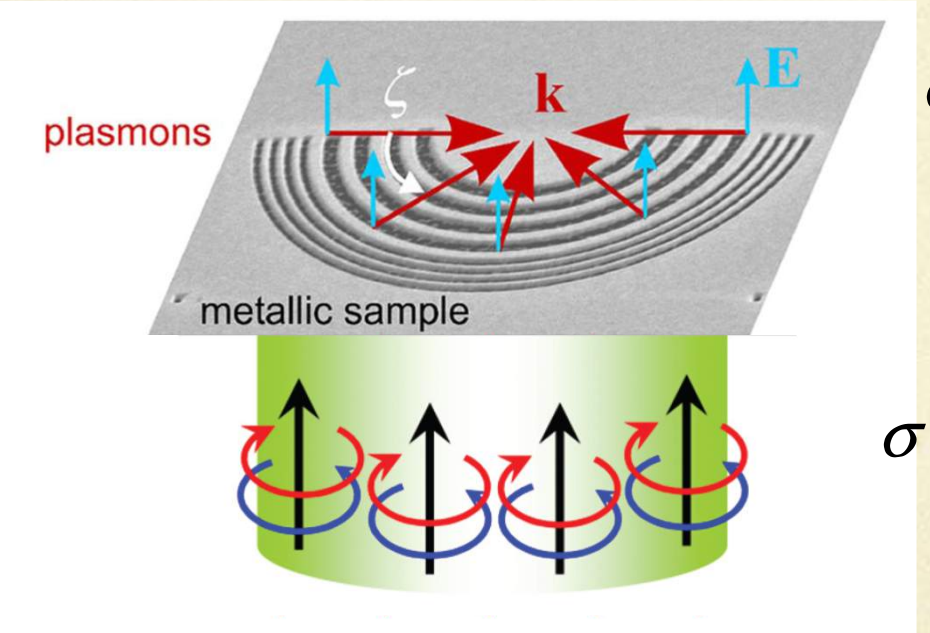


B. Zel'dovich *et al.* (1994)
K.Y. Bliokh *et al.* (2008)
K.Y. Bliokh *et al.* (2010)

Application to Bessel beams

Such spin Hall effect was observed in a plasmonic semicircular lens:

$\theta_0 = \pi / 2\ ,\quad \Phi_B = 2\pi$

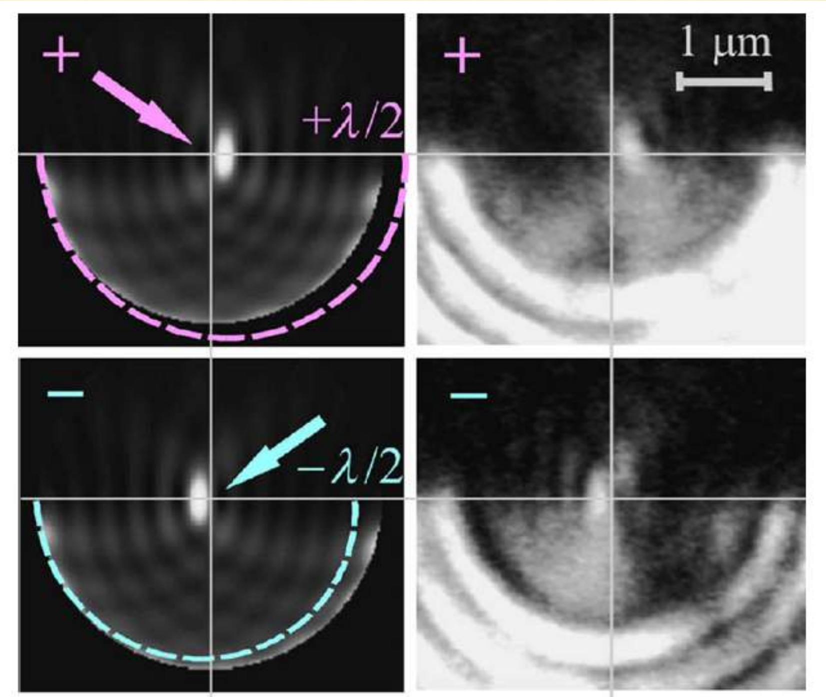


$\phi \in \left(-\pi / 2, \pi / 2\right)$

$\ell = 0$

$\sigma = 1$

$\sigma = -1$



$\langle y \rangle \sim \sigma k^{-1}$

K.Y. Bliokh *et al.*, PRL (2008)

Summary for quantum approach

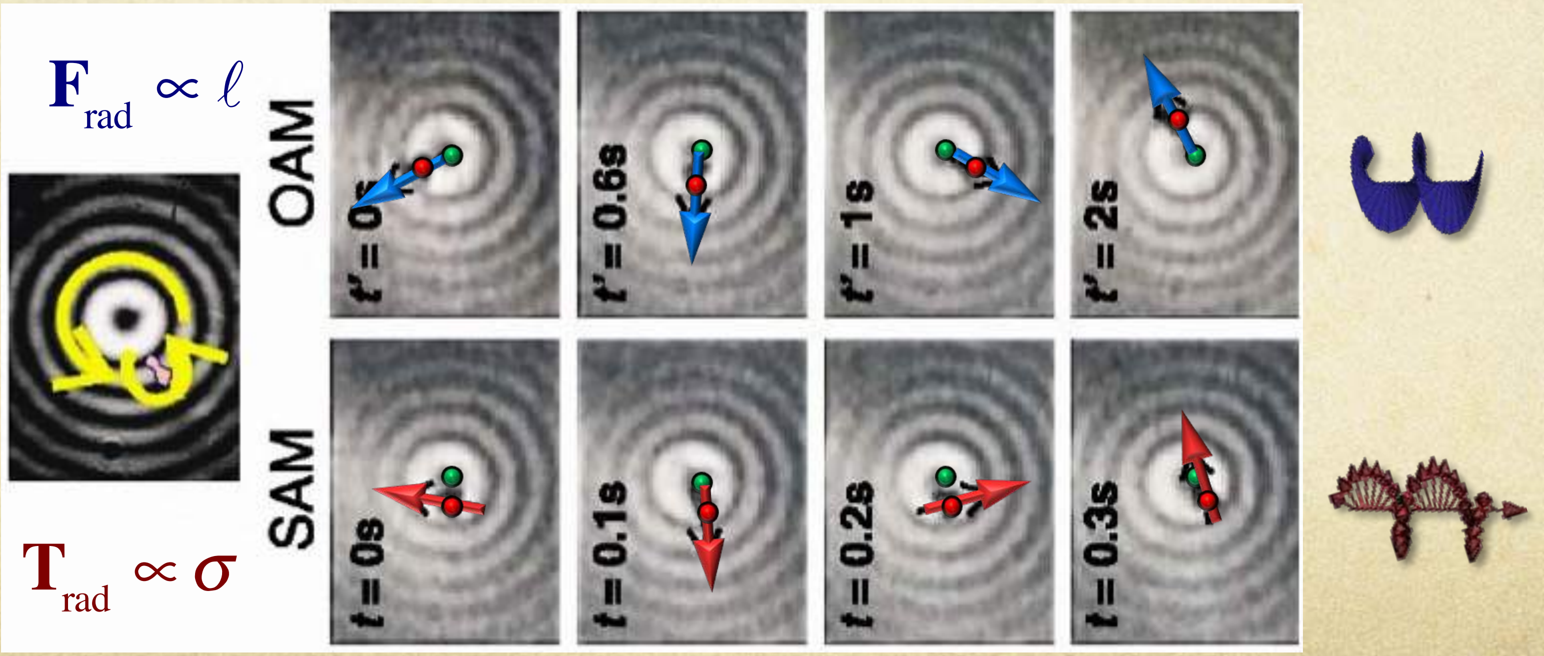
- All difficulties with the transversality and canonical SAM and OAM operators can be overcomed.
- Spin and orbital AM are meaningful and separately measurable properties of light.
- They exhibit nontrivial spin–orbit interaction features (stemming from the tranversality): helicity–dependent OAM and position.

S.J. Van Enk and G. Nienhuis, *J. Mod. Opt.* **41**, 963 (1994)
K.Y. Bliokh *et al.*, *Phys. Rev. A* **82**, 063825 (2010)
S.M. Barnett, *J. Mod. Opt.* **57**, 1339 (2010)
I. Bialynicky–Birula & Z. Bialynicky–Birula, *J. Opt.* **13**, 064014 (2011)
I. Fernandez–Corbaton *et al.*, arXiv:1308.1729 (2013)
K.Y. Bliokh *et al.*, *New J. Phys.* (2014, in press)

Field–theory approach to SAM and OAM

Field theory approach

Quantum approach describes the SAM and OAM in the **k**–space or via their integral expectation values. However, in optics we can measure the SAM and OAM locally in real space. To describe the spin and orbital densities and currents, we have to use the field theory.



Canonical Noether currents

Poincaré symmetries and electromagnetic free-space Langrangian lead to the canonical Noether currents:

$$\partial_\beta T^{\alpha\beta} = 0, \quad T^{\alpha 0} = (W, \mathbf{P}) \qquad T^{\alpha\beta} \neq T^{\beta\alpha}$$

– Canonical stress–energy tensor (nonsymmetric)

$$\partial_\gamma M^{\alpha\beta\gamma} = 0, \qquad \mathbf{M} = \mathbf{r} \times \mathbf{P} + \mathbf{S} \equiv \mathbf{L} + \mathbf{S}$$

$$M^{\alpha\beta\gamma} = r^\alpha T^{\beta\gamma} - r^\beta T^{\alpha\gamma} + S^{\alpha\beta\gamma} \equiv L^{\alpha\beta\gamma} + S^{\alpha\beta\gamma}$$

$$\partial_\gamma S^{\alpha\beta\gamma} = -\partial_\gamma L^{\alpha\beta\gamma} = T^{\alpha\beta} - T^{\beta\alpha} \neq 0$$

– Canonical AM tensor: OAM + SAM parts (nonconserved)

Canonical Noether currents

Explicitly:

$$\mathbf{P} = \mathbf{E} \cdot \left(\nabla\right) \mathbf{A}$$

– Canonical momentum density

$$\mathbf{L} = \mathbf{r} \times \mathbf{P} = \mathbf{E} \cdot \left(\mathbf{r} \times \nabla\right) \mathbf{A}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{A}$$

– OAM and SAM densities

These quantities are gauge–dependent, and, therefore, physically meaningless in traditional field theory:

Not unique, nonobservable, only gravitation matters, ...

Any field–theory textbook or expert: S. Deser, I. Bialynicky-Birula, F. Hehl

Belinfante symmetrized currents

Belinfante's symmetrization procedure results in gauge-invariant and properly symmetric tensors:

$$\mathcal{T}^{\alpha\beta} = T^{\alpha\beta} + \partial_\gamma K^{\alpha\beta\gamma}, \quad \mathcal{T}^{\alpha 0} = \left(W, \boldsymbol{\mathcal{P}}\right) \quad \mathcal{T}^{\alpha\beta} = \mathcal{T}^{\beta\alpha}$$

$$\boldsymbol{\mathcal{P}} = \mathbf{E}\cdot\left(\nabla\right)\mathbf{A} - \left(\mathbf{E}\cdot\nabla\right)\mathbf{A} = \mathbf{E}\times\mathbf{B}$$

Poynting vector = canonical momentum + spin momentum

$$\mathcal{M}^{\alpha\beta\gamma} = r^\alpha \mathcal{T}^{\beta\gamma} - r^\beta \mathcal{T}^{\alpha\gamma}, \quad \boldsymbol{\mathcal{M}} = \mathbf{r}\times\boldsymbol{\mathcal{P}}$$

This total AM density includes both orbital and spin parts (plane-wave paradox) but they are nonseparable.

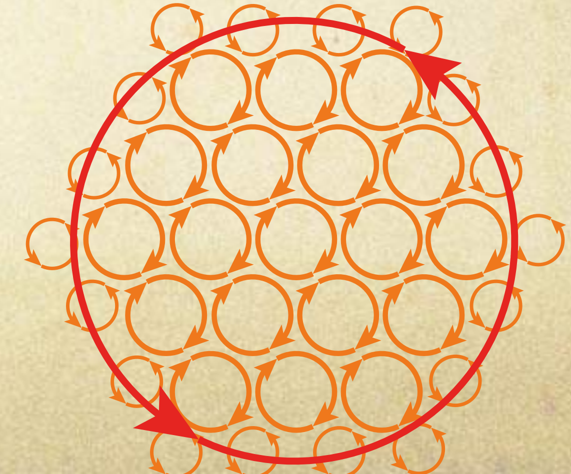
Belinfante (1940), Rosenfeld (1940)

Belinfante symmetrized currents

Belinfante's procedure 'improves' stress–energy tensor but eliminates one degree of freedom: spin.

Instead of the SAM density it introduces an enigmatic spin momentum current. This is an analgoue of the boundary magnetization current, which does not transport energy and is not observable *per se*.

$$\int \mathbf{S}\, d^3 r = \int \mathbf{r} \times \mathbf{P}_S\, d^3 r, \quad \nabla \cdot \mathbf{P}_S = 0$$



What is spin?

Hans C Ohanian

Virtual probability current associated with the spin

Katsunori Mita

Am. J. Phys. (1986, 2000)

Gauge invariance issue

To make the canonical spin and orbital quantities gauge–invariant, typically the transverse part of the vector–potential is assumed (= Coulomb gauge):

$$\mathbf{A} \to \mathbf{A}_{\perp}, \quad \nabla \cdot \mathbf{A} = 0$$

But the transverse vector–potential is nonlocal:

$$\mathbf{A}(\mathbf{r}) \propto \int \frac{\nabla' \times \mathbf{B}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'$$

However, it becomes local and meaningful in the most important case of monochromatic optical fields:

$$\boxed{\mathbf{A}(\mathbf{r}) \propto -i\omega \mathbf{E}(\mathbf{r})} \qquad \mathbf{O}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{O}(\mathbf{r})e^{-i\omega t}\right]$$

Application to optical fields

Thus, the gauge–invariant local momentum, OAM, and SAM densities follow from the canonical Noether tensors for monochromatic fields:

$$\mathbf{P} = \frac{1}{2\omega}\,\mathrm{Im}\left[\mathbf{E}^{*}\cdot(\nabla)\mathbf{E}\right] \quad \propto \left(\mathbf{E}\middle|\hat{\mathbf{p}}\middle|\mathbf{E}\right)$$

$$\mathbf{L} = \mathbf{r}\times\mathbf{P}\,, \quad \mathbf{S} = \frac{1}{2\omega}\,\mathrm{Im}\left(\mathbf{E}^{*}\times\mathbf{E}\right) \quad \propto \left(\mathbf{E}\middle|\hat{\mathbf{S}}\middle|\mathbf{E}\right)$$

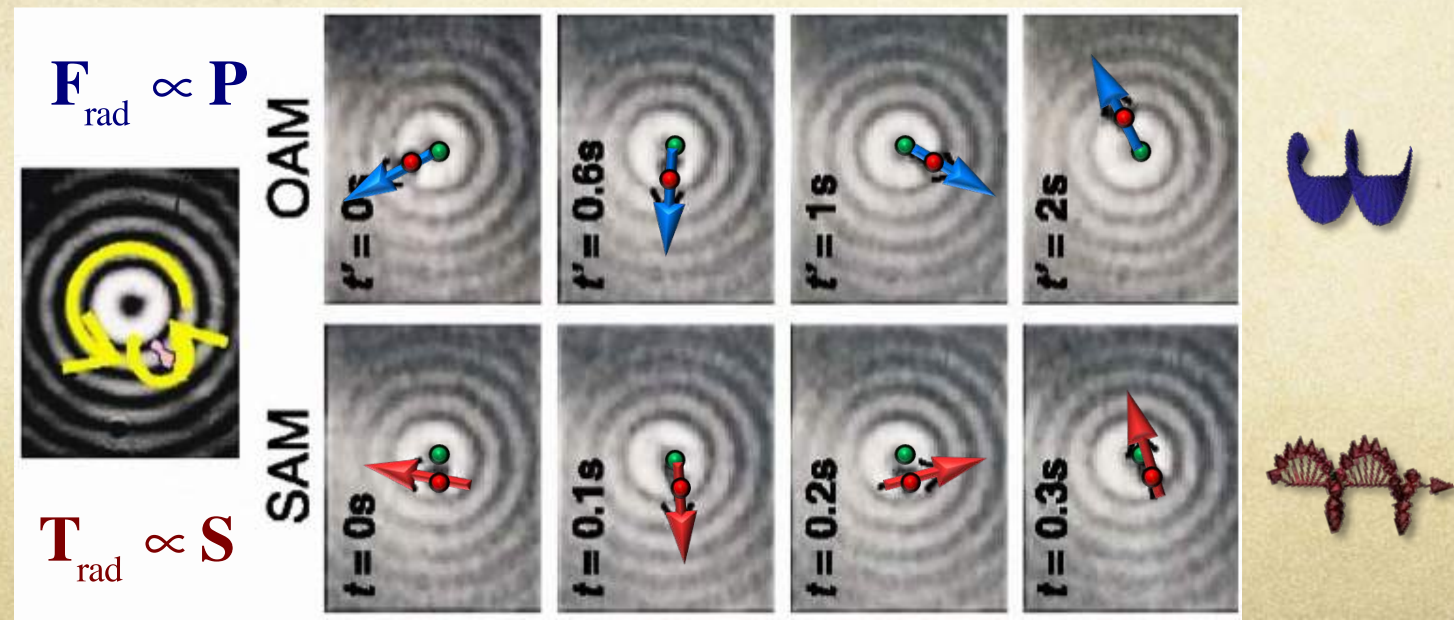
These have very clear physical meanings:
the polarization-independent phase gradient $\mathbf{P}$ and the normal to the polarization ellipse times its ellipticity $\mathbf{S}$.

Application to optical fields

Most importantly, the canonical momentum and spin densities **immediately appear in optical experiments**. They determine the radiation pressure and torque on a point electric dipole:

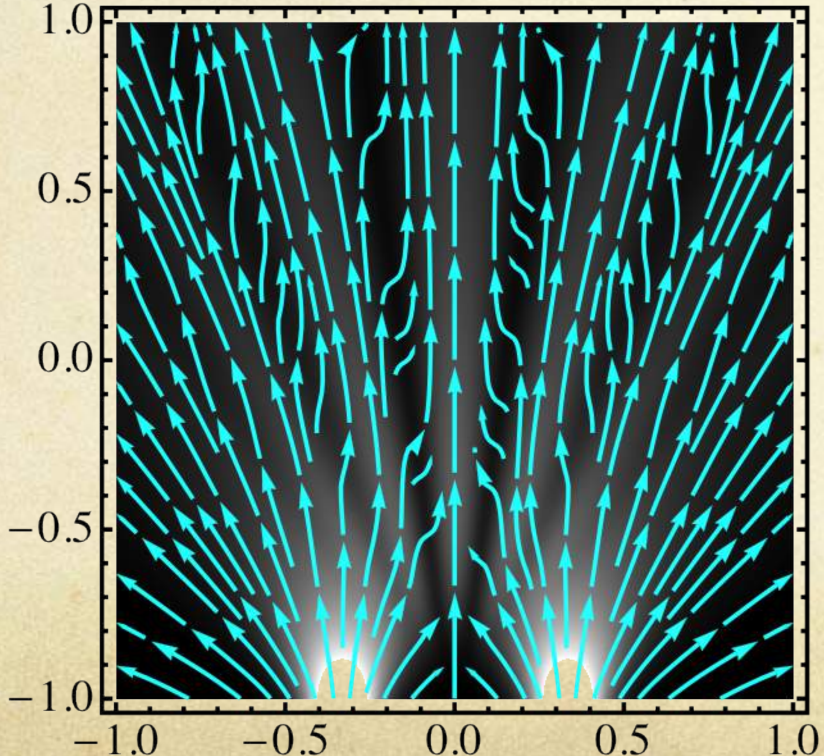
$$\mathbf{F}^{\mathrm{rad}} \propto \mathrm{Im}(\alpha)\mathbf{P}\,, \quad \mathbf{T}^{\mathrm{rad}} \propto \mathrm{Im}(\alpha)\mathbf{S}$$

Ashkin & Gordon (1983)
Canaguier-Durand (2013)
Bliokh *et al.* (2013, 2104)



Application to optical fields

Furthermore, other measurements of the optical momentum density reveal the canonical momentum rather than the Poynting vector! E.g. quantum weak measurements of photon trajectories = streamlines of $\mathbf{P}(\mathbf{r})$.



Physics World:
The top breakthrough 2011

$$\frac{\mathbf{P}(\mathbf{r})}{W(\mathbf{r})} \propto \mathrm{Re}\frac{\langle \mathbf{r}|\hat{\mathbf{p}}|\psi\rangle}{\langle \mathbf{r}|\psi\rangle} \equiv \langle \mathbf{p}\rangle_{\mathrm{weak}}$$

Berry, JO (2009), Wiseman, NJP (2009)
Kocsis *et al.*, Science (2011)
Bliokh *et al.*, NJP (2013)

Application to optical fields

Thus, despite the common optical and field–theory belief, it is the components of the canonical Noether tensors (in Coulomb gauge) rather than symmetrized Belinfante tensors that naturally appear in optical and quantum experiments measuring momentum and AM densities in optical fields.

In particular, the Poynting vector and its Belinfante's spin part are unobservable in optics.

Surprisingly, it seems that we only very recently drew the proper attention to these remarkable facts!

Bliokh *et al.*, *New J. Phys.* (2013, 2013, 2014); *Nature Commun.* (2014)

Conformity with the quantum approach

Let us check the correspondence between the field-theory and quantum approaches.

The integral OAM and SAM are separately conserved :

$$\partial_t \int \mathbf{L}\, d^3 r = 0 \;, \quad \partial_t \int \mathbf{S}\, d^3 r = 0$$

These are proportional to the expectation values of the OAM and SAM operators, $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$.

These quantities can be quantized via $\mathbf{A}_\perp$ to obtain the second-quantization OAM and SAM field operators that act on Fock states of photons: $\hat{\mathbf{L}}$ and $\hat{\mathbf{S}}$.

S.J. van Enk and G. Nienhuis, *J. Mod. Opt.* **41**, 963 (1994)

Conformity with the quantum approach

Remarkably, the second–quantization OAM and SAM operators obey the same "strange" commutation relations as the modified operators $\hat{\mathbf{L}}'$ and $\hat{\mathbf{S}}'$:

$$\left[\hat{\mathsf{S}}_i, \hat{\mathsf{S}}_j\right] = 0, \qquad \left[\hat{\mathsf{L}}_i, \hat{\mathsf{S}}_j\right] = i\varepsilon_{ijl}\hat{\mathsf{S}}_l,$$

$$\left[\hat{\mathsf{L}}_i, \hat{\mathsf{L}}_j\right] = i\varepsilon_{ijl}\left(\hat{\mathsf{L}}_l - \hat{\mathsf{S}}_l\right)$$

Considering the quantum photon–atom interaction (akin to the light–particle interaction), van Enk & Neinhuis conclude: "both SAM and OAM of a photon are well defined and separately measurable."

S.J. van Enk and G. Nienhuis, *J. Mod. Opt.* **41**, 963 (1994)

Local OAM and SAM conservation

Thus, there is a perfect agreement between the field–theory, 1st–quantization, and 2nd–quantization approaches.

There is only one remaining issue. The field–theory OAM and SAM Noether currents are not conserved locally:

$$\partial_\gamma S^{\alpha\beta\gamma} = -\partial_\gamma L^{\alpha\beta\gamma} = T^{\alpha\beta} - T^{\beta\alpha} \neq 0$$

Belinfante 'improved' the stress–energy tensor $T^{\alpha\beta}$, but lost the spin and orbital properties. Instead, we will improve the SAM and OAM Noether currents $S^{\alpha\beta\gamma}$, $L^{\alpha\beta\gamma}$.

K.Y. Bliokh, J. Dressel, F. Nori, *New J. Phys.* (2014)

Local OAM and SAM conservation

We modify the separation of the spin and orbital parts in the canonical AM current (akin to the OAM and SAM operators), $M^{\alpha\beta\gamma} = L'^{\alpha\beta\gamma} + S'^{\alpha\beta\gamma}$:

$$L'^{\alpha\beta\gamma} = L^{\alpha\beta\gamma} + \Delta^{\alpha\beta\gamma}, \quad S'^{\alpha\beta\gamma} = S^{\alpha\beta\gamma} - \Delta^{\alpha\beta\gamma}$$

$$T^{\alpha\beta} - T^{\beta\alpha} = \partial_\gamma \Delta^{\alpha\beta\gamma}$$ – using transversality

This makes the modifies currents conserved:

$$\partial_\gamma S'^{\alpha\beta\gamma} = -\partial_\gamma L'^{\alpha\beta\gamma} = 0$$

Here $\Delta^{\alpha\beta\gamma}$ modifies the AM fluxes but not densities:

$$\Delta^{\alpha\beta 0} = 0\,, \quad \Delta^{\alpha\beta\gamma} = -\Delta^{\beta\alpha\gamma}$$

Local OAM and SAM conservation

Explicitly, we obtain the new local OAM and SAM conservation laws ($\mathbf{A} = \mathbf{A}_\perp$):

$$\partial_t S_i + \partial_j \Sigma'_{ij} = 0:$$

$$S_i = \left(\mathbf{E} \times \mathbf{A}\right)_i, \;\; \Sigma'_{ij} = \delta_{ij}\left(\mathbf{B} \cdot \mathbf{A}\right) - B_i A_j - B_j A_i$$

$$\partial_t L_i + \partial_j \Lambda'_{ij} = 0:$$

$$L_i = \left[\mathbf{E} \cdot (\mathbf{r} \times \nabla)\mathbf{A}\right]_i,$$

$$\Lambda'_{ij} = -\left[\mathbf{B} \times (\mathbf{r} \times \nabla)_i \mathbf{A}\right]_j - \frac{1}{2}\varepsilon_{ijk} r_k \left(E^2 - B^2\right) + B_j A_i$$

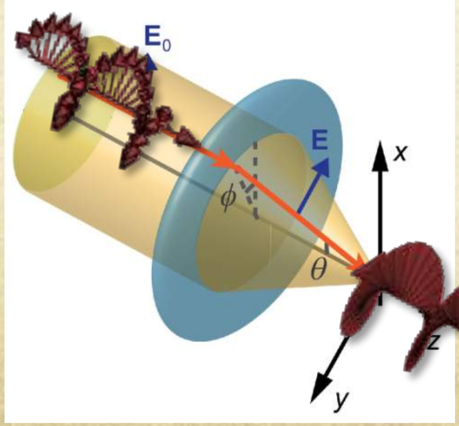
K.Y. Bliokh, J. Dressel, F. Nori, *New J. Phys.* (2014)

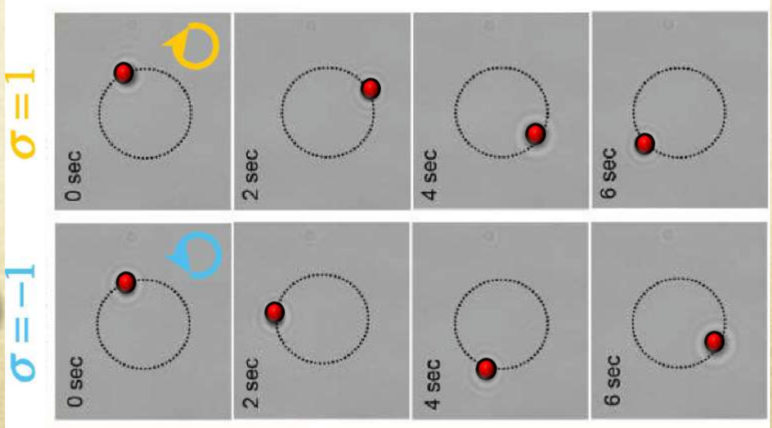
Local OAM and SAM conservation

The modified OAM and SAM fluxes not only provide formal conservation laws, but also correspond to observable spin–orbit interaction effects.

Normalized OAM and SAM fluxes in nonparaxial beams:

$$\frac{\langle \Lambda_{zz} \rangle}{\langle P_z \rangle} = \frac{1}{k}\left(\ell + \sigma\,\Phi_B\right) \propto \langle L_z \rangle, \quad \frac{\langle \Sigma_{zz} \rangle}{\langle P_z \rangle} = \frac{\sigma}{k}\left(1 - \Phi_B\right) \propto \langle S_z \rangle$$





Summary for field–theory approach

- Canonical Noether tensors (in Coulomb gauge) rather than the Belinfante tensors are meaningful and observable for optical fields.
- The OAM and SAM following from these tensors are perfectly consistent with quantum approaches.
- The canonical OAM and SAM fluxes should be corrected with a spin–orbit term providing local conservation laws and corresponding to observable effects.

S.J. Van Enk and G. Nienhuis, *J. Mod. Opt.* **41**, 963 (1994)
K.Y. Bliokh *et al.*, *New J. Phys.* **15**, 033026 (2013); *New J. Phys.* **15**, 073022 (2013); *Nature Commun.* 5, 3300 (2014); *New J. Phys.* (2014, in press)

Outline

Optical (monochromatic) fields and laboratory experiments

observability and measurements

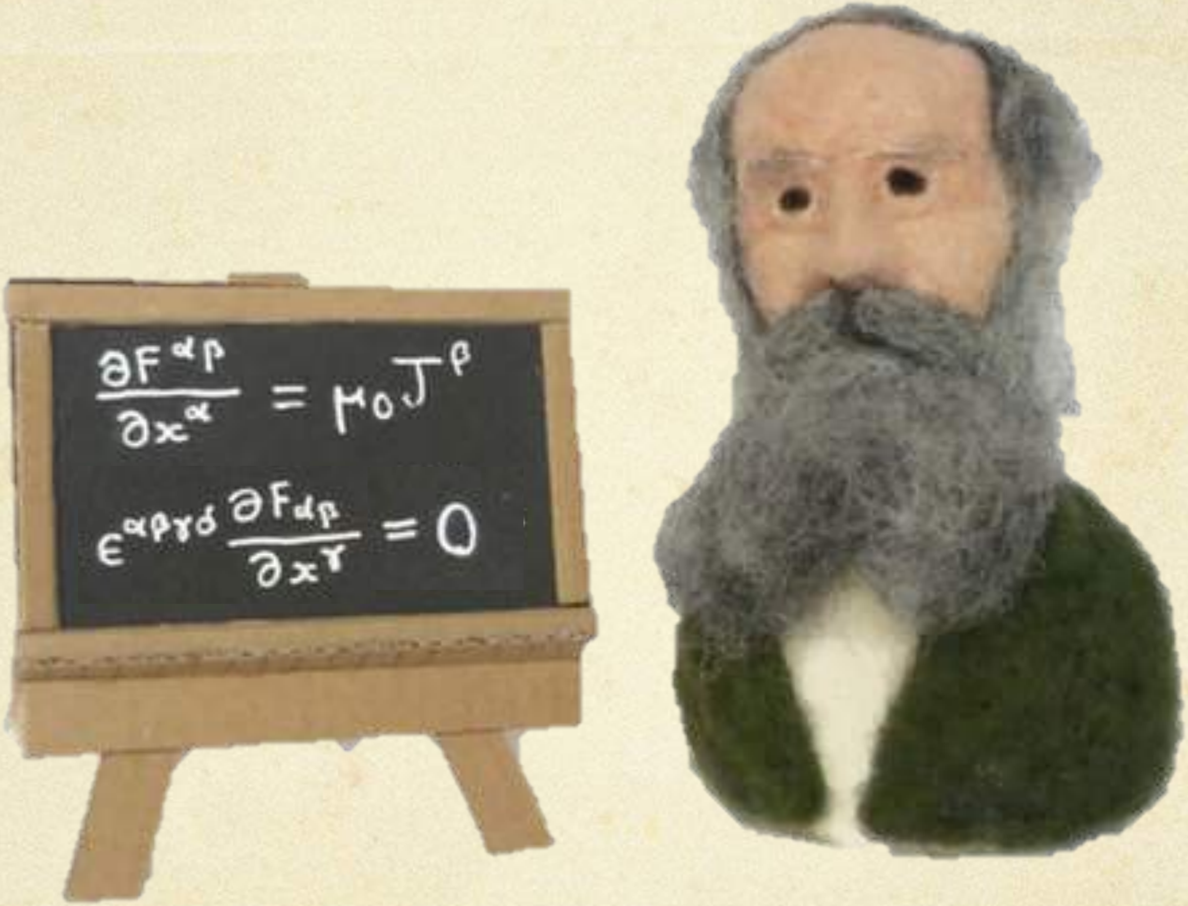
Quantum (operator) approach and expectation values

spin–orbit interactions

Electromagnetic field theory and local currents

SAM, OAM, helicity, vortices, position, momentum

Thank you!


$\frac{\partial F^{\alpha\beta}}{\partial x^{\alpha}} = \mu_0 J^{\beta}$
$\epsilon^{\alpha\beta\gamma\delta} \frac{\partial F_{\alpha\beta}}{\partial x^{\gamma}} = 0$